

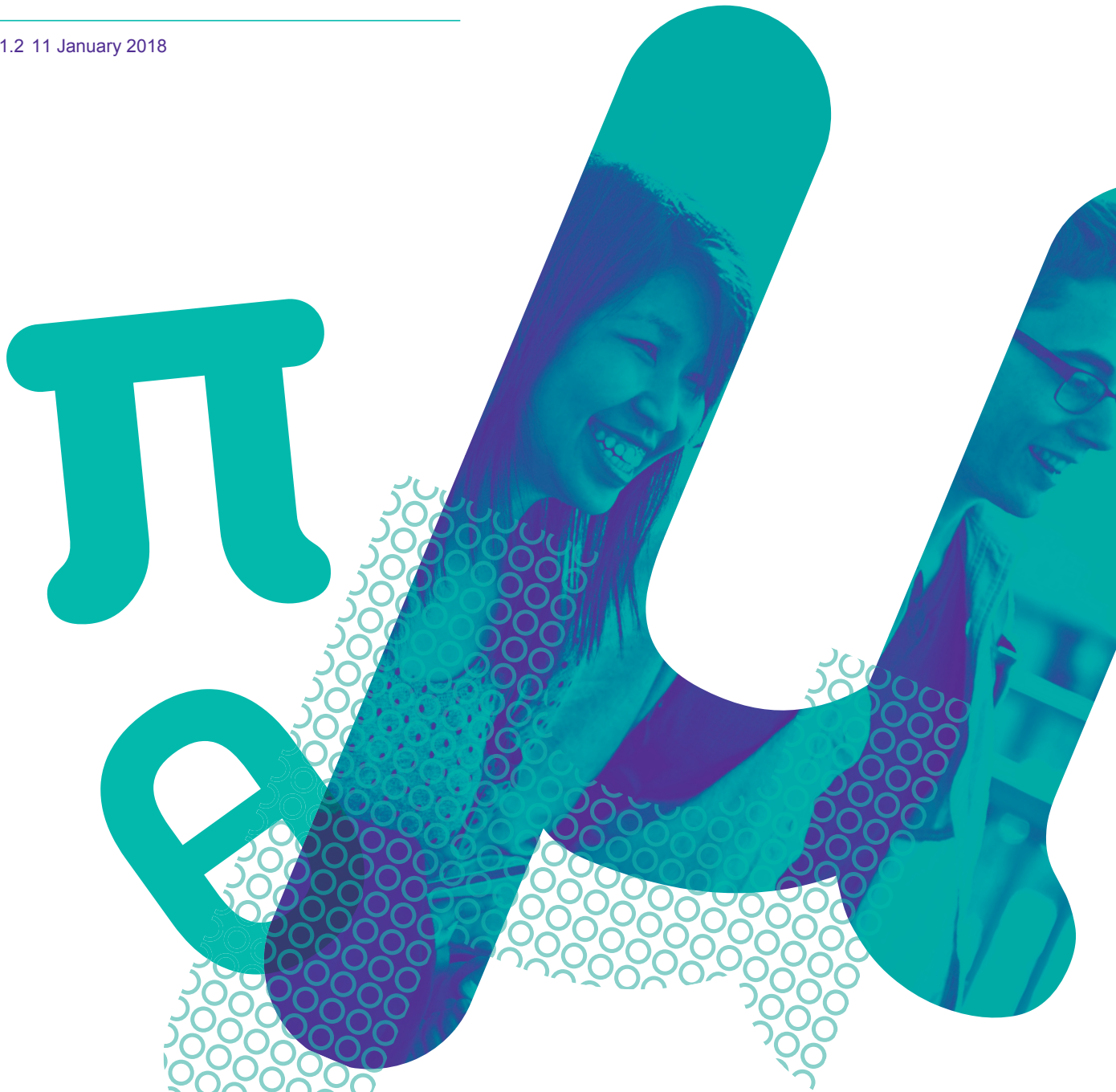


AS
**FURTHER
MATHEMATICS**
(7366)

Specification

For teaching from September 2017 onwards
For exams in 2018 onwards

Version 1.2 11 January 2018



Contents

| | |
|---|-----------|
| 1 Introduction | 5 |
| 1.1 Why choose AQA for AS Further Mathematics | 5 |
| 1.2 Support and resources to help you teach | 5 |
| 2 Specification at a glance | 7 |
| 2.1 Subject content | 7 |
| 2.2 Assessments | 8 |
| 3 Subject content | 11 |
| 3.1 Overarching themes | 11 |
| 3.2 Compulsory content | 12 |
| 3.3 Optional application 1 – mechanics | 16 |
| 3.4 Optional application 2 – statistics | 18 |
| 3.5 Optional application 3 – discrete mathematics | 21 |
| 4 Scheme of assessment | 25 |
| 4.1 Aims | 25 |
| 4.2 Assessment objectives | 26 |
| 4.3 Assessment weightings | 27 |
| 5 General administration | 29 |
| 5.1 Entries and codes | 29 |
| 5.2 Overlaps with other qualifications | 29 |
| 5.3 Awarding grades and reporting results | 29 |
| 5.4 Re-sits and shelf life | 29 |
| 5.5 Previous learning and prerequisites | 30 |
| 5.6 Access to assessment: diversity and inclusion | 30 |
| 5.7 Working with AQA for the first time | 30 |
| 5.8 Private candidates | 31 |
| 5.9 Use of calculators | 31 |
| 6 Appendix A: mathematical notation | 33 |
| 6.1 Set notation | 33 |
| 6.2 Miscellaneous symbols | 34 |
| 6.3 Operations | 35 |
| 6.4 Functions | 36 |
| 6.5 Exponential and logarithmic functions | 37 |
| 6.6 Trigonometric functions | 38 |
| 6.7 Complex numbers (Further Maths only) | 38 |
| 6.9 Vectors | 39 |
| 6.10 Differential equations (Further Maths only) | 40 |

| | |
|---------------------------------|----|
| 6.11 Probability and statistics | 40 |
| 6.12 Mechanics | 42 |

| | |
|---|----|
| 7 Appendix B: mathematical formulae and identities | 43 |
|---|----|

Are you using the latest version of this specification?

- You will always find the most up-to-date version of this specification on our website at
- We will write to you if there are significant changes to the specification.

1 Introduction

1.1 Why choose AQA for AS Further Mathematics

Maths is essential for many higher education courses and careers. We've worked closely with higher education to ensure this qualification gives your students the best possible chance to progress and realise their potential.

Assessment design that rewards understanding

We want students to see the links between different areas of maths and to apply their maths skills across all areas.

Consistent assessments are essential, which is why we've worked hard to ensure our papers are clear and reward your students for their mathematical skills and knowledge.

You can find out about all our Further Mathematics qualifications at aqa.org.uk/maths

1.2 Support and resources to help you teach

We've worked with experienced teachers to provide you with a range of resources that will help you confidently plan, teach and prepare for exams.

Teaching resources

Visit aqa.org.uk/7366 to see all our teaching resources. They include:

- route maps to allow you to plan how to deliver the specification in the way that will best suit you and your students
- teaching guidance to outline clearly the possible scope of teaching and learning
- lesson plans and homework sheets tailored to this specification
- tests and assessments that will allow you to measure the development of your students as they work through the content
- textbooks that are approved by AQA
- training courses to help you deliver AQA Mathematics qualifications
- subject expertise courses for all teachers, from newly-qualified teachers who are just getting started, to experienced teachers looking for fresh inspiration.
- training courses to help you deliver AQA Further Mathematics qualifications
- subject expertise courses for all teachers, from newly qualified teachers who are just getting started to experienced teachers looking for fresh inspiration.

Preparing for exams

Visit aqa.org.uk/7366 for everything you need to prepare for our exams, including:

- past papers, mark schemes and examiners' reports
- specimen papers and mark schemes for new courses

- Exampro: a searchable bank of past AQA exam questions
- example student answers with examiner commentaries.

Analyse your students' results with Enhanced Results Analysis (ERA)

Find out which questions were the most challenging, how the results compare to previous years and where your students need to improve. ERA, our free online results analysis tool, will help you see where to focus your teaching. Register at aqa.org.uk/era

For information about results, including maintaining standards over time, grade boundaries and our post-results services, visit aqa.org.uk/results

Keep your skills up-to-date with professional development

Wherever you are in your career, there's always something new to learn. As well as subject specific training, we offer a range of courses to help boost your skills.

- Improve your teaching skills in areas including differentiation, teaching literacy and meeting Ofsted requirements.
- Prepare for a new role with our leadership and management courses.

You can attend a course at venues around the country, in your school or online – whatever suits your needs and availability. Find out more at coursesandevents.aqa.org.uk

Help and support

Visit our website for information, guidance, support and resources at aqa.org.uk/7366

If you'd like us to share news and information about this qualification, sign up for emails and updates at aqa.org.uk/from-2017

Alternatively, you can call or email our subject team direct.

E: maths@aqa.org.uk

T: 0161 957 3852

2 Specification at a glance

This qualification is linear. Linear means that students will sit all their exams at the end of the course.

This AS qualification builds on the skills, knowledge and understanding set out in the whole GCSE (9–1) subject content for mathematics and the subject content for AS and A-level mathematics.

This AS Further Mathematics qualification can be taught alongside the AS Mathematics qualification as the content does not significantly overlap with or depend upon the other content of A-level Mathematics.

2.1 Subject content

All students must study this core content.

- [Compulsory content](#) (page 12)
- [OT1: Mathematical argument, language and proof](#) (page 11)
- [OT2: Mathematical problem solving](#) (page 11)
- [OT3: Mathematical modelling](#) (page 12)

Students must study two of these options.

- [Optional application 1 – mechanics](#) (page 16)
- [Optional application 2 – statistics](#) (page 18)
- [Optional application 3 – discrete mathematics](#) (page 21)

2.2 Assessments

| Paper 1 |
|--|
| <p>What's assessed</p> <p>May assess content from the following sections:</p> <ul style="list-style-type: none"> • A: Proof • B: Complex numbers • C: Matrices • D: Further algebra and functions • E: Further calculus • F: Further vectors • G: Polar coordinates • H: Hyperbolic functions |
| <p>How it's assessed</p> <ul style="list-style-type: none"> • Written exam: 1 hour 30 minutes • 80 marks • 50% of AS |
| <p>Questions</p> <p>A mix of question styles, from short, single-mark questions to multi-step problems.</p> |



| Paper 2 |
|--|
| <p>What's assessed</p> <p>One question paper answer booklet on Discrete and one question paper answer booklet on Statistics.</p> |
| <p>How it's assessed</p> <ul style="list-style-type: none"> • Written exam: 1 hour 30 minutes • 80 marks • 50% of AS |
| <p>Questions</p> <p>A mix of question styles, from short, single-mark questions to multi-step problems.</p> |

OR

| Paper 2 |
|---|
| What's assessed One question paper answer booklet on Statistics and one question paper answer booklet on Mechanics. |
| How it's assessed <ul style="list-style-type: none">• Written exam: 1 hour 30 minutes• 80 marks• 50% of AS |
| Questions A mix of question styles, from short, single-mark questions to multi-step problems. |

OR

| Paper 2 |
|---|
| What's assessed One question paper answer booklet on Mechanics and one question paper answer booklet on Discrete. |
| How it's assessed <ul style="list-style-type: none">• Written exam: 1 hour 30 minutes• 80 marks• 50% of AS |
| Questions A mix of question styles, from short, single-mark questions to multi-step problems. |

3 Subject content

The subject content in sections A to H is compulsory for all students. Students must study two of the optional applications. The optional applications are mechanics (MA to MD), statistics (SA to SH) and discrete (DA to DG).

3.1 Overarching themes

AS specifications in further mathematics must require students to demonstrate the overarching knowledge and skills contained in sections **OT1**, **OT2** and **OT3**. These must be applied, along with associated mathematical thinking and understanding, across the whole of the detailed content set out in sections **A** to **DG**.

Appendix A sets out the mathematical notation that students are required to understand for this qualification. Appendix B sets out the mathematical formulae and identities students are required to use in this qualification. Further information is provided in the appendices.

3.1.1 OT1: Mathematical argument, language and proof

| | Content |
|-------|--|
| OT1.1 | Construct and present mathematical arguments through appropriate use of diagrams; sketching graphs; logical deduction; precise statements involving correct use of symbols and connecting language, including: constant, coefficient, expression, equation, function, identity, index, term, variable. |
| OT1.2 | Understand and use mathematical language and syntax as set out in the content. |
| OT1.3 | Understand and use language and symbols associated with set theory, as set out in the content. |
| OT1.5 | Comprehend and critique mathematical arguments, proofs and justifications of methods and formulae, including those relating to applications of mathematics. |

3.1.2 OT2: Mathematical problem solving

| | Content |
|-------|---|
| OT2.1 | Recognise the underlying mathematical structure in a situation and simplify and abstract appropriately to enable problems to be solved. |
| OT2.2 | Construct extended arguments to solve problems presented in an unstructured form, including problems in context. |
| OT2.3 | Interpret and communicate solutions in the context of the original problem. |

| | Content |
|-------|--|
| OT2.6 | Understand the concept of a mathematical problem solving cycle, including specifying the problem, collecting information, processing and representing information and interpreting results, which may identify the need to repeat the cycle. |
| OT2.7 | Understand, interpret and extract information from diagrams and construct mathematical diagrams to solve problems, including in mechanics. |

3.1.3 OT3: Mathematical modelling

| | Knowledge/skill |
|-------|---|
| OT3.1 | Translate a situation in context into a mathematical model, making simplifying assumptions. |
| OT3.2 | Use a mathematical model with suitable inputs to engage with and explore situations (for a given model or a model constructed or selected by the student). |
| OT3.3 | Interpret the outputs of a mathematical model in the context of the original situation (for a given model or a model constructed or selected by the student). |
| OT3.4 | Understand that a mathematical model can be refined by considering its outputs and simplifying assumptions; evaluate whether the model is appropriate. |
| OT3.5 | Understand and use modelling assumptions. |

3.2 Compulsory content

3.2.1 A: Proof

| | Content |
|----|---|
| A1 | Construct proofs using mathematical induction; contexts include sums of series, divisibility, and powers of matrices. |

3.2.2 B: Complex numbers

| | Content |
|----|---|
| B1 | Solve any quadratic equation with real coefficients; solve cubic or quartic equations with real coefficients (given sufficient information to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics). |

| | Content |
|----|---|
| B2 | Add, subtract, multiply and divide complex numbers in the form $x + iy$ with x and y real; understand and use the terms 'real part' and 'imaginary part'. |

| | Content |
|----|---|
| B3 | Understand and use the complex conjugate; know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs. |

| | Knowledge/skill |
|----|------------------------------------|
| B4 | Use and interpret Argand diagrams. |

| | Content |
|----|---|
| B5 | Convert between the Cartesian form and the modulus-argument form of a complex number (knowledge of radians is assumed). |

| | Content |
|----|---|
| B6 | Multiply and divide complex numbers in modulus-argument form (knowledge of radians and compound angle formulae is assumed). |

| | Content |
|----|---|
| B7 | Construct and interpret simple loci in the Argand diagram such as $ z - a > r$ and $\arg(z - a) = \theta$ (knowledge of radians is assumed). |

3.2.3 C: Matrices

| | Content |
|----|---|
| C1 | Add, subtract and multiply conformable matrices; multiply a matrix by a scalar. |

| | Content |
|----|--|
| C2 | Understand and use zero and identity matrices. |

| | Content |
|----|---|
| C3 | Use matrices to represent linear transformations in 2D; successive transformations; single transformations in 3D (3D transformations confined to reflection in one of $x = 0$, $y = 0$, $z = 0$ or rotation about one of the coordinate axes) (knowledge of 3D vectors is assumed). |

| | Content |
|----|--|
| C4 | Find invariant points and lines for a linear transformation. |

| | Content |
|----|--|
| C5 | Calculate determinants of 2×2 matrices. |

| | Content |
|----|--|
| C6 | Understand and use singular and non-singular matrices; properties of inverse matrices. Calculate and use the inverse of non-singular 2×2 matrices. |

3.2.4 D: Further algebra and functions

| | Content |
|----|---|
| D1 | Understand and use the relationship between roots and coefficients of polynomial equations up to quartic equations. |

| | Content |
|----|--|
| D2 | Form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial equation (of at least cubic degree). |

| | Content |
|----|--|
| D3 | Understand and use formulae for the sums of integers, squares and cubes and use these to sum other series. |

| | Content |
|----|---|
| D4 | Understand and use the method of differences for summation of series. |

| | Content |
|----|---|
| D6 | Recognise and use the Maclaurin series for e^x , $\ln(1+x)$, $\sin x$, $\cos x$, and $(1+x)^n$, and be aware of the range of values of x for which they are valid (proof not required). |

| | Content |
|----|--|
| D8 | Inequalities involving polynomial equations (cubic and quartic). |

| | Content |
|----|--|
| D9 | Solving inequalities such as $\frac{ax+b}{cx+d} < ex+f$ algebraically. |

| | Content |
|-----|--|
| D12 | Graphs of rational functions of form $\frac{ax+b}{cx+d}$; asymptotes, points of intersection with coordinate axes or other straight lines; associated inequalities. |

| | Content |
|-----|--|
| D13 | Graphs of rational functions of form $\frac{ax^2+bx+c}{dx^2+ex+f}$, including cases when some of these coefficients are zero; asymptotes parallel to coordinate axes. |

| | Content |
|-----|--|
| D14 | Using quadratic theory (not calculus) to find the possible values of the function and coordinates of the stationary points of the graph for rational functions of form $\frac{ax^2 + bx + c}{dx^2 + ex + f}$ |

| | Content |
|-----|---|
| D15 | Sketching graphs of curves with equation $y^2 = 4ax$, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $xy = c^2$ including intercepts with axes and equations of asymptotes of hyperbolas. |

| | Content |
|-----|---|
| D16 | Single transformations of curves involving translations, stretches parallel to coordinate axes and reflections in the coordinate axes and the lines $y = \pm x$. |

3.2.5 E: Further calculus

| | Content |
|----|--|
| E2 | Derive formulae for and calculate volumes of revolution. |

| | Content |
|----|---|
| E3 | Understand and evaluate the mean value of a function. |

3.2.6 F: Further vectors

| | Content |
|----|--|
| F1 | Understand and use the vector and Cartesian forms of an equation of a straight line in 3D. |

| | Content |
|----|---|
| F3 | Calculate the scalar product and use it to calculate the angle between two lines. |

| | Content |
|----|--|
| F4 | Check whether vectors are perpendicular by using the scalar product. |

| | Content |
|----|---|
| F6 | Find the intersection of two lines. Calculate the perpendicular distance between two lines and from a point to a line. |

3.2.7 G: Polar coordinates

| | Content |
|----|--|
| G1 | Understand and use polar coordinates and be able to convert between polar and Cartesian coordinates. |

| | Content |
|----|--|
| G2 | Sketch curves with r given as a function of θ , including use of trigonometric functions. |

3.2.8 H: Hyperbolic functions

| | Content |
|----|--|
| H1 | Understand the definitions of hyperbolic functions $\sinh x$, $\cosh x$ and $\tanh x$, and be able to sketch their graphs. |

| | Content |
|----|--|
| H3 | Understand and be able to use the definitions of the inverse hyperbolic functions. |

| | Content |
|----|---|
| H4 | Derive and use the logarithmic forms of the inverse hyperbolic functions. |

| | Content |
|----|--|
| H6 | Understand and use $\tanh x \equiv \frac{\sinh x}{\cosh x}$ Understand and use $\cosh^2 x - \sinh^2 x \equiv 1$ |

3.3 Optional application 1 – mechanics

3.3.1 MA: Dimensional analysis

| | Content |
|-----|---|
| MA1 | Finding dimensions of quantities; checking for dimensional consistency. |

| | Content |
|-----|---|
| MA2 | Prediction of formulae; finding powers in potential formulae. |

3.3.2 MB: Momentum and collisions

| | Content |
|-----|---|
| MB1 | Conservation of momentum for linear motion and cases where velocities are given as one or two dimensional vectors (resolving will not be required). |

| | Content |
|-----|--|
| MB2 | Coefficient of restitution and Newton's Experimental Law. Use in direct collisions and impacts with a fixed smooth surface (resolving will not be required). |

| | Content |
|-----|---|
| MB3 | Impulse and its relation to momentum (in one- or two-dimensions) (resolving will not be required). Use of $Ft = mv - mu$ |

| | Content |
|-----|--|
| MB4 | Impulse for variable forces. One dimension only. Use of $I = \int Fdt$. |

3.3.3 MC: Work, energy and power

| | Content |
|-----|---|
| MC1 | Work done by a force acting in the direction of motion or directly opposing the motion. |

| | Content |
|-----|---|
| MC2 | Gravitational potential energy. Use in conservation of energy problems. |

| | Content |
|-----|---|
| MC3 | Kinetic energy. Use in conservation of energy problems. |

| | Content |
|-----|---|
| MC4 | Hooke's Law including using modulus of elasticity. Use of $T = kx$ or $T = \frac{\lambda}{l}x$ |

| | Content |
|-----|--|
| MC5 | Work done by a variable force. Use of $WD = \int Fdx$. Use in conservation of energy problems. |

| | Content |
|-----|---|
| MC6 | Elastic potential energy using modulus of elasticity. Use of $EPE = \frac{kx^2}{2}$ and $EPE = \frac{\lambda x^2}{2l}$. Use in conservation of energy problems. |

| | Content |
|-----|---|
| MC7 | Power (resolving will not be required). Use of $P = Fv$ |

3.3.4 MD: Circular motion

| | Content |
|-----|---|
| MD1 | Motion of a particle moving in a circle with constant speed (knowledge of radians assumed). |

| | Content |
|-----|--|
| MD2 | Understand the definition of angular speed. Use both radians and revolutions per unit time. |

| | Content |
|-----|---|
| MD3 | Relationships between speed, angular speed, radius and acceleration. Use of $v = r\omega$, $a = r\omega^2$ and $a = \frac{v^2}{r}$ |

3.4 Optional application 2 – statistics

3.4.1 SA: Discrete random variables (DRVs) and expectation

| | Content |
|-----|--|
| SA1 | Understand DRVs with distributions given in the form of a table or function. |

| | Content |
|-----|-----------------------------------|
| SA2 | Evaluate probabilities for a DRV. |

| | Content |
|-----|---|
| SA3 | Evaluate measures of average and spread for a DRV to include mean, variance, standard deviation, mode and median. |

| | Content |
|-----|--|
| SA4 | Understand expectation and know the formulae: $E(X) = \sum x_i p_i$; $E(X^2) = \sum x_i^2 p_i$; $\text{Var}(X) = E(X^2) - (E(X))^2$ |

| | Content |
|-----|---|
| SA5 | Understand expectation of linear functions of DRVs and know the formulae: $E(aX + b) = aE(X) + b$ and $\text{Var}(aX + b) = a^2 \text{Var}(X)$ |

| | Content |
|-----|--|
| SA6 | Know the discrete uniform distribution defined on the set $\{1, 2, \dots, n\}$. Understand when this distribution can be used as a model. |
| | Content |
| SA7 | Proof of mean and variance of discrete uniform distribution. |

3.4.2 SB: Poisson distribution

| | Content |
|-----|---|
| SB1 | Understand conditions for a Poisson distribution to model a situation. Understand terminology $X \sim \text{Po}(\lambda)$. |
| | Content |
| SB2 | Know the Poisson formula and calculate Poisson probabilities using the formula or equivalent calculator function. |
| | Content |
| SB3 | Know mean, variance and standard deviation of a Poisson distribution. Use the result that, if $X \sim \text{Po}(\lambda)$ then the mean and variance of X are equal. |
| | Content |
| SB4 | Understand the distribution of the sum of independent Poisson distributions. |
| | Content |
| SB5 | Formulate hypotheses and carry out a hypothesis test of a population mean from a single observation from a Poisson distribution using direct evaluation of Poisson probabilities. |

3.4.3 SC: Type I and Type II errors

| | Content |
|-----|--|
| SC1 | Understand Type I and Type II errors and define in context. Calculate the probability of making a Type I error from tests based on a Poisson or Binomial distribution. |

3.4.4 SD: Continuous random variables (CRVs)

| | Content |
|-----|---|
| SD1 | Understand and use a probability density function, $f(x)$, for a continuous distribution and understand the differences between discrete and continuous distributions. |

| | Content |
|-----|---|
| SD2 | Find the probability of an observation lying in a specified interval. |

| | Content |
|-----|--|
| SD3 | Find the median and quartiles for a given probability density function, $f(x)$. |

| | Content |
|-----|---|
| SD4 | Find the mean, variance and standard deviation for a given pdf, $f(x)$. Know the formulae $E(X) = \int xf(x)dx, E(X^2) = \int x^2f(x)dx, \text{Var}(X) = E(X^2) - (E(X))^2$ |

| | Content |
|-----|--|
| SD5 | Understand the expectation and variance of linear functions of CRVs and know the formulae: $E(aX + b) = aE(X) + b \text{ and } \text{Var}(aX + b) = a^2\text{Var}(X)$ Know the formula $E(g(X)) = \int g(x)f(x)dx$ Find the mean, variance and standard deviation of functions of a continuous random variable such as $E(5X^3), E(18X^{-3}), \text{Var}(6X^{-1})$ |

| | Content |
|-----|--|
| SD8 | Know that if X and Y are independent (discrete or continuous) random variables then $E(X + Y) = E(X) + E(Y)$ and $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$ |

3.4.5 SE: Chi squared tests for association

| | Content |
|-----|--|
| SE1 | Construction of $n \times m$ contingency tables. |

| | Content |
|-----|---|
| SE2 | Use of $\sum \frac{(O_i - E_i)^2}{E_i}$ as an approximate χ^2 statistic with appropriate degrees of freedom. |

| | Content |
|-----|--|
| SE3 | Know and use the convention that all E_i should be greater than 5. |

| | Content |
|-----|--|
| SE4 | Identification of sources of association in the context of a question. |

3.4.6 SH: Confidence intervals

| | Content |
|-----|---|
| SH1 | Construct symmetric confidence intervals for the mean of a normal distribution with known variance. |

| | Content |
|-----|---|
| SH2 | Construct symmetric confidence intervals from large samples, for the mean of a normal distribution with unknown variance. |

| | Content |
|-----|---|
| SH3 | Make inferences from constructed or given confidence intervals. |

3.5 Optional application 3 – discrete mathematics

3.5.1 DA: Graphs

| | Content |
|-----|--|
| DA1 | Understand and use the language of graphs including: vertex, edge, trail, cycle, connected, degree, subgraph, subdivision, multiple edge and loop. |

| | Content |
|-----|---|
| DA2 | Identify or prove properties of a graph including that a graph is Eulerian, semi-Eulerian or Hamiltonian. |

| | Content |
|-----|---|
| DA3 | Understand and use Euler’s formula for connected planar graphs. |

| | Content |
|-----|--|
| DA5 | Understand and use complete graphs and bipartite graphs, including adjacency matrices and the complement of a graph. |

| | Content |
|-----|--|
| DA6 | Understand and use simple graphs, simple-connected graphs and trees. |

3.5.2 DB: Networks

| | Content |
|-----|--|
| DB1 | Understand and use the language of networks including: node, arc and weight. |

| | Content |
|-----|---|
| DB2 | Solve network optimisation problems using spanning trees. |

| | Content |
|-----|----------------------------------|
| DB3 | Solve route inspection problems. |

| | Content |
|-----|--|
| DB4 | Find and interpret upper bounds and lower bounds for the travelling salesperson problem. |

| | Content |
|-----|--|
| DB5 | Evaluate, modify and refine models which use networks. |

3.5.3 DC: Network flows

| | Content |
|-----|--|
| DC1 | Interpret flow problems represented by a network of directed arcs. |

| | Content |
|-----|---|
| DC2 | Find the value of a cut and understand its meaning. |

| | Content |
|-----|---|
| DC3 | Use and interpret the maximum flow-minimum cut theorem. |

| | Content |
|-----|---|
| DC4 | Introduce supersources and supersinks to a network. |

3.5.4 DD: Linear programming

| | Content |
|-----|--|
| DD1 | Formulate constrained optimisation problems. |

| | Content |
|-----|--|
| DD2 | Solve constrained optimisation problems via graphical methods. |

3.5.5 DE: Critical path analysis

| | Content |
|-----|--|
| DE1 | Construct, represent and interpret a precedence (activity) network using activity-on-node. |

| | Content |
|-----|---|
| DE2 | Determine earliest and latest start and finish times for an activity network. |

| | Content |
|-----|--|
| DE3 | Identify critical activities, critical paths and the float of non-critical activities. |

| | Content |
|-----|---|
| DE4 | Refine models and understand the implications of possible changes in the context of critical path analysis. |

3.5.6 DF: Game theory for zero-sum games

| | Content |
|-----|---|
| DF1 | Understand, interpret and construct pay-off matrices. |

| | Content |
|-----|--|
| DF2 | Find play-safe strategies and the value of the game. |

| | Content |
|-----|--|
| DF3 | Prove the existence or non-existence of a stable solution. |

| | Content |
|-----|--|
| DF4 | Identify and make use of dominated strategies. |

| | Content |
|-----|--|
| DF5 | Find optimal mixed strategies for a game including use of graphical methods. |

3.5.7 DG: Binary operations

| | Content |
|-----|---|
| DG1 | Understand and use binary operations including use of modular arithmetic and matrix multiplication. |

| | Content |
|-----|--|
| DG2 | Understand, use and prove the commutativity of a binary operation. |

| | Content |
|-----|--|
| DG3 | Understand, use and prove the associativity of a binary operation. |

| | Content |
|-----|--|
| DG4 | Construct a Cayley table for a given set under a given binary operation. |

| | Content |
|-----|---|
| DG5 | Understand and prove the existence of an identity element for a given set under a given binary operation. |

| | Content |
|-----|---|
| DG6 | Find the inverse of an element belonging to a given set under a given binary operation. |

4 Scheme of assessment

Find past papers and mark schemes, and specimen papers for new courses, on our website at aqa.org.uk/pastpapers

This specification is designed to be taken over one or two years.

This is a linear qualification. In order to achieve the award, students must complete all assessments at the end of the course and in the same series.

AS exams and certification for this specification are available for the first time in May/June 2018 and then every May/June for the life of the specification.

All materials are available in English only.

Our AS exams in Further Mathematics include questions that allow students to demonstrate their ability to:

- recall information.
- draw together information from different areas of the specification.
- apply their knowledge and understanding in practical and theoretical contexts.

4.1 Aims

Courses based on this specification must encourage students to:

- understand mathematics and mathematical processes in ways that promote confidence, foster enjoyment and provide a strong foundation for progress to further study
- extend their range of mathematical skills and techniques
- understand coherence and progression in mathematics and how different areas of mathematics are connected
- apply mathematics in other fields of study and be aware of the relevance of mathematics to the world of work and to situations in society in general
- use their mathematical knowledge to make logical and reasoned decisions in solving problems both within pure mathematics and in a variety of contexts, and communicate the mathematical rationale for these decisions clearly
- reason logically and recognise incorrect reasoning
- generalise mathematically
- construct mathematical proofs
- use their mathematical skills and techniques to solve challenging problems which require them to decide on the solution strategy
- recognise when mathematics can be used to analyse and solve a problem in context
- represent situations mathematically and understand the relationship between problems in context and mathematical models that may be applied to solve them
- draw diagrams and sketch graphs to help explore mathematical situations and interpret solutions
- make deductions and inferences and draw conclusions by using mathematical reasoning
- interpret solutions and communicate their interpretation effectively in the context of the problem

- read and comprehend mathematical arguments, including justifications of methods and formulae, and communicate their understanding
- read and comprehend articles concerning applications of mathematics and communicate their understanding
- use technology such as calculators and computers effectively, and recognise when such use may be inappropriate
- take increasing responsibility for their own learning and the evaluation of their own mathematical development.

4.2 Assessment objectives

Assessment objectives (AOs) are set by Ofqual and are the same across all AS Further Mathematics specifications and all exam boards.

The exams will measure how students have achieved the following assessment objectives.

- AO1: Use and apply standard techniques. Students should be able to:
 - select and correctly carry out routine procedures
 - accurately recall facts, terminology and definitions
- AO2: Reason, interpret and communicate mathematically. Students should be able to:
 - construct rigorous mathematical arguments (including proofs)
 - make deductions and inferences
 - assess the validity of mathematical arguments
 - explain their reasoning
 - use mathematical language and notation correctly.

Where questions/tasks targeting this assessment objective will also credit students for the ability to 'use and apply standard techniques' (AO1) and/or to 'solve problems within mathematics and in other contexts' (AO3) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).

- AO3: Solve problems within mathematics and in other contexts. Students should be able to:
 - translate problems in mathematical and non-mathematical contexts into mathematical processes
 - interpret solutions to problems in their original context, and, where appropriate, evaluate their accuracy and limitations
 - translate situations in context into mathematical models
 - use mathematical models
 - evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them.

Where questions/tasks targeting this assessment objective will also credit students for the ability to 'use and apply standard techniques' (AO1) and/or to 'reason, interpret and communicate mathematically' (AO2) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).

4.2.1 Assessment objective weightings for AS Further Mathematics

| Assessment objectives (AOs) | Component weightings (approx %) | | Overall weighting (approx %) |
|-----------------------------|---------------------------------|---------|------------------------------|
| | Paper 1 | Paper 2 | |
| AO1 | 65 | 55 | 60 |

| Assessment objectives (AOs) | Component weightings (approx %) | | Overall weighting (approx %) |
|---------------------------------|---------------------------------|---------|------------------------------|
| | Paper 1 | Paper 2 | |
| AO2 | 20 | 20 | 20 |
| AO3 | 15 | 25 | 20 |
| Overall weighting of components | 50 | 50 | 100 |

4.3 Assessment weightings

The marks awarded on the papers will be scaled to meet the weighting of the components. Students' final marks will be calculated by adding together the scaled marks for each component. Grade boundaries will be set using this total scaled mark. The scaling and total scaled marks are shown in the table below.

Students' final marks will be calculated by adding together the scaled marks for each component, this includes the two optional topics chosen as part of paper 2. At qualification level different grade boundaries will be published to reflect the different routes through the qualification.

| Component | Maximum raw mark | Scaling factor | Maximum scaled mark |
|--------------------|------------------|----------------|---------------------|
| Paper 1 | 80 | x1 | 80 |
| Paper 2 | 80 | x1 | 80 |
| Total scaled mark: | | | 160 |

5 General administration

You can find information about all aspects of administration, as well as all the forms you need, at aqa.org.uk/examsadmin

5.1 Entries and codes

You only need to make one entry for each qualification – this will cover all the question papers, non-exam assessment and certification.

Every specification is given a national discount (classification) code by the Department for Education (DfE), which indicates its subject area.

If a student takes two specifications with the same discount code, further and higher education providers are likely to take the view that they have only achieved one of the two qualifications. Please check this before your students start their course.

| Qualification title | AQA entry code | DfE discount code |
|--|----------------|-------------------|
| AQA Advanced Subsidiary GCE in Further Mathematics | 7366 | RB1A |

This specification complies with:

- Ofqual *General conditions of recognition* that apply to all regulated qualifications
- Ofqual GCE qualification level conditions that apply to all GCEs
- Ofqual GCE subject level conditions that apply to all GCEs in this subject
- all other relevant regulatory documents.

The Ofqual qualification accreditation number (QAN) is 603/1604/4.

5.2 Overlaps with other qualifications

There is overlapping content in the AS and A-level Further Mathematics specifications. This helps you teach the AS and A-level together.

5.3 Awarding grades and reporting results

The AS qualification will be graded on a five-point scale: A, B, C, D and E.

Students who fail to reach the minimum standard for grade E will be recorded as U (unclassified) and will not receive a qualification certificate.

5.4 Re-sits and shelf life

Students can re-sit the qualification as many times as they wish, within the shelf life of the qualification.

5.5 Previous learning and prerequisites

There are no previous learning requirements. Any requirements for entry to a course based on this specification are at the discretion of schools and colleges.

However, we recommend that students should have the skills and knowledge associated with a GCSE Mathematics or equivalent.

5.6 Access to assessment: diversity and inclusion

General qualifications are designed to prepare students for a wide range of occupations and further study. Therefore our qualifications must assess a wide range of competences.

The subject criteria have been assessed to see if any of the skills or knowledge required present any possible difficulty to any students, whatever their ethnic background, religion, sex, age, disability or sexuality. Tests of specific competences were only included if they were important to the subject.

As members of the Joint Council for Qualifications (JCQ) we participate in the production of the JCQ document *Access Arrangements and Reasonable Adjustments: General and Vocational qualifications*. We follow these guidelines when assessing the needs of individual students who may require an access arrangement or reasonable adjustment. This document is published at jcq.org.uk

5.6.1 Students with disabilities and special needs

We're required by the Equality Act 2010 to make reasonable adjustments to remove or lessen any disadvantage that affects a disabled student.

We can make arrangements for disabled students and students with special needs to help them access the assessments, as long as the competences being tested aren't changed. Access arrangements must be agreed **before** the assessment. For example, a Braille paper would be a reasonable adjustment for a Braille reader.

To arrange access arrangements or reasonable adjustments, you can apply using the online service at aqa.org.uk/eaqa

5.6.2 Special consideration

We can give special consideration to students who have been disadvantaged at the time of the assessment through no fault of their own – for example a temporary illness, injury or serious problem such as family bereavement. We can only do this **after** the assessment.

Your exams officer should apply online for special consideration at aqa.org.uk/eaqa

For more information and advice visit aqa.org.uk/access or email accessarrangementsqueries@aqa.org.uk

5.7 Working with AQA for the first time

If your school or college hasn't previously offered our specifications, you need to register as an AQA centre. Find out how at aqa.org.uk/becomeacentre

5.8 Private candidates

This specification is available to private candidates.

A private candidate is someone who enters for exams through an AQA approved school or college but is not enrolled as a student there.

A private candidate may be self-taught, home schooled or have private tuition, either with a tutor or through a distance learning organisation. They must be based in the UK.

If you have any queries as a private candidate, you can:

- speak to the exams officer at the school or college where you intend to take your exams
- visit our website at aqa.org.uk/privatecandidates
- email privatecandidates@aqa.org.uk

5.9 Use of calculators

A calculator is required for use in all assessments in this specification. Details of the requirements for calculators can be found in the Joint Council for General Qualifications document *Instructions for conducting examinations*.

For AS Further Mathematics exams, calculators should have the following as a required minimum:

- an iterative function
- the ability to perform calculations with matrices up to order 3×3
- the ability to compute summary statistics and access probabilities from standard statistical distributions.

For the purposes of this specification, a 'calculator' is any electronic or mechanical device which may be used for the performance of mathematical computations. However, only those permissible in the guidance in the *Instructions for conducting examinations* are allowed in AS Further Mathematics exams.

6 Appendix A: mathematical notation

The tables below set out the notation that must be used by AS and A-level mathematics and further mathematics specifications. Students will be expected to understand this notation without need for further explanation.

Mathematics students will not be expected to understand notation that relates only to further mathematics content. Further mathematics students will be expected to understand all notation in the list.

For further mathematics, the notation for the core content is listed under sub headings indicating 'further mathematics only'. In this subject, awarding organisations are required to include, in their specifications, content that is additional to the core content. They will therefore need to add to the notation list accordingly.

AS students will be expected to understand notation that relates to AS content, and will not be expected to understand notation that relates only to A-level content.

6.1 Set notation

| 1 | Set notation | Meaning |
|------|-----------------------|---|
| 1.1 | \in | is an element of |
| 1.2 | \notin | is not an element of |
| 1.3 | \subseteq | is a subset of |
| 1.4 | \subset | is a proper subset of |
| 1.5 | $\{x_1, x_2, \dots\}$ | the set with elements x_1, x_2, \dots |
| 1.6 | $\{x: \dots\}$ | the set of all x such that ... |
| 1.7 | $n(A)$ | the number of elements in set A |
| 1.8 | \emptyset | the empty set |
| 1.9 | ε | the universal set |
| 1.10 | A' | the complement of the set A |
| 1.11 | \mathbb{N} | the set of natural numbers $\{1, 2, 3, \dots\}$ |
| 1.12 | \mathbb{Z} | the set of integers $\{0, \pm 1, \pm 2, \pm 3, \dots\}$ |

| 1 | Set notation | Meaning |
|------|------------------|---|
| 1.13 | \mathbb{Z}^+ | the set of positive integers $\{1, 2, 3, \dots\}$ |
| 1.14 | \mathbb{Z}_0^+ | the set of non-negative integers $\{0, 1, 2, 3, \dots\}$ |
| 1.15 | \mathbb{R} | the set of real numbers |
| 1.16 | \mathbb{Q} | the set of rational numbers $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$ |
| 1.17 | \cup | union |
| 1.18 | \cap | intersection |
| 1.19 | (x, y) | the ordered pair x, y |
| 1.20 | $[a, b]$ | the closed interval $\{x \in \mathbb{R} : a \leq x \leq b\}$ |
| 1.21 | $[a, b)$ | the interval $\{x \in \mathbb{R} : a \leq x < b\}$ |
| 1.22 | $(a, b]$ | the interval $\{x \in \mathbb{R} : a < x \leq b\}$ |
| 1.23 | (a, b) | the open interval $\{x \in \mathbb{R} : a < x < b\}$ |

Set notation (Further Maths only)

| 1 | Set notation | Meaning |
|------|--------------|----------------------------|
| 1.24 | \mathbb{C} | the set of complex numbers |

6.2 Miscellaneous symbols

| 2 | Miscellaneous symbols | Meaning |
|-----|-----------------------|------------------------------------|
| 2.1 | $=$ | is equal to |
| 2.2 | \neq | is not equal to |
| 2.3 | \equiv | is identical to or is congruent to |
| 2.4 | \approx | is approximately equal to |
| 2.5 | ∞ | infinity |
| 2.6 | \propto | is proportional to |

| 2 | Miscellaneous symbols | Meaning |
|------|-----------------------|--|
| 2.7 | \therefore | therefore |
| 2.8 | \because | because |
| 2.9 | $<$ | is less than |
| 2.10 | \leq, \leq | is less than or equal to, is not greater than |
| 2.11 | $>$ | is greater than |
| 2.12 | \geq, \geq | is greater than or equal to, is not less than |
| 2.13 | $p \Rightarrow q$ | p implies q (if p then q) |
| 2.14 | $p \Leftarrow q$ | p is implied by q (if q then p) |
| 2.15 | $p \Leftrightarrow q$ | p implies and is implied by q (p is equivalent to q) |
| 2.16 | a | first term of an arithmetic or geometric sequence |
| 2.17 | l | last term of an arithmetic sequence |
| 2.18 | d | common difference of an arithmetic sequence |
| 2.19 | r | common ratio of a geometric sequence |
| 2.20 | S_n | sum to n terms of a sequence |
| 2.21 | S_∞ | sum to infinity of a sequence |

Miscellaneous symbols (Further Maths only)

| 2 | Miscellaneous symbols | Meaning |
|------|-----------------------|------------------|
| 2.22 | \cong | is isomorphic to |

6.3 Operations

| 3 | Operations | Meaning |
|-----|------------|---------------|
| 3.1 | $a + b$ | a plus b |
| 3.2 | $a - b$ | a minus b |

| 3 | Operations | Meaning |
|------|------------------------------------|---|
| 3.3 | $a \times b, ab, a \cdot b$ | a multiplied by b |
| 3.4 | $a \div b, \frac{a}{b}$ | a divided by b |
| 3.5 | $\sum_{i=1}^n a_i$ | $a_1 + a_2 + \dots + a_n$ |
| 3.6 | $\prod_{i=1}^n a_i$ | $a_1 \times a_2 \times \dots \times a_n$ |
| 3.7 | \sqrt{a} | the non-negative square root of a |
| 3.8 | $ a $ | the modulus of a |
| 3.9 | $n!$ | n factorial: $n! = n \times (n-1) \times \dots \times 2 \times 1, n \in \mathbb{N}; 0! = 1$ |
| 3.10 | $\binom{n}{r}, {}^n C_r, {}_n C_r$ | the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n, r \in \mathbb{Z}_0^+, r \leq n$ or $\frac{n(n-1)\dots(n-r+1)}{r!}$ for $n \in \mathbb{Q}, r \in \mathbb{Z}_0^+$ |

Operations (Further Maths only)

| 3 | Operations | Meaning |
|------|----------------------------|---|
| 3.11 | $a \times_n b$ | multiplication modulo n of a by b |
| 3.12 | $a +_n b$ | addition modulo n of a and b |
| 3.13 | $G = \langle n, * \rangle$ | n is the generator of a given group G under the operation $*$ |

6.4 Functions

| 4 | Functions | Meaning |
|-----|------------------|--|
| 4.1 | $f(x)$ | the value of the function f at x |
| 4.2 | $f: x \mapsto y$ | the function f maps the element x to the element y |

| 4 | Functions | Meaning |
|------|------------------------------------|---|
| 4.3 | f^{-1} | the inverse function of the function f |
| 4.4 | gf | the composite function of f and g which is defined by $gf(x) = g(f(x))$ |
| 4.5 | $\lim_{x \rightarrow a} f(x)$ | the limit of $f(x)$ as x tends to a |
| 4.6 | $\Delta x, \delta x$ | an increment of x |
| 4.7 | $\frac{dy}{dx}$ | the derivative of y with respect to x |
| 4.8 | $\frac{d^n y}{dx^n}$ | the n th derivative of y with respect to x |
| 4.9 | $f'(x), f''(x), \dots, f^{(n)}(x)$ | the first, second, ..., n th derivatives of $f(x)$ with respect to x |
| 4.10 | \dot{x}, \ddot{x}, \dots | the first, second, ... derivatives of x with respect to t |
| 4.11 | $\int y \, dx$ | the indefinite integral of y with respect to x |
| 4.12 | $\int_a^b y \, dx$ | the definite integral of y with respect to x between the limits $x = a$ and $x = b$ |

6.5 Exponential and logarithmic functions

| 5 | Exponential and logarithmic functions | Meaning |
|-----|---------------------------------------|----------------------------------|
| 5.1 | e | base of natural logarithms |
| 5.2 | $e^x, \exp x$ | exponential function of x |
| 5.3 | $\log_a x$ | logarithm to the base a of x |
| 5.4 | $\ln x, \log_e x$ | natural logarithm of x |

6.6 Trigonometric functions

| 6 | Trigonometric functions | Meaning |
|-----|--|-------------------------------------|
| 6.1 | $\sin, \cos, \tan,$ $\operatorname{cosec}, \sec, \cot$ | the trigonometric functions |
| 6.2 | $\sin^{-1}, \cos^{-1}, \tan^{-1}$ $\arcsin, \arccos, \arctan$ | the inverse trigonometric functions |
| 6.3 | ° | degrees |
| 6.4 | rad | radians |

Trigonometric functions (Further Maths only)

| 6 | Trigonometric functions | Meaning |
|-----|--|-------------------------------------|
| 6.5 | $\operatorname{cosec}^{-1}, \sec^{-1}, \cot^{-1},$ $\operatorname{arccosec}, \operatorname{arcsec}, \operatorname{arccot}$ | the inverse trigonometric functions |
| 6.6 | $\sinh, \cosh, \tanh,$ $\operatorname{cosech}, \operatorname{sech}, \operatorname{coth}$ | the hyperbolic functions |
| 6.7 | $\sinh^{-1}, \cosh^{-1}, \tanh^{-1}$ $\operatorname{cosech}^{-1}, \operatorname{sech}^{-1}, \operatorname{coth}^{-1}$ $\operatorname{arcsinh}, \operatorname{arccosh}, \operatorname{arctanh},$ $\operatorname{arccosech}, \operatorname{arcsech}, \operatorname{arcoth}$ | the inverse hyperbolic functions |

6.7 Complex numbers (Further Maths only)

| 7 | Complex numbers | Meaning |
|-----|----------------------------------|--|
| 7.1 | i, j | square root of -1 |
| 7.2 | $x + iy$ | complex number with real part x and imaginary part y |
| 7.3 | $r(\cos \theta + i \sin \theta)$ | modulus argument form of a complex number with modulus r and argument θ |
| 7.4 | z | a complex number, $z = x + iy = r(\cos \theta + i \sin \theta)$ |

| 7 | Complex numbers | Meaning |
|-----|-----------------|---|
| 7.5 | $\text{Re}(z)$ | the real part of z , $\text{Re}(z) = x$ |
| 7.6 | $\text{Im}(z)$ | the imaginary part of z , $\text{Im}(z) = y$ |
| 7.7 | $ z $ | the modulus of z , $ z = r = \sqrt{x^2 + y^2}$ |
| 7.8 | $\arg(z)$ | the argument of z , $\arg(z) = \theta$, $-\pi < \theta \leq \pi$ |
| 7.9 | z^* | the complex conjugate of z , $x - iy$ |

Matrices (Further Maths only)

| 8 | Matrices | Meaning |
|-----|--|--|
| 8.1 | \mathbf{M} | a matrix \mathbf{M} |
| 8.2 | $\mathbf{0}$ | zero matrix |
| 8.3 | I | identity matrix |
| 8.4 | \mathbf{M}^{-1} | the inverse of the matrix \mathbf{M} |
| 8.5 | \mathbf{M}^T | the transpose of the matrix \mathbf{M} |
| 8.6 | Δ , $\det \mathbf{M}$ or $ \mathbf{M} $ | the determinant of the square matrix \mathbf{M} |
| 8.7 | $\mathbf{M}\mathbf{r}$ | image of column vector \mathbf{r} under the transformation associated with the matrix \mathbf{M} |

6.9 Vectors

| 9 | Vectors | Meaning |
|-----|--|---|
| 9.1 | \mathbf{a} , \underline{a} , \tilde{a} | the vector \mathbf{a} , \underline{a} , \tilde{a} ; these alternatives apply throughout section 9 |
| 9.2 | \vec{AB} | the vector represented in magnitude and direction by the directed line segment AB |
| 9.3 | \hat{a} | a unit vector in the direction of \mathbf{a} |
| 9.4 | \mathbf{i} , \mathbf{j} , \mathbf{k} | unit vectors in the directions of the cartesian coordinate axes |

| 9 | Vectors | Meaning |
|------|---|--|
| 9.5 | $ \mathbf{a} , a$ | the magnitude of \mathbf{a} |
| 9.6 | $ \vec{AB} , AB$ | the magnitude of \vec{AB} |
| 9.7 | $\begin{pmatrix} a \\ b \end{pmatrix}, a\mathbf{i} + b\mathbf{j}$ | column vector and corresponding unit vector notation |
| 9.8 | \mathbf{r} | position vector |
| 9.9 | \mathbf{s} | displacement vector |
| 9.10 | \mathbf{v} | velocity vector |
| 9.11 | \mathbf{a} | acceleration vector |

Vectors (Further Maths only)

| 9 | Vectors | Meaning |
|------|-------------------------------|---|
| 9.12 | $\mathbf{a} \cdot \mathbf{b}$ | the scalar product of \mathbf{a} and \mathbf{b} |

6.10 Differential equations (Further Maths only)

| 10 | Differential equations | Meaning |
|------|------------------------|---------------|
| 10.1 | ω | angular speed |

6.11 Probability and statistics

| 11 | Probability and statistics | Meaning |
|------|----------------------------|---|
| 11.1 | A, B, C etc. | events |
| 11.2 | $A \cup B$ | union of the events A and B |
| 11.3 | $A \cap B$ | intersection of the events A and B |
| 11.4 | $P(A)$ | probability of the event A |
| 11.5 | A' | complement of the event A |
| 11.6 | $P(A B)$ | probability of the event A conditional on the event B |

| 11 | Probability and statistics | Meaning |
|-------|----------------------------|--|
| 11.7 | X, Y, R etc. | random variables |
| 11.8 | x, y, r etc. | values of the random variables X, Y, R etc. |
| 11.9 | x_1, x_2, \dots | values of observations |
| 11.10 | f_1, f_2, \dots | frequencies with which the observations x_1, x_2, \dots occur |
| 11.11 | $p(x), P(X = x)$ | probability function of the discrete random variable X |
| 11.12 | p_1, p_2, \dots | probabilities of the values x_1, x_2, \dots of the discrete random variable X |
| 11.13 | $E(X)$ | expectation of the random variable X |
| 11.14 | $\text{Var}(X)$ | variance of the random variable X |
| 11.15 | \sim | has the distribution |
| 11.16 | $B(n, p)$ | binomial distribution with parameters n and p , where n is the number of trials and p is the probability of success in a trial |
| 11.17 | q | $q = 1 - p$ for binomial distribution |
| 11.18 | $N(\mu, \sigma^2)$ | Normal distribution with mean μ and variance σ^2 |
| 11.19 | $Z \sim N(0, 1)$ | standard Normal distribution |
| 11.20 | ϕ | probability density function of the standardised Normal variable with distribution $N(0, 1)$ |
| 11.21 | Φ | corresponding cumulative distribution function |
| 11.22 | μ | population mean |
| 11.23 | σ^2 | population variance |
| 11.24 | σ | population standard deviation |
| 11.25 | \bar{x} | sample mean |
| 11.26 | s^2 | sample variance |
| 11.27 | s | sample standard deviation |

| 11 | Probability and statistics | Meaning |
|-------|----------------------------|---|
| 11.28 | H_0 | null hypothesis |
| 11.29 | H_1 | alternative hypothesis |
| 11.30 | r | product moment correlation coefficient for a sample |
| 11.31 | ρ | product moment correlation coefficient for a population |

6.12 Mechanics

| 12 | Mechanics | Meaning |
|-------|-----------------------|---|
| 12.1 | kg | kilogram |
| 12.2 | m | metre |
| 12.3 | km | kilometre |
| 12.4 | m/s, $m\ s^{-1}$ | metre(s) per second (velocity) |
| 12.5 | m/s^2 , $m\ s^{-2}$ | metre(s) per second per second (acceleration) |
| 12.6 | F | Force or resultant force |
| 12.7 | N | newton |
| 12.8 | Nm | newton metre (moment of a force) |
| 12.9 | t | time |
| 12.10 | s | displacement |
| 12.11 | u | initial velocity |
| 12.12 | v | velocity or final velocity |
| 12.13 | a | acceleration |
| 12.14 | g | acceleration due to gravity |
| 12.15 | μ | coefficient of friction |

7 Appendix B: mathematical formulae and identities

Students must be able to use the following formulae and identities for AS and A-level further mathematics, without these formulae and identities being provided, either in these forms or in equivalent forms. These formulae and identities may only be provided where they are the starting point for a proof or as a result to be proved.

Pure mathematics

Quadratic equations

$$ax^2 + bx + c = 0 \text{ has roots } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Laws of indices

$$a^x a^y \equiv a^{x+y}$$

$$a^x \div a^y \equiv a^{x-y}$$

$$(a^x)^y \equiv a^{xy}$$

Laws of logarithms

$$x = a^n \Leftrightarrow n = \log_a x \text{ for } a > 0 \text{ and } x > 0$$

$$\log_a x + \log_a y \equiv \log_a(xy)$$

$$\log_a x - \log_a y \equiv \log_a\left(\frac{x}{y}\right)$$

$$k \log_a x \equiv \log_a(x^k)$$

Coordinate geometry

A straight line graph, gradient m passing through (x_1, y_1) has equation

$$y - y_1 = m(x - x_1)$$

Straight lines with gradients m_1 and m_2 are perpendicular when $m_1 m_2 = -1$

Sequences

General term of an arithmetic progression: $u_n = a + (n - 1)d$

General term of a geometric progression: $u_n = ar^{n-1}$

Trigonometry

In the triangle ABC

$$\text{Sine rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Cosine rule: } a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area} = \frac{1}{2}ab \sin C$$

$$\cos^2 A + \sin^2 A \equiv 1$$

$$\sec^2 A \equiv 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$$

$$\sin 2A \equiv 2 \sin A \cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A$$

$$\tan 2A \equiv \frac{2 \tan A}{1 - \tan^2 A}$$

Mensuration

Circumference and Area of circle, radius r and diameter d :

$$C = 2\pi r = \pi d$$

$$A = \pi r^2$$

Pythagoras' Theorem: In any right-angled triangle where a , b and c are the lengths of the sides and c is the hypotenuse:

$$c^2 = a^2 + b^2$$

Area of a trapezium = $\frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides and h is their perpendicular separation.

Volume of a prism = area of cross section \times length

For a circle of radius r , where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A :

$$s = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Complex numbers

For two complex numbers $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$:

$$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$$

Loci in the Argand diagram:

$|z - a| = r$ is a circle radius r centred at a

$\arg(z - a) = \theta$ is a half line drawn from a at angle θ to a line parallel to the positive real axis.

Exponential form:

$$e^{i\theta} = \cos \theta + i \sin \theta$$

Matrices

For a 2 by 2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the determinant $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

the inverse is $\frac{1}{\Delta} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

The transformation represented by matrix **AB** is the transformation represented by matrix **B** followed by the transformation represented by matrix **A**.

For matrices **A**, **B**:

$$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$$

Algebra

$$\sum_{r=1}^n r = \frac{1}{2}n(n+1)$$

For $ax^2 + bx + c = 0$ with roots α and β :

$$\alpha + \beta = \frac{-b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

For $ax^3 + bx^2 + cx + d = 0$ with roots α , β and γ :

$$\sum \alpha = \frac{-b}{a}$$

$$\sum \alpha\beta = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

Hyperbolic functions

$$\cosh x \equiv \frac{1}{2}(e^x + e^{-x})$$

$$\sinh x \equiv \frac{1}{2}(e^x - e^{-x})$$

$$\tanh x \equiv \frac{\sinh x}{\cosh x}$$

Calculus and differential equations

Differentiation

| Function | Derivative |
|---------------|-------------------------|
| x^n | nx^{n-1} |
| $\sin kx$ | $k\cos kx$ |
| $\cos kx$ | $-k\sin kx$ |
| e^{kx} | ke^{kx} |
| $\ln x$ | $\frac{1}{x}$ |
| $f(x) + g(x)$ | $f'(x) + g'(x)$ |
| $f(x)g(x)$ | $f'(x)g(x) + f(x)g'(x)$ |
| $f(g(x))$ | $f'(g(x))g'(x)$ |

Integration

| Function | Integral |
|-----------------|---------------------------------------|
| x^n | $\frac{1}{n+1}x^{n+1} + c, n \neq -1$ |
| $\cos kx$ | $\frac{1}{k}\sin kx + c$ |
| $\sin kx$ | $-\frac{1}{k}\cos kx + c$ |
| e^{kx} | $\frac{1}{k}e^{kx} + c$ |
| $\frac{1}{x}$ | $\ln x + c, x \neq 0$ |
| $f'(x) + g'(x)$ | $f(x) + g(x) + c$ |
| $f'(g(x))g'(x)$ | $f(g(x)) + c$ |

Area under a curve = $\int_a^b y \, dx$ ($y \geq 0$)

Volumes of revolution about the x and y axes:

$$V_x = \pi \int_a^b y^2 dx$$

$$V_y = \pi \int_c^d x^2 dy$$

Simple Harmonic Motion:

$$\ddot{x} = -\omega^2 x$$

Vectors

$$|xi + yj + zk| = \sqrt{(x^2 + y^2 + z^2)}$$

Scalar product of two vectors $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ is

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}| |\mathbf{b}| \cos \theta$$

where θ is the acute angle between the vectors \mathbf{a} and \mathbf{b} .

The equation of the line through the point with position vector \mathbf{a} parallel to vector \mathbf{b} is:

$$\mathbf{r} = \mathbf{a} + t\mathbf{b}$$

The equation of the plane containing the point with position vector \mathbf{a} and perpendicular to vector \mathbf{n} is:

$$(\mathbf{r} - \mathbf{a}) \cdot \mathbf{n} = 0$$

Mechanics

Forces and equilibrium

Weight = mass $\times g$

Friction: $F \leq \mu R$

Newton's second law in the form: $F = ma$

Kinematics

For motion in a straight line with variable acceleration:

$$v = \frac{dr}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$$

$$r = \int v \, dt$$

$$v = \int a \, dt$$

Statistics

The mean of a set of data: $\bar{x} = \frac{\Sigma x}{n} = \frac{\Sigma fx}{\Sigma f}$

The standard Normal variable: $Z = \frac{X - \mu}{\sigma}$ where $X \sim N(\mu, \sigma^2)$

Get help and support

Visit our website for information, guidance, support and resources at

You can talk directly to the Further Mathematics subject team:

E: maths@aqa.org.uk

T: 0161 957 3852