AQA

AS
Further Mathematics

7366/2S - Statistics

Mark scheme

7366

June 2018

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
dM	mark is dependent on one or more M marks and is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.”

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	AO1.2	B1	0
	Total		**1**	
2	Circles correct answer	AO1.1b	B1	0.801
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3	Evaluates $E(X^2)$ by calculating $1^2 \times 0.2 + 2^2 \times 0.4 + 4^2 \times 0.35 + 9^2 \times 0.05$	AO1.1a	M1	$E(X^2) = 1^2 \times 0.2 + 2^2 \times 0.4 + 4^2 \times 0.35 + 9^2 \times 0.05$ $= 11.45$ or $\frac{229}{20}$
	Evaluates $E(Y^2)$ by integrating $\int y^2 \times f(y)\ \ dy$ Must see the integral	AO1.1a	M1	$E(Y^2) = \int_0^4 y^2 \times \frac{1}{64} y^3\ \ dy$ $= \frac{1}{64}\left[\frac{1}{6}y^6\right]_0^4 = \left[\frac{1}{384}y^6\right]_0^4 = \frac{32}{3}$
	Finds $E(X^2) = 11.45$ and $E(Y^2) = \frac{32}{3}$	AO1.1b	A1	$E(X^2 + Y^2) = E(X^2) + E(Y^2)$ $= 11.45 + \frac{32}{3}$
	Uses $E(X^2 + Y^2) = E(X^2) + E(Y^2)$ to show that $E(X^2 + Y^2) = \frac{1327}{60}$ AG Mark awarded if they have a completely correct solution, which is clear, easy to follow and contains no slips	AO2.1	R1	$= \frac{1327}{60}$
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(a)	Calculates the correct sample mean	AO1.1b	B1	$\bar{x} = \frac{3540}{100}$ = 35.4
	Finds correct *z* value to at least 3 significant figures. Condone -2.326… Can be implied by a correct confidence interval.	AO1.1a	M1	z = 2.32634787 $\bar{x} \pm z\sqrt{\frac{\sigma^2}{n}}$
	Uses formula for confidence interval with $\sqrt{\frac{10}{100}}$	AO1.1a	M1	$= 35.4 \pm 2.326\sqrt{\frac{10}{100}}$ = (34.66, 36.14) or 35.4 ± 0.7357
	Obtains confidence interval CAO	AO1.1b	A1	
4(b)	States null hypothesis is rejected as the confidence interval does not contain 38. OE Follow through their confidence interval.	AO3.5a	E1F	Dante rejects the null hypothesis because 38 is outside the confidence interval.
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	States the mode	AO1.2	B1	1.75
5(b)	Uses the area under a p.d.f. is equal to 1 to form an equation to find the y-coordinate of the maximum point of the p.d.f.	AO3.1a	M1	$\frac{2k}{2} = 1$ $k = 1$ Maximum point of the p.d.f. is (1.75, 1)
	Finds the equation of one line using their k, the y-coordinate of the maximum point of the p.d.f. May be in terms of k	AO1.1a	M1	$y = \frac{k}{1.75}x$ $y = \frac{1}{1.75}x$ $y = \frac{4}{7}x$
	Finds the equation of both lines Follow through their k May be in terms of k	AO1.1b	A1F	$y = -\frac{k}{0.25}(x-2)$ $y = -\frac{1}{0.25}(x-2)$ $y = -4x + 8$
	States fully defined probability density function (OE) CAO	AO3.2a	A1	$f(x) = \begin{cases} \frac{4}{7}x & 0 \le x < 1.75 \\ -4x+8 & 1.75 \le x < 2 \\ 0 & \text{otherwise} \end{cases}$
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
6	Uses sum of probabilities = 1 to correctly find k	AO1.1b	B1	$2k + 4k + 6k + 8k = 1$ $20k = 1$ $k = \frac{1}{20}$ or 0.05
	Calculates $E(Y)$ or $E(Y^2)$ correctly	AO1.1a	M1	$E(Y) = 1 \times 2k + 2 \times 4k + 3 \times 6k + 4 \times 8k$ $= 60k$ or 3
	Calculates both E(Y) and $E(Y^2)$ correctly	AO1.1b	A1	$E(Y^2) = 1^2 \times 2k + 2^2 \times 4k + 3^2 \times 6k + 4^2 \times 8k$ $= 200k$ or 10
	Calculates Var (Y)	AO1.1a	M1	Var $(Y) = E(Y^2) - E(Y)^2$ $= 200k - (60k)^2$ or $10 - 3^2$ $= 200k - 3600k^2$ or 1
	Uses formula for Var $(aY \pm b)$	AO1.1a	M1	Var $(5Y - 2) = 5^2$ Var (Y) $= 5^2 \times 1$ $= 25$
	Completes a fully correct, logical argument to show that Var $(5Y - 2) = 25$ AG	AO2.1	R1	

6 ALT	Uses sum of probabilities = 1 to correctly find k	AO1.1b	B1	$2k + 4k + 6k + 8k = 1$ $20k = 1$ $k = \frac{1}{20}$ or 0.05
	Finds values of $5Y - 2$ for each value of y	AO1.1a	M1	<table><tr><td>y</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$5y$-2</td><td>3</td><td>8</td><td>13</td><td>18</td></tr></table>
	Calculates $E(5Y - 2)$ or $E((5Y - 2)^2)$ correctly	AO1.1a	M1	$E(5Y - 2) = 3 \times 2k + 8 \times 4k$ $+ 13 \times 6k + 18 \times 8k$ $= 260k$ or 13
	Calculates $E(5Y - 2)$ and $E((5Y - 2)^2)$ correctly	AO1.1b	A1	$E((5Y - 2)^2) = 3^2 \times 2k + 8^2 \times 4k$ $+ 13^2 \times 6k + 18^2 \times 8k$ $= 3880k$ or 194
	Calculates Var $(5Y - 2)$	AO1.1a	M1	Var $(5Y - 2) = E((5Y-2)^2) - E(5Y-2)^2$ $= 3880k - (260k)^2$ or $194 - 13^2$ $= 3880k - 67600k^2$
	Completes a fully correct, logical argument to show that Var (5Y – 2) = 25 AG	AO2.1	R1	$= 25$
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)	States both hypotheses using correct language	AO2.5	B1	H_0: $\lambda = 50$ H_1: $\lambda < 50$ $X \sim \text{Po}(50)$ $P(X \leq 30) = 0.002$ $(P(X \leq 33) = 0.007$ and $P(X \leq 34) = 0.0108)$ p-value = 0.002 < 0.01 (30 < 34 = critical value) Reject H_0 in favour of H_1 Significant evidence to suggest that the mean number of vehicles passing the service station per minute has reduced
	Selects and uses Poisson model with $\lambda = 50$ to find P(total vehicles ≤ 30) or P(total vehicles < 30) or P(total vehicles ≤ 33) and P(total vehicles ≤ 34)	AO3.3	M1	
	Obtains AWRT $P(X \leq 30) = 0.002$ (0.001594) or AWRT $P(X \leq 33) = 0.007$ (0.0069788) and AWRT $P(X \leq 34) = 0.0108$ (0.01078145)	AO1.1b	A1	
	Evaluates the Poisson model by comparing 'their' p-value with 0.01 or 30 with their critical value	AO3.5a	R1	
	Infers H_0 rejected	AO2.2b	E1	
	Concludes in context (not definite)	AO3.2a	E1	
7(b)	States meaning in context of Type I error	AO3.2a	E1	Type I error is to conclude that the mean number of vehicles passing the service station per minute using the junction has reduced when it has not.
7(c)	States that rate at which events occur is unlikely to be constant over time in context or States that the events do not occur independently in context Must be consistent	AO3.5b	E1	The rate at which vehicles pass the service station is unlikely to be constant over time. or Vehicles may not pass the service station independently, eg convoy
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Obtains one correct missing expected value e.g. 0 and car = $\frac{28 \times 35}{80}$	AO1.1a	M1	<table><tr><th></th><th>0</th><th>1</th><th>2</th><th>3+</th></tr><tr><td>**C**</td><td>**12.25**</td><td>10.0625</td><td>**8.3125**</td><td>4.375</td></tr><tr><td>**MB**</td><td>**15.75**</td><td>**12.9375**</td><td>10.6875</td><td>**5.625**</td></tr></table>
	Obtains all expected values Must not be rounded CAO Condone 12.937 for 12.9375	AO1.1b	A1	

<table>
<tr><td>8(b)</td><td>States both hypotheses using correct language
Accept equivalent wording</td><td>AO2.5</td><td>B1</td><td>H_0: There is no association between number of claims and type of insurance policy
H_1: There is an association between number of claims and type of insurance policy</td></tr>
<tr><td></td><td>Combines expected and observed values for 2 and 3 or more correctly
Follow through their answers to part (a)</td><td>AO1.1b</td><td>B1F</td><td><table><tr><td></td><td colspan="2">2 or more</td></tr><tr><td></td><td>Observed</td><td>Expected</td></tr><tr><td>C</td><td>16</td><td>12.6875</td></tr><tr><td>MB</td><td>13</td><td>16.3125</td></tr></table></td></tr>
<tr><td></td><td>Calculates χ^2-test statistic
Can be awarded even if 2 and 3 or more are not combined</td><td>AO1.1a</td><td>M1</td><td rowspan="2">$\sum \frac{(O-E)^2}{E} =$
$\frac{(9-12.25)^2}{12.25} + \frac{(19-15.75)^2}{15.75} +$
$\frac{(10-10.0625)^2}{10.0625} + \frac{(13-12.9375)^2}{12.9375} +$
$+ \frac{(16-12.6875)^2}{12.6875} + \frac{(13-16.3125)^2}{16.3125}$
= 3.07</td></tr>
<tr><td></td><td>Calculates χ^2-test statistic correctly.
Condone given to 1 s.f.
CAO</td><td>AO1.1b</td><td>A1</td></tr>
<tr><td></td><td>States critical value (or p-value, follow through their χ^2 value)
Condone χ^2 cv for 3 df = 6.251 If 2 and 3 or more are not combined</td><td>AO1.1b</td><td>B1F</td><td rowspan="4">χ^2 cv for 2 df = 4.605
(p = 0.215)

3.07 < 4.605
(0.215 > 0.1)

Accept H_0

No significant evidence to suggest that there is an association between number of claims and type of insurance policy</td></tr>
<tr><td></td><td>Evaluates χ^2-test statistic by comparing the cv with their ts (or their p value with 0.1)</td><td>AO3.5a</td><td>R1</td></tr>
<tr><td></td><td>Infers H_0 not rejected (OE)</td><td>AO2.2b</td><td>E1</td></tr>
<tr><td></td><td>Concludes in context
(not definite)
Accept equivalent wording</td><td>AO3.2a</td><td>E1</td></tr>
<tr><td></td><td>Total</td><td></td><td>10</td><td></td></tr>
<tr><td></td><td>TOTAL</td><td></td><td>40</td><td></td></tr>
</table>