AQA

LEVEL 3
TECHNICAL LEVEL ENGINEERING

Mathematics for Engineers

Mark scheme

Unit number: J/506/5953
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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

MARKING METHODS

In fairness to candidates, all examiners **must** use the same marking methods. The following advice may seem obvious, but all examiners **must** follow it as closely as possible.

1. If you have any doubt about how to allocate marks to an answer, consult your Team Leader.
2. Refer constantly to the mark scheme and standardising scripts throughout the marking period.
3. Use the full range of marks. Don't hesitate to give full marks when the answer merits them.
4. The key to good and fair marking is **consistency**.

INTRODUCTION

The information provided for each question is intended to be a guide to the kind of answers anticipated and is neither exhaustive nor prescriptive. **All appropriate responses should be given credit.**

Where literary or linguistic terms appear in the Mark Scheme, they do so generally for the sake of brevity. Knowledge of such terms, other than those given in the specification, is **not** required. However, when determining the level of response for a particular answer, examiners should take into account any instances where the candidate uses these terms effectively to aid the clarity and precision of the argument.

DESCRIPTIONS OF LEVELS OF RESPONSE

The following procedure must be adopted in marking by levels of response:

- read the answer as a whole
- work up through the descriptors to find the one which best fits
- where there is more than one mark available in a level, determine the mark from the mark range judging whether the answer is nearer to the level above or to the one below.

Since answers will rarely match a descriptor in all respects, examiners must allow good performance in some aspects to compensate for shortcomings in other respects. Consequently, the level is determined by the 'best fit' rather than requiring every element of the descriptor to be matched. Examiners should aim to use the full range of levels and marks, taking into account the standard that can reasonably be expected of candidates.

0 1 An engineering company is to fabricate 20 tool storage containers. One of the containers is shown in **Figure 1**.

1.1 Determine the volume of **one** tool storage container in both mm^3 and m^3.

[5 marks]

Volume in mm^3: $V_{cuboid} = 400mm \times 500mm \times 1300mm = 260 \times 10^6 mm^3$ or equivalent.
Volume in m^3: $V_{cuboid} = 0.4m \times 0.5m \times 1.3m = 0.260m^3$ or equivalent.
1 mark for the correct method/formula.
1 mark for the correct values in mm.
1 mark for the correct answer in mm^3.
1 mark for the correct values in m.
1 mark for the correct answer in m^3.

1.2 Determine the surface area of the steel necessary to manufacture the 20 tool storage containers. Your answer must be in standard units.

[4 marks]

Surface area of a cuboid $= 2(bh + hl + lb)$

Surface area of one cuboid = $2([0.5 \times 0.4] + [0.4 \text{ x } 1.3] + [1.3 \text{ x } 0.5]) = 2.74m^2$

Total surface area for 20 containers = $20 \times 2.74m^2 = 54.8m^2$

1 mark for the correct values.
1 mark for the correct answer of one cuboid.
1 mark for the correct units.
1 mark for the correct total surface area.

1.3 Determine the mass of the total steel requirements to one decimal place.
The mass per unit area of sheet steel = 12.2 kg m^{-2}

[2 marks]

Total mass of steel = $12.2 \text{ kg m}^{-2} \times 54.8m^2 = 668.6kg$ to one decimal place.

1 mark for the correct answer.
1 mark for the correct decimal place answer.

Allow follow through if the answers to parts 1.1 and/or 1.2 are incorrect.

11 marks in total for Question 1.

0 2 A space exploration company is testing a component for a prototype space craft. The component is fired vertically into the air where:

$$u = 25\ m\,s^{-1}$$

2.1 Determine the time *t* taken to reach 15 m above the firing position, and determine the time *t* taken to get back 15 m to the firing position.

[7 marks]

$15 = 25t - \frac{1}{2} \times 9.81t^2$ then

$15 = 25t - 4.905t^2$

$\therefore 4.905t^2 - 25t + 15 = 0$

Using the quadratic equation

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ filling in the values:

$$t = \frac{- - 25 \pm \sqrt{25^2 - (4)(4.905)(15)}}{2 \times 4.905}$$

$t = \frac{25 + 18.18516\ldots}{9.81}$ and $t = \frac{25 - 18.18516\ldots}{9.81}$

$t = 0.695$ ascent and $t = 4.40$ descent seconds to 3 significant figures

1 mark for inserting the known values into the equation.
1 mark for equating to zero.
1 mark for selection of quadratic equation.
1 mark for inserting the correct values into the quadratic equation.
1 mark for correct "+" solution.
1 mark for correct "–" solution.
1 mark for the correct answers.

7 marks in total for Question 2.1.

2.2 The company engineers needed to know when to ignite the rockets to slow the component down. From tests they got the following results:

$$2^{t+1} = 3^{2t-5}$$

Determine the value of *t*.

[6 marks]

Taking logs of both sides we have:

$(t+1)\log 2 = (2t-5)\log 3$ then

$t\log 2 + \log 2 = 2t\log 3 - 5\log 3$

$\log 2 + 5\log 3 = 2t\log 3 - t\log 2$

$2.6866 \ldots = 0.6532..t$

$\therefore t = \frac{2.6866\ldots}{0.6532\ldots} =$ 4.11 seconds to 3 significant figures

1 mark for taking logs of both sides.
1 mark for correctly removing brackets.
1 mark for collecting like terms.
1 mark for the correct enumeration of log values.
1 mark for the transposition.
1 mark for correct value of "t"

Any form of logarithm will suffice
13 marks in total for Question 2.

0 3 Two voltage phasors are shown in **Figure 2**.

Determine the value of their resultant phasor **CB**.

[6 marks]

Using the cosine rule of the type:

$a^2 = b^2 + c^2 - 2bc\cos A$

$\hat{A} = 180° - 33° = 147°$ By inserting the correct values, we have:

$a^2 = 50^2 + 110^2 - (2)(50)(110)\cos 147°$

$a^2 = 23825.37625$

$a = \sqrt{23825.37625} = 154.355\ V$

$\therefore BC = 154\ V.$

1 mark for the correct form of the cosine rule.
1 mark for the correct calculation of the angle at A.
1 mark for the correct values used in the equation.
1 mark for the double negative becoming positive.
1 mark for the correct evaluation of a^2.
1 mark for the correct evaluation of the phasor BC.
Any other suitable method.

6 marks for Question 3.

0 4 A CNC (Computer Numerical Control) programmer needs to convert from (8, 7) Cartesian coordinates into polar coordinates before a cutting operation can begin. Perform that calculation.

[4 marks]

$\theta = \tan^{-1}\left(\frac{7}{8}\right) = 41.2°$ OR 0.719 radians

$r = \sqrt{7^2 + 8^2} = 10.6$

1 mark for correct angle formula.
1 mark for the correct value.
1 mark for the correct radius formula.
1 mark for the correct value.

4 marks in total for Question 4.

0 5 A set of 20 ingots has been cast and their masses (kg) are shown in **Table 1**.

Table 1

8.0	8.6	8.2	7.5	8.0	9.0	8.5	7.6	8.2	7.8
8.3	7.1	8.1	8.3	8.7	7.8	8.7	8.5	8.4	8.5

5.1 Fill out the table below and determine the median mass of the ingots.

[2 marks]

7.1	7.5	7.6	7.8	7.8	8.0	8.0	8.1	8.2	8.2
8.3	8.3	8.4	8.5	8.5	8.5	8.6	8.7	8.7	9.0

Median value $= \frac{8.2+8.3}{2} = 8.25\text{kg}$

1 mark for the table.
1 mark for the answer.

2 marks in total for Question 5.1

5.2 Determine the mean mass of the ingots. **[3 marks]**

The mean mass can be determined by:

$$\overline{m} = \frac{\sum m}{n} = \frac{7.1+7.5+7.6+7.8+7.8+8.0+8.0+8.1+8.2+8.2+8.3+8.3+8.4+8.5+8.5+8.5+8.6+8.7+8.7+9.0}{20}$$

Therefore, we have:

$$\overline{m} = \frac{163.8}{20} = 8.19 \text{ kg}$$

1 mark for using the correct method.
1 mark for using the correct values.
1 mark for the correct answer.

3 marks in total for Question 5.2

5.3 Determine the variance of the ingots.

[3 marks]

The variance of the sample can be found by using:

$\sigma^2 = \frac{\sum(m-\overline{m})^2}{n}$ Therefore, we have:

$$\frac{\sum(m-\overline{m})^2}{20} = ((7.1-8.19)^2+(7.5-8.19)^2+(7.6-8.19)^2+(7.8-8.19)^2+(7.8-8.19)^2+(8.0-8.19)^2+(8.0-8.19)^2+(8.1-8.19)^2+(8.2-8.19)^2+(8.2-8.19)^2+(8.3-8.19)^2+(8.3-8.19)^2+(8.4-8.19)^2+(8.5-8.19)^2+(8.5-8.19)^2+(8.5-8.19)^2+(8.6-8.19)^2+(8.7-8.19)^2+(8.7-8.19)^2+(9.0-8.19)^2)\ /\ 20 = 0.205.$$

1 mark for correct use of mean.
1 mark for correct values in the formula.
1 mark for the correct answer.

3 marks in total for Question 5.3.

Allow follow-through from part 5.1 and part 5.2.

8 marks in total for Question 5.

0 6 A space craft has a position function: $s = -6\cos(2t) + 5t^3$

6.1 By using the process of differentiation, determine a function for the space craft's acceleration.

[6 marks]

$$\frac{ds}{dt} = 12sin(2t) + 15t^2$$

$$\frac{d^2s}{dt^2} = 24cos(2t) + 30t$$

1 mark for the $--$ becoming $+$.

1 mark for $\frac{ds}{dt}$ and $\frac{d^2s}{dt^2}$

1 mark for $12sin(2t)$
1 mark for $15t^2$
1 mark for $24cos(2t)$
1 mark for $30t$

6 marks in total for Question 6.1

6.2 Calculate the acceleration when $t = 5$ seconds.

$$\frac{d^2s}{dt^2} = 24cos(2t) + 30t$$

$$\frac{d^2s}{dt^2} = 24cos(2 \times 5) + 30 \times 5 = 130\ m\ s^{-2}.$$

[2 marks]

1 mark for the correct values.
1 mark for the correct units.

2 marks in total for Question 6.2

8 marks in total for Question 6.

0 7 The velocity of a satellite is given by the function

$V = 3t^3 + 6e^{4t}$ m s^{-1}, where t is the time in seconds.

[10 marks]

By using the process of integration, determine the distance travelled by the satellite in the first 3 seconds

$\left[\frac{3t^4}{4} + \frac{3e^{4t}}{2}\right]_0^3$ Inserting the values, we have:

$$\left[\frac{3\times3^4}{4} + \frac{3e^{4\times3}}{2}\right] - \left[\frac{3\times0^4}{4} + \frac{3e^{4\times0}}{2}\right]$$

$$[60.75 + 244.132 \times 10^3] - \left[\frac{3}{2}\right]$$

Distance travelled = 244 km to the nearest kilometre.

1 mark for each of the integrals **(2 marks in total)**.
1 mark for inserting the correct limits **(4 marks in total)**.
1 mark for the correct evaluation of each sum **(3 marks in total)**.
1 mark for the correct answer.

10 marks in total for Question 7.

0 8 The decay voltage v across a capacitor at time t seconds is given by:

$v = 250e^{-t/3}$

8.1 Complete the cells in **Table 2**.

[4 marks]

Table 2

t	0	1	2	3	4	5	6
$e^{-t/3}$	1	0.716	0.513	0.368	0.264	0.189	0.135
$v = 250e^{-t/3}$	250	179	128	92.0	65.9	47.2	33.8

Marking example 1:
$e^{-t/3} = e^{-0/3} = 1$

Marking example 2:
$v = 250e^{-0/3} = 250\ v$

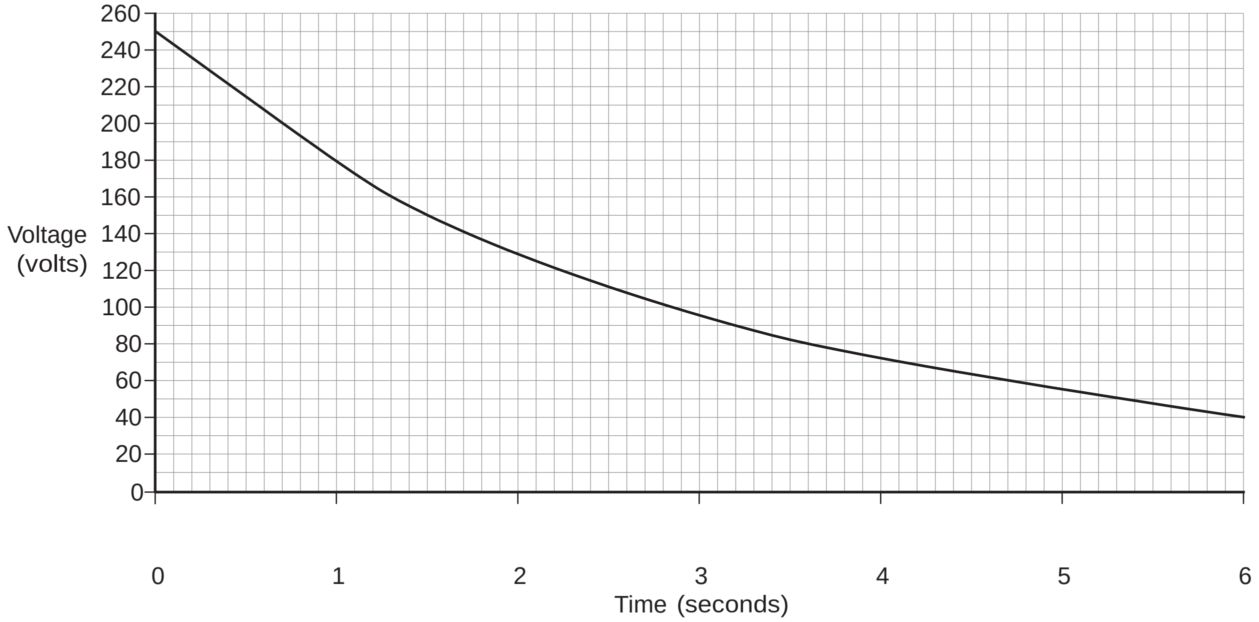
4 marks for a similar complete table **(2 marks** per row**)**.
0 values in each row = **0 mark**
1–4 correct values in each row = **1 mark**
5 or more correct values in each row = **2 marks**.

4 marks in total for Question 8.1.

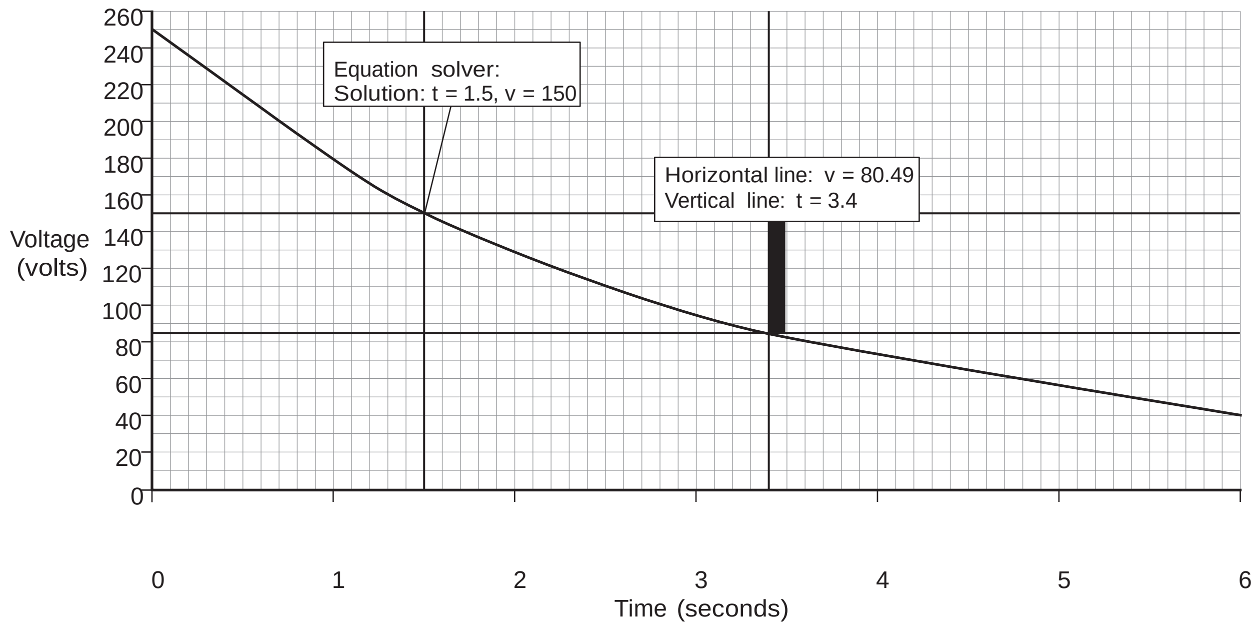
8.2 Using your values in **Table 2**, plot the decay voltage against time on **Graph 1**.

[4 marks]

Graph 1



4 marks for a graph similar to the one above. For full marks a smooth, decaying curve must be shown.



4 marks in total for Question 8.2.

8.3 From **Graph 1**, determine the time when $v = 150\ V$ and when $v = 80\ V$.

[2 marks]

$v = 150\ V$ when $t \approx 1.5$ seconds.

$v = 80\ V$ when $t \approx 3.4$ seconds

2 marks in total for Question 8.3

10 marks in total for Question 8.

0 9 **Figure 3** shows the gable end of a house.

9.1 Determine the area of the gable end. Area of the rectangle $= 8\text{ m} \times 7\text{ m} = 56\text{ m}^2$.

Using Pythagoras, we have:
$5^2 = 4^2 + h^2 \therefore h^2 = 5^2 - 4^2 = 9\text{ m}^2$

Thus: $h = 3\text{ m}$.

Area of the triangle: **[8 marks]**

Area $= \frac{1}{2} \times b \times h = \frac{8 \times 3}{2} = 12\text{ m}^2$

Total area:
Area $= 56\text{m}^2 + 12\text{ m}^2 = 68\text{ m}^2$

1 mark for the rectangular area formula.
1 mark for the correct answer.
1 mark for using Pythagoras.
1 mark for the correct transposition.
1 mark for the correct value for *h*.

NB: allow 3 marks if the candidate recognises that this is a 3–4–5 triangle and determines the correct value for *h* that way.
1 mark for the correct values in triangular area formula.
1 mark for the correct value of the area.
1 mark for the correct total area.

8 marks in total for Question 9.1.

9.2 The gable end requires painting. If 1.45 litres of paint cover 1 m^2 calculate the number of litres of paint required for complete coverage of the gable end. Answer to the nearest litre.
Total volume of paint required: $68 \times 1.45 = 98.6\ (99\text{ litres})$ **[2 marks]**

1 mark for correct formula.
1 mark for the correct answer to the nearest litre.

2 marks in total for Question 9.2.

10 marks in total for Question 9.

Assessment Outcomes coverage

Assessment Outcomes	Marks and % of marks available in section A	Marks and % of marks available in section B	Total marks
AO1	0 marks 0%	30 marks 100%	30 marks
AO2	11 marks 22%	0 marks 0%	11 marks
AO3	13 marks 26%	0 marks 0%	13 marks
AO4	10 marks 20%	0 marks 0%	10 marks
AO5	8 marks 16%	0 marks 0%	8 marks
AO6	8 marks 16%	0 marks 0%	8 marks
Total marks	50	30	80

Question	AO1	AO2	AO3	AO4	AO5	AO6
1		11				
2			13			
3				6		
4				4		
5					8	
6						8
7	10					
8	10					
9	10					
Totals	**30**	**11**	**13**	**10**	**8**	**8**