AQA

Surname ______________________________

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Centre Number ______________________________

Candidate Number ______________________________

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# AS
# FURTHER MATHEMATICS

## Paper 1

## 7366/1

**Monday 14 May 2018 Afternoon**

**Time allowed: 1 hour 30 minutes**

**At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.**

**[Turn over]**

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**For this paper:**
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

## INSTRUCTIONS

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Answer ALL questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do not write on blank pages.
- Show all necessary working; otherwise marks for method may be lost.

- Do all rough work in this book. Cross through any work that you do not want to be marked.

## INFORMATION

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

## ADVICE

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

**DO NOT TURN OVER UNTIL TOLD TO DO SO**

**Answer ALL questions in the spaces provided.**

**1** $z = 3 - \mathrm{i}$

**Determine the value of $zz^*$**

**Circle your answer. [1 mark]**

$10$ $\sqrt{10}$ $10 - 2\mathrm{i}$ $10 + 2\mathrm{i}$

2 Three matrices A, B and C are given by

$$\mathbf{A} = \begin{bmatrix} 5 & 2 & -3 \\ 0 & 7 & 6 \\ 4 & 1 & 0 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 3 & -5 \\ -2 & 6 \end{bmatrix}$$

$$\text{and } \mathbf{C} = \begin{bmatrix} 6 & 4 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

Which of the following CANNOT be calculated?

Circle your answer. [1 mark]

AB AC BC $A^2$

[Turn over]

**3** **Which of the following functions has the fourth term $-\frac{1}{720}x^6$ in its Maclaurin series expansion?**

**Circle your answer. [1 mark]**

**$\sin x$ $\cos x$ $e^x$ $\ln(1 + x)$**

**4** **Sketch the graph given by the polar equation**

$$r = \frac{a}{\cos \theta}$$

**where $a$ is a positive constant.**
**[2 marks]**

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5 Describe fully the transformation given by the matrix

$$\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[3 marks]

[Turn over]

6 (a) Matthew is finding a formula for the inverse function arsinh $x$.
He writes his steps as follows:

Let $y = \sinh x$

$$y = \frac{1}{2}(e^x - e^{-x})$$

$$2y = e^x - e^{-x}$$

$$0 = e^x - 2y - e^{-x}$$

$$0 = (e^x)^2 - 2ye^x - 1$$

$$0 = (e^x - y)^2 - y^2 - 1$$

$$y^2 + 1 = (e^x - y)^2$$

$$\pm\sqrt{y^2 + 1} = e^x - y$$

$$y \pm \sqrt{y^2 + 1} = e^x$$

To find the inverse function, swap $x$ and $y$: $x \pm \sqrt{x^2 + 1} = e^y$

$$\ln\left(x \pm \sqrt{x^2 + 1}\right) = y$$

$$\text{arsinh}\, x = \ln\left(x \pm \sqrt{x^2 + 1}\right)$$

[Turn over]

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**Identify, and explain, the error in Matthew's proof. [2 marks]**

**[Turn over]**

**6(b)** **Solve** $\ln\left(x + \sqrt{x^2 + 1}\right) = 3$

**[1 mark]**

**7** **Find two invariant points under the transformation given by**

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

**[2 marks]**

**[Turn over]**

**8** $2 - 3\text{i}$ is one root of the equation

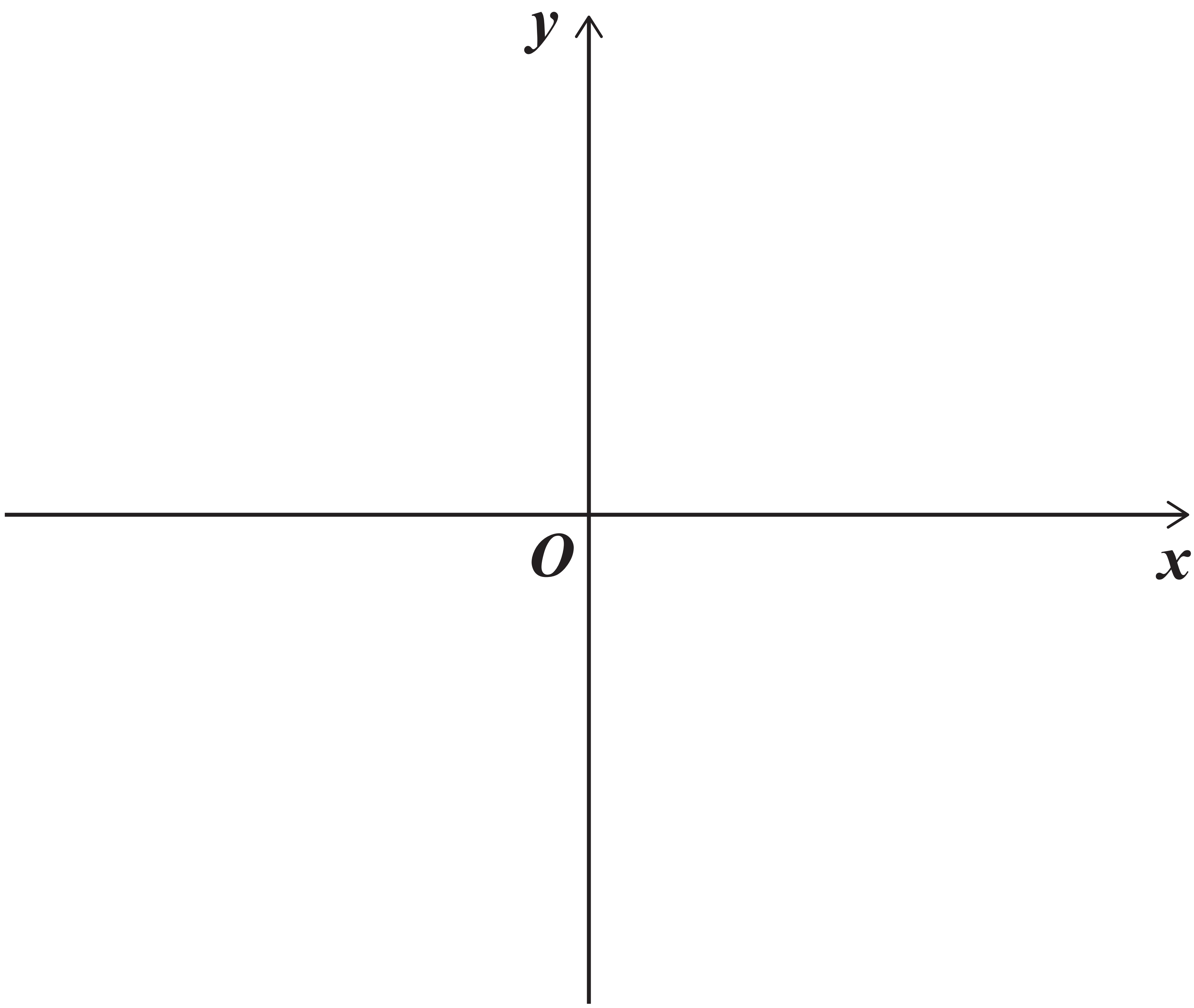
$$z^3 + mz + 52 = 0$$

where $m$ is real.

**8 (a)** Find the other roots. [3 marks]

8 (b) Determine the value of $m$.
[2 marks]

**9 (a)** Sketch the graph of $y^2 = 4x$
[1 mark]




**9 (b)** Ben is using a 3D printer to make a plastic bowl which holds exactly 1000 $cm^3$ of water.

Ben models the bowl as a region which is rotated through $2\pi$ radians about the $x$-axis.

He uses the finite region enclosed by the lines $x = d$ and $y = 0$ and the curve with equation $y^2 = 4x$ for $y \geq 0$

9 (b) (i) Find the depth of the bowl to the nearest millimetre. [4 marks]

9 (b) (ii) **What assumption has Ben made about the bowl? [1 mark]**

**[Turn over]**

**10 (a)** **Prove by induction that, for all integers $n \geq 1$,**

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

**[4 marks]**

[Turn over]

**10 (b)** Hence show that

$$\sum_{r=1}^{2n} r(r-1)(r+1)$$

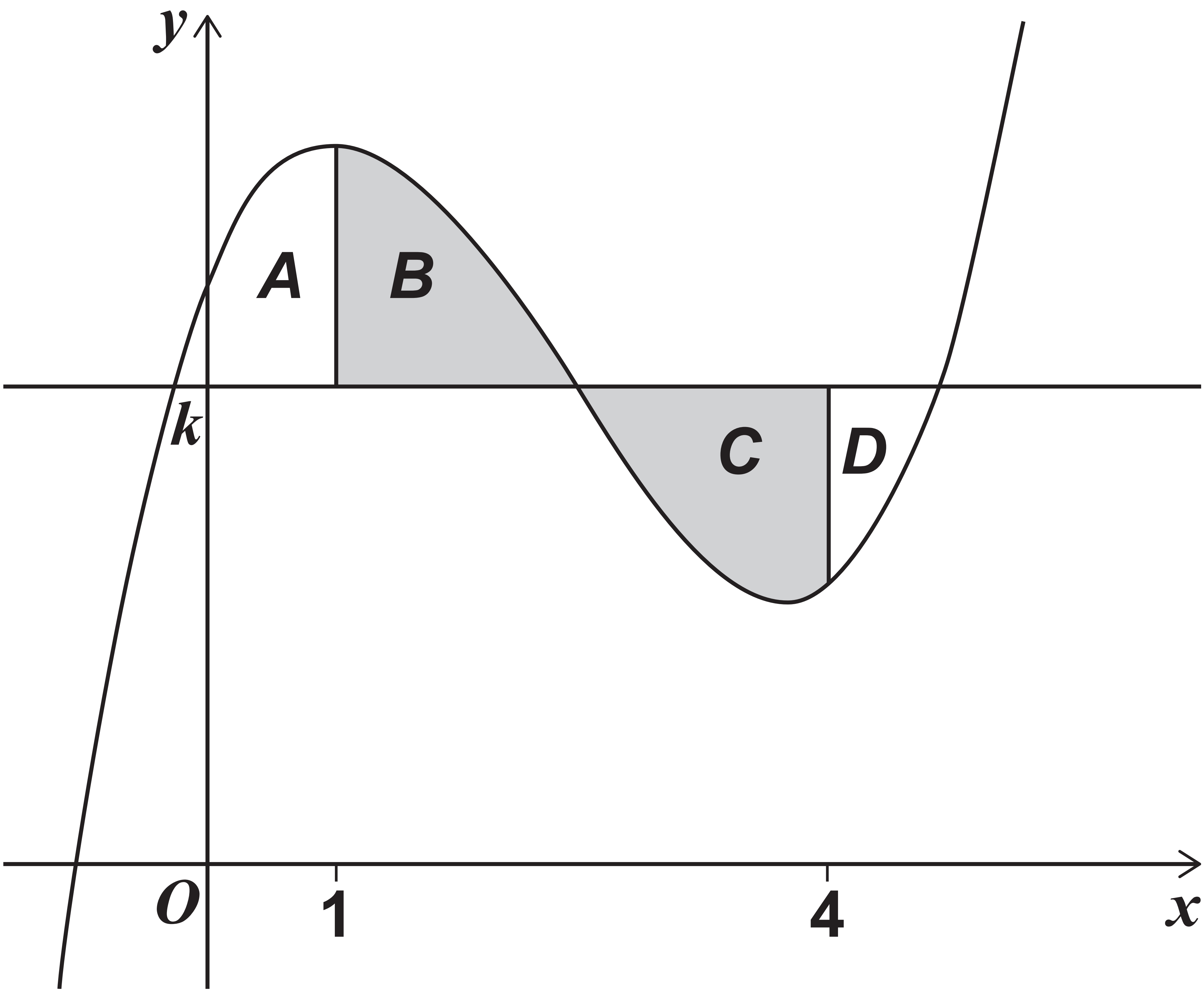
$$= n(n+1)(2n-1)(2n+1)$$

**[4 marks]**

**11** Four finite regions *A*, *B*, *C* and *D* are enclosed by the curve with equation

$$y = x^3 - 7x^2 + 11x + 6$$

and the lines $y = k$, $x = 1$ and $x = 4$, as shown in the diagram below.



**The areas of *B* and *C* are equal.**

**Find the value of $k$. [3 marks]**

**12 (a)** Show that the matrix $\begin{bmatrix} 5-k & 2 \\ k^3+1 & k \end{bmatrix}$ is singular when $k = 1$. [1 mark]

**12 (b)** Find the values of $k$ for which the matrix $\begin{bmatrix} 5-k & 2 \\ k^3+1 & k \end{bmatrix}$ has a negative determinant.

**Fully justify your answer.**
**[5 marks]**

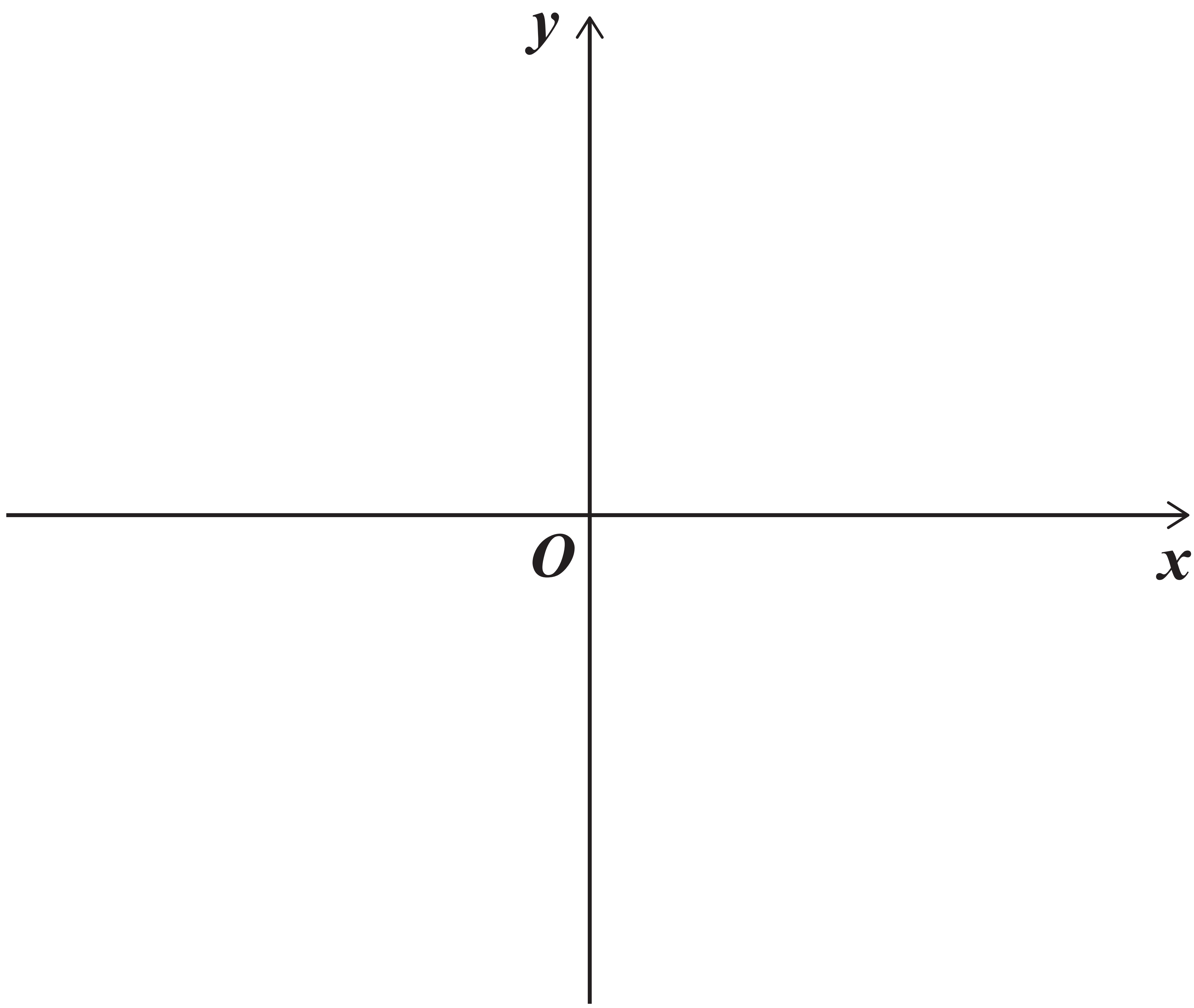
**[Turn over]**

**13** The graph of the rational function $y = \mathrm{f}(x)$ intersects the $x$-axis exactly once at $(-3, 0)$

The graph has exactly two asymptotes, $y = 2$ and $x = -1$

**13 (a)** Find $\mathrm{f}(x)$ [2 marks]

13(b) **Sketch the graph of the function.**
**[3 marks]**




**[Turn over]**

**13 (c)** **Find the range of values of $x$ for which $f(x) \leq 5$ [4 marks]**

[Turn over]

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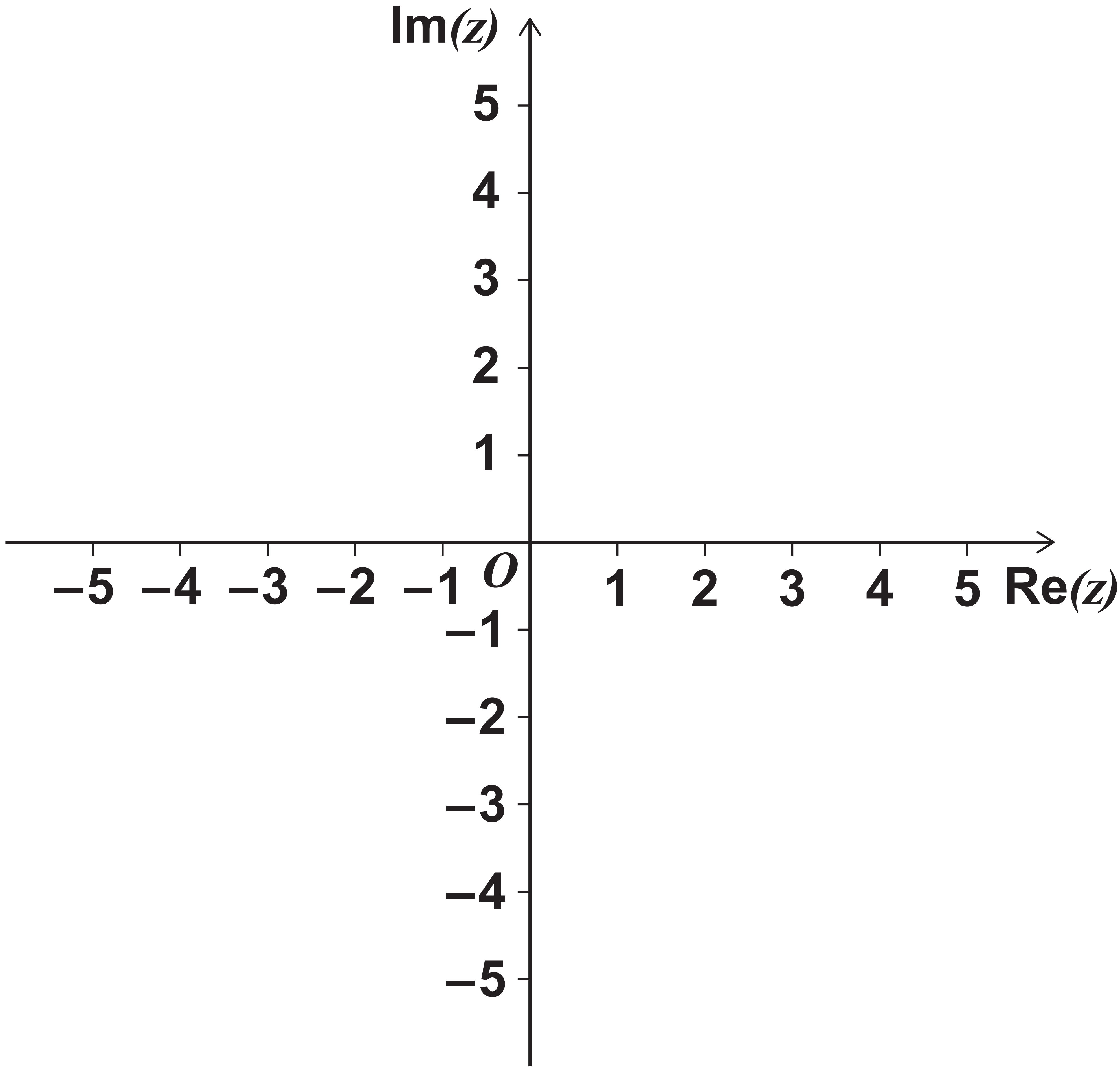
**14 (a)** Sketch, on the Argand diagram below, the locus of points satisfying the equation

$|z - 3| = 2$

[1 mark]



14 (b) There is a unique complex number $w$ that satisfies both

$$|w - 3| = 2 \quad \text{and} \quad \arg(w + 1) = \alpha$$

where $\alpha$ is a constant such that $0 < \alpha < \pi$

14 (b) (i) Find the value of $\alpha$. [2 marks]

**14 (b) (ii) Express $w$ in the form $r(\cos\theta + \mathrm{i}\sin\theta)$.**

**Give each of $r$ and $\theta$ to two significant figures. [4 marks]**

**[Turn over]**

**15 (a)** Show that

$$\frac{1}{r+2} - \frac{1}{r+3} = \frac{1}{(r+2)(r+3)}$$

[1 mark]

**15 (b)** Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{(r+2)(r+3)} = \frac{n}{3(n+3)}$$

[3 marks]

**16** **Two matrices A and B satisfy the equation**

$$\mathbf{AB} = I + 2\mathbf{A}$$

**where $I$ is the identity matrix**

**and** $\mathbf{B} = \begin{bmatrix} 3 & -2 \\ -4 & 8 \end{bmatrix}$

**Find A. [3 marks]**

17 Find the exact solution to the equation

$$\sinh\theta(\sinh\theta + \cosh\theta) = 1$$

[4 marks]

18 $\alpha$, $\beta$ and $\gamma$ are the real roots of the cubic equation

$$x^3 + mx^2 + nx + 2 = 0$$

By considering $(\alpha - \beta)^2 + (\gamma - \alpha)^2 + (\beta - \gamma)^2$, prove that

$$m^2 \geq 3n$$

[4 marks]

**19** **A theme park has two zip wires.**

**Sarah models the two zip wires as straight lines using coordinates in metres.**

**The ends of one wire are located at (0, 0, 0) and (0, 100, –20)**

**The ends of the other wire are located at (10, 0, 20) and (–10, 100, –5)**

**19 (a)** **Use Sarah's model to find the shortest distance between the zip wires. [7 marks]**

[Turn over]

**19 (b)** **State one way in which Sarah's model could be refined. [1 mark]**

**END OF QUESTIONS**

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