LEVEL 3 TECHNICAL LEVEL
Design Engineering
Mechatronic Engineering

J/506/5953 - Unit 3 Mathematics for Engineers

Mark scheme

June 2018

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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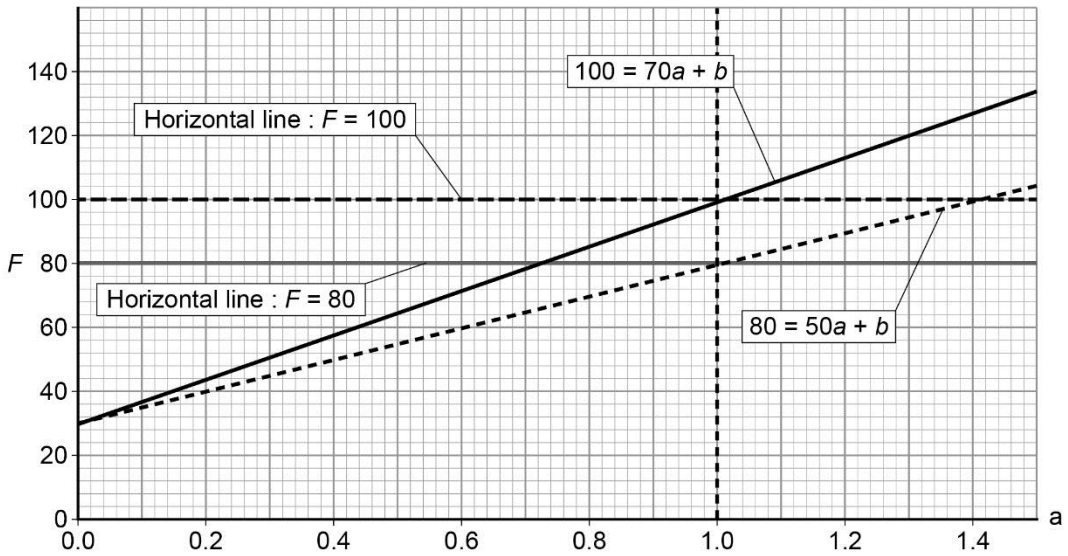
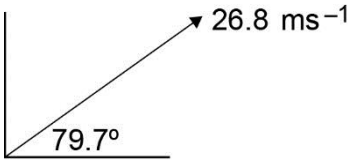
Question	Guidance	Mark
01.1	**Determine the area of one plate in both m^2 and mm^2** **Area of trapezium in m^2** $A = \left(\frac{0.55 + 1.15}{2}\right) 0.75 = 637.5 \times 10^{-3} m^2$ **Area of trapezium in mm^2** $A = \left(\frac{550 + 1150}{2}\right) 750 = 637.5 \times 10^{3} mm^2$ **1 mark** using for the correct values in m. **1 mark** for the correct answer in m^2 (or other suitable method). **1 mark** for using the correct values in mm. **1 mark** for the correct answer in mm^2 **Maximum 4 marks**	4

01.2	**Each plate was initially cut from a rectangular sheet of dimensions 1150 mm × 750 mm.** **Calculate the percentage of waste material for each finished plate in terms of its area.** **Answer to the nearest whole percentage.** $\tan(90° - 62°) \times 750 \text{ mm} = 398.7820\ldots \text{ mm}$ $1150 \text{ mm} - 398.7820\ldots \text{ mm} - 550 \text{ mm} = 201.2179\ldots \text{ mm}$ **Area of triangle 1** $\frac{1}{2} \times 398.7820\ldots \times 750 = 149.5432\ldots \times 10^3 \text{ mm}^2$ **Area of triangle 2** $\frac{1}{2} \times 201.2179\ldots \times 750 = 75.4567\ldots \times 10^3 \text{ mm}^2$ **Total area of rectangle** $1150 \times 750 = 862.5 \times 10^3 \text{ mm}^2$ **Total area of trapezium** $A_t = 862.5 \times 10^3 \text{ mm}^2 - 149.5432\ldots \times 10^3 \text{ mm}^2 - 75.4567\ldots \times 10^3 \text{ mm}^2$ $= 637.5 \times 10^3 \text{ mm}^2$ **Percentage waste material** $\left(1 - \left(\frac{637.5 \times 10^3 \text{ mm}^2}{862.5 \times 10^3 \text{ mm}^2}\right)\right) \times 100\% = 26\%$ Or other suitable methods will do. **1 mark** each for determining the lengths of each triangle's baseline. **1 mark** each for determining each triangles area. **1 mark** for the calculation of the rectangles area. **1 mark** for calculation of the percentage of waste material. **1 mark** for nearest whole number. **Maximum 7 marks**	7

Total marks for Question 1	**11**

Question	Guidance	Mark
02.1	**Determine the values for *a* and *b* when: *F* = 100, when *L* = 70 and *F* = 80 when *L* = 50** $100 = 70a + b$ (1) and $80 = 50a + b$ (2) From equation (2) we have: $b = 80 - 50a$ (3) Now, substitute equation (3) into equation (1) $100 = 70a + 80 - 50a \therefore 20 = 20a$ and $a = 1$ Now, from equation (2) substitute in for *a* $80 = (50)(1) + b \therefore b = 30$ **2 marks** for setting up both simultaneous equations. **1 mark** for writing one of the constants in terms of the other. **1 mark** for substituting into one of the original equations. **1 mark** for finding the value of one of the constants. **1 mark** for substituting this value into one of the original equations. **1 mark** for finding the last value. **Maximum 7 marks**	7
02.2	**Confirm your values for *a* and *b* by plotting a graph on Figure 2 where a and b are values to be determined.**  **1 mark** for the line's start point – the vertical intercept at 30 **1 mark** for each equation and recognising these are straight lines. **1 mark** for each horizontal line at 100 and 80 respectively. **1 mark** for the vertical line at $a = 100$ **Maximum 6 marks**	6
	Total marks for Question 2	**13**

Question	Guidance	Mark
03	**Determine the resultant velocity vector for the new autonomous robot by finding the resultant sum of the three velocity vectors of the old robots. Draw a diagram of the resultant vector.** $\rightarrow \quad 17\cos25° + 22\cos90° - 11\cos15° = 4.7820\ldots \text{m s}^{-1}$ $\uparrow \quad 17\sin25° + 22\sin90° - 11\sin15° = 26.3375\ldots \text{m s}^{-1}$ Magnitude $= \sqrt{(\rightarrow^2 + \uparrow^2)} = 26.8 \text{ m s}^{-1}$ $\theta = \tan^{-1}\left(\frac{\uparrow}{\rightarrow}\right) = 79.7°$ to the horizontal  **1 mark** for recognising cosine ratio on the horizontal component. **1 mark** for the correct horizontal component value. **1 mark** for recognising the sine ratio on the vertical component. **1 mark** for the correct vertical component value. **1 mark** for the correct method for calculating the magnitude. **1 mark** for the correct value of the magnitude. **1 mark** for the correct method of calculating the angle. **1 mark** for the correct value of the angle. **2 marks** for drawing the correct resultant vector. **Maximum 10 marks**	10
	Total marks for Question 3	**10**

<table>
<tr><th>Question</th><th>Guidance</th><th>Mark</th></tr>
<tr><td>04.1</td><td>Determine the mean length of the sample.

The mean length of the sample can be determined by

$\bar{L} = \frac{\sum L}{n} = \frac{12.5+12.2+12.7+12.9+11.9+12.5+12.6+12.5+11.9+12.8+11.9+12.0+12.7+12.7+12.5+12.6+12.4+11.9+12.0+12.1}{20}$

Therefore, we have $\bar{L} = \frac{247.3}{20} = 12.4$ mm

1 mark for using the correct method.
1 mark for using the correct values.
1 mark for the correct answer.
Maximum 3 marks</td><td>3</td></tr>
<tr><td>04.2</td><td>Determine the median length of the sample.

The median of the sample can be calculated.

<table><tr><td>11.9</td><td>11.9</td><td>11.9</td><td>11.9</td><td>12.0</td><td>12.0</td><td>12.1</td><td>12.2</td><td>12.4</td><td>12.5</td></tr><tr><td>12.5</td><td>12.5</td><td>12.5</td><td>12.6</td><td>12.6</td><td>12.7</td><td>12.7</td><td>12.7</td><td>12.8</td><td>12.9</td></tr></table>
Therefore, the median value is calculated by $\frac{12.5+12.5}{2} = 12.5$

1 mark for the correct value.
Maximum 1 mark</td><td>1</td></tr>
</table>

04.3	**Determine the standard deviation of the sample.** The standard deviation of the sample can be found by using $\sigma = \sqrt{\frac{\sum(L-\bar{L})^2}{n}}$ Therefore, we have $\frac{\sum(L-\bar{L})^2}{20} =$ $((12.5-12.365)^2 + (12.2-12.365)^2 + (12.7-12.365)^2 + (12.9-12.365)^2 + (11.9-12.365)^2 + (12.5-12.365)^2$ $+ (12.6-12.365)^2 + (12.5-12.365)^2 + (11.9-12.365)^2 + (12.8-12.365)^2 + (11.9-12.365)^2$ $+ (12.0-12.365)^2 + (12.7-12.365)^2 + (12.7-12.365)^2 + (12.5-12.365)^2 + (12.6-12.365)^2$ $+ (12.4-12.365)^2 + (11.9-12.365)^2 + (12.0-12.365)^2 + (12.1-12.365)^2)/\ 20 = 0.111275$ $\sigma = \sqrt{\text{variance}} = \sqrt{0.111275} = 0.334$ mm **1 mark** for the correct formula. **1 mark** for correct use of mean. **1 mark** for correct values in the formula. **1 mark** for the correct answer. **Maximum 4 marks** **Allow** follow-through from Question **04.1**.	4

Total marks for Question 4	**8**

Question	Guidance	Mark
05	**Calculate the value of the definite integral.** $\int_0^{\pi/2} -3\sin(2\theta).d\theta$ $-3\sin(2\theta) = \left[\frac{3}{2}\cos(2\theta)\right]_0^{\pi/2}$ $\int_0^{\pi/2} = \left(\frac{3}{2}\cos\left(2\times\frac{\pi}{2}\right)\right) - \left(\frac{3}{2}\cos(2\times 0)\right)$ $= -\frac{3}{2} - \frac{3}{2} = -3 \text{ square units.}$ **1 mark** for use of square brackets. **1 mark** for "–" × "–" becoming "+". **1 mark** for cos. **1 mark** $\frac{3}{2}$ **1 mark** for subtracting lower limit from higher limit. **1 mark** for first part of the sum. **1 mark** for the final answer. **1 mark** for correct units. **Maximum 8 marks**	8
	Total marks for Question 5	**8**

Question	Guidance	Mark
06	**Calculate the three angles in degrees of the triangle.** **Give your answer to 1 decimal place.** Using any form of the cosine rule to determine any angle: $b^2 = a^2 + c^2 - 2ac$ cosB Transpose to determine angle B: $\text{cosB} = \frac{a^2 + c^2 - b^2}{2ac} = \frac{6^2 + 8^2 - 12^2}{2 \times 6 \times 8} = -0.4583\ldots$ Therefore $\text{B} = 117.2796\ldots°$ Either of the cosine or sine rules can now be used, allow marks for either method. This solution uses the sine rule as an exemplar: $\frac{a}{\text{sinA}} = \frac{b}{\text{sinB}} \therefore \text{sinA} = \frac{a\ \text{sinB}}{b} = \frac{8\ \text{sin}117.2796\ldots°}{12} = 0.4443\ldots°$ Now $\text{A} = 36.3360\ldots°$ The final angle can be found using: $\text{C} = 180° - \text{A} - \text{B} = 180° - 36.3360\ldots° - 117.2796\ldots° = 26.3843\ldots°$ Therefore, we have: $\text{A} = 36.3°$ $\text{B} = 117.3°$ $\text{C} = 26.4°$ All to 1 dp as required. **1 mark** use of cosine rule. **3 marks** for the transposition. **1 mark** for the correct angle value. **1 mark** for use of the sine rule / cosine rule (depending on the student preference). **1 mark** for correct transposition. **1 mark** for correct solution. **1 mark** for the final angle value. **1 mark** for correct decimal places. **Maximum 10 marks**	10

Total marks for Question 6	**10**

Question	Guidance	Mark
07.1	**Determine the value for of the cooling constant *k*.** $T_2 = T_0 + (T_1 - T_0)e^{-k\,t}$ $T_2 - T_0 = (T_1 - T_0)e^{-k\,t}$ $e^{-k\,t} = \frac{T_2 - T_0}{T_1 - T_0}$ Now, apply ln to both sides: $-k\,t = ln\left(\frac{T_2 - T_0}{T_1 - T_0}\right)$ Then: $k = -\frac{1}{t}\,ln\left(\frac{T_2 - T_0}{T_1 - T_0}\right)$ Put in the values: $k = -\frac{1}{10}\,ln\left(\frac{329.7 - 280.0}{1280.0 - 280.0}\right) = 0.300\ s^{-1}$ **1 mark** for each step in the transposition – total **5 marks**. **1 mark** for use of the correct values. **1 mark** for the correct answer. **Maximum 7 marks**	7
07.2	**The bar has initial temperature of 1280 K and it is immersed in chilled water at a temperature of 280.0 K.** **Find the expected temperature of the bar after 5 seconds. Use the formula $T_2 = T_0 + (T_1 - T_0)e^{-k\,t}$** Input the known values: $T_2 = 280.0 + (1280.0 - 280.0)e^{-0.3\times5}$ $T_2 = 280.0 + 223.1301\ldots$ $T_2 = 503\ K$ **1 mark** for correct use of values. **1 mark** for correct answer. **1 mark** for correctly answering to the nearest whole number. **Maximum 3 marks**	3
	Total marks for Question 7	**10**

07.2	**The bar has initial temperature of 128.0 K and it is immersed in chilled water at a temperature of 280.0 K.** **Find the expected temperature of the bar after 5 seconds. Use the formula** $\mathbf{T_2 = T_0 + (T_1 - T_0)e^{-k\,t}}$ Input the known values: $T_2 = 280.0 + (128.0 - 280.0)e^{-0.3\times5}$ $T_2 = 280.0 - 33.9157\ldots$ $T_2 = 246\ \text{K}$ **1 mark** for correct use of values. **1 mark** for correct answer. **1 mark** for correctly answering to the nearest whole number. **Maximum 3 marks** **Accept either solution due to the misprint / typo on the exam paper.**	3

Question	Guidance	Mark
08.1	**Calculate the change in potential energy of the truck after this climb.** Slope $= 10\% = \frac{10}{100}$ Therefore, we have: $\theta = \tan^{-1}\left(\frac{10}{100}\right) = 5.7105\ldots°$ To find the height (h): $\sin\theta = \frac{\text{opp } (h)}{\text{hyp}} \therefore h = \sin\theta \times 125 = 12.4379\ldots\text{m}$ The potential energy is calculated by: $\text{PE} = \text{mgh} = 40\,000 \times 9.81 \times 12.4379 \ldots = 4.8806\ldots\text{Mj}$ Better as: 4.88 Mj to 3 sig. fig. **1 mark** for recognition of a 10% slope. **1 mark** for recognising the tangent ratio. **1 mark** for correct angle calculation. **1 mark** for use of the sine ratio. **1 mark** for the correct transposition. **1 mark** for the correct height. **1 mark** for the correct value of the potential energy. **Maximum 7 marks**	7
08.2	**If the truck was allowed to free-wheel back down the slope from rest what would be its velocity?** **Assume there is no resistance to the motion.** $\text{KE} = {}^1/_2\,\text{mv}^2 \therefore v = \sqrt{\frac{2\,\text{PE}}{\text{m}}} = \sqrt{\frac{2 \times 4.8806\ldots}{40000}}\,15.6215\ldots\text{m s}^{-1}$ Better as: $v = 15.6\ \text{m s}^{-1}$ The student could use the conservation of energy formula and calculate the value. Allow this method also. $\text{mgh} = {}^1/_2\,\text{mv}^2 \therefore v = \sqrt{2\text{gh}} = \sqrt{2 \times 9.81 \times 12.4379\ldots} = 15.6215\ldots\text{m s}^{-1}$ Better as: $v = 15.6\ \text{m s}^{-1}$ **1 mark** for the transposition. **1 mark** for the values. **1 mark** for the final answer to 3 sig. fig. **Maximum 3 marks**	3

Total marks for Question 8	**10**

Assessment outcomes coverage

Assessment Outcomes	Marks and % of marks available in section A	Marks and % of marks available in section B	Total Marks
AO1:	0 marks 0%	30 marks 100%	30 marks
AO2:	11 Marks 22%	0 marks 0%	11 marks
AO3:	13 Marks 26%	0 marks 0%	13 marks
AO4:	10 Marks 20%	0 marks 0%	10 marks
AO5:	8 Marks 16%	0 Marks 0%	8 marks
AO6:	8 Marks 16%	0 Marks 0%	8 marks
Total Marks	**50**	**30**	**80**

Question	AO1	AO2	AO3	AO4	AO5	AO6
1		11				
2			13			
3				10		
4					8	
5						8
6	10					
7	10					
8	10					
Totals	**30**	**11**	**13**	**10**	**8**	**8**