AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

Level 3 Technical Level
IT: PROGRAMMING

Unit 5 Mathematics for programmers

Tuesday 12 June 2018 Afternoon Time allowed: 2 hours

Materials
For this paper you must have:
- a ruler
- scientific calculator (non-programmable)
- stencils or other drawing equipment (eg flowchart stencils)
- formula sheet.

Instructions
- Use black ink or black ball-point pen.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.
- If you need more space use the additional pages at the back of this booklet.
- Include units in all answers, where required, as marks are given for units in some questions.

Information
- The marks for questions are shown in brackets.
- There are 50 marks in **Section A** and 30 marks in **Section B**. Both sections should be attempted.

Advice
- In all calculations, show clearly how you work out your answer.
- Use diagrams, where appropriate, to clarify your answers.
- You are expected to use a calculator where appropriate.
- You are reminded of the need for good English and clear presentation in your answers.

For Examiner's Use	
Question	Mark
1–5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
TOTAL	

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Section A

Answer **all** questions in this section.

In multiple choice questions only **one** answer per question is allowed.

For each answer completely fill in the circle alongside the appropriate answer.

CORRECT METHOD 

WRONG METHODS 

If you want to change your answer you must cross out your original answer as shown. 

If you wish to return to an answer previously crossed out, ring the answer you now wish to select as shown. 

You may do your working in the blank space around each question but this will not be marked. Do **not** use additional sheets for this working.

0 1 What is the minimum number of binary bits required to represent the decimal number 35?

[1 mark]

A 4 ⬭

B 5 ⬭

C 6 ⬭

D 7 ⬭

0 2 Which **one** of the following hexadecimal numbers is equivalent to the binary number 010111000111?

[1 mark]

A 5C7 ⬭

B 2707 ⬭

C A3E ⬭

D 7C5 ⬭

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0 3 If the probability of an event happening is 0.2, what is the probability of it **not** happening?

[1 mark]

A 99.8 ⬭

B 0.8 ⬭

C 9.8 ⬭

D 0 ⬭

0 4 What two-input (A, B) and one-output (P) logic operator does the truth table in **Table 1** represent?

Table 1

A	B	P
0	0	0
0	1	0
1	0	0
1	1	1

[1 mark]

A OR ⬭

B AND ⬭

C XOR ⬭

D NOT ⬭

Do not write outside the box

0 5 Which **one** of the infix expressions below does the postfix expression 8 2 * 5 + represent?

[1 mark]

A 8 * 2 + 5 ⬭

B 8 + 2 * 5 ⬭

C 5 * 8 + 2 ⬭

D 5 + 2 * 8 ⬭

5

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0 6 . 1 Perform the following subtraction operation on two signed 8-bit binary numbers:

00000111 – 00001010

[1 mark]

0 6 . 2 Convert the following signed binary number into two's complement form:

10001000

[1 mark]

0 6 . 3 Explain the effect of the conversion in Question **06.2** on the signed number.

[1 mark]

3

Turn over for the next question

Do not write outside the box

0 7 There are three coloured balls in a bag: one blue, one red and one green.

0 7 . 1 What is the probability of taking out a red ball?

[1 mark]

0 7 . 2 What is the probability of taking out a green ball followed by a red ball without replacing the green ball?

[2 marks]

3

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0 8 **Table 2** represents a sorted list of numbers in an array.

Table 2

0	4	7	10	14	23	45	47	53

A binary search is used to search for number 47.

Describe the first three steps of this search.

[3 marks]

3

Turn over for the next question

Do not write outside the box

0 9 Let set N = {1, 2, 3, 4, 5, 6}, set A = {2, 3} and set B = {3, 4, 5}

Using Venn diagrams, demonstrate that (A U B)' = A' ∩ B' where:

- U is the set union operator and
- ∩ is the set intersection operator.

Clearly identify the sets in your diagrams.

[4 marks]

4

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1 0 In the Linux operating system, file access permissions are represented in three groups of 3-bit values as shown in **Table 3**. The groups are identified as **A**, **B** and **C** from left to right.

Table 3

A	B	C
rwx – rwx – rwx		

- if r=1 the file can be read
- if w=1 the file can be written to
- if x=1 the file can be executed,

otherwise they are set to 0 (zero).

The Linux command 'chmod' uses this information as three octal digits to modify file access rights. A programmer wishes to set:

- group **A** as read, write and execute
- group **B** as read and execute
- group **C** as read only.

1 0 . 1 Work out the bit values for **A**, **B** and **C**.

[1 mark]

1 0 . 2 What are the three octal digits that make up the chmod value?

[1 mark]

2

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1 1 . 1 Give the unit (or identity) matrix for a '2 by 2' matrix.

[1 mark]

1 1 . 2 The square matrix $\mathbf{A} = \begin{pmatrix} 8 & 3 \\ 5 & 2 \end{pmatrix}$. If the inverse matrix, $\mathbf{A}^{-1}$, is expressed as $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then calculate the values a, b, c and d and give the inverse matrix.

[4 marks]

1 1 . 3 Prove that, in Question **11.2**, the matrix $\mathbf{A}^{-1}$ is the inverse of matrix **A**.

[1 mark]

6

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1 2 The equation below gives the relationship between the improvement, i_s, in the speed of a computer system and the number of CPUs, n, when running a program:

$$0.8i_s\left(1 + \frac{1}{n}\right) = 1$$

1 2 . 1 Express i_s as a function of n using the function notation.

[2 marks]

1 2 . 2 Calculate the ratio of improvements in the execution speeds of the program when the number of CPUs is increased from 2 to 4.

[3 marks]

5

Turn over for the next question

1 3 A half-adder is the basic logic block of a CPU's binary numbers adder hardware.

The half-adder:

- adds two binary bits, A and B
- produces a single bit sum S as output
- produces a single bit carry C as a second output.

The two logic equations are shown below:

$S = \overline{A} \,.\, B + A \,.\, \overline{B}$
$C = A \,.\, B$

Here, $\overline{A}$ = NOT A and $\overline{B}$ = NOT B

1 3 . 1 Using the above equations, complete the truth table, **Table 4**.

[2 marks]

Table 4

A	B	S	C
0	0		
0	1		
1	0		
1	1		

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1 3 . 2 Using the truth table in **Table 4**, draw the corresponding logic circuit.

[2 marks]

1 3 . 3 Using De Morgan's law, prove that the complement of the logic equation

$$P = A \,.\, B + A \,.\, C \text{ is } \overline{P} = \overline{A} + \overline{B} \,.\, \overline{C}$$

[3 marks]

7

Turn over ▶

Do not write outside the box

1 4 . 1 What is the result of the following series?

$$R = \sum_{i=0}^{i=6} 2^i$$

Show how you calculated your answer.

[1 mark]

1 4 . 2 Consider the following first six numbers of a sequence:

−1, 0, 3, 8, 15, 24

Deduce the *n*th term where *n* = 0 refers to the first term.

[1 mark]

1 4 . 3 Calculate the value of the 10th number in the sequence from Question **14.2**.

[1 mark]

1 4 . 4 Explain the need for a 'base case' in a recursive function.

[2 marks]

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1 4 . 5 A recursive function S(N), representing the sum of every number from 1 to N, is defined below:

S(0) = 0
For all values of N > 0, S(N) = S(N–1) + N

Explain what makes this a recursive function.

[2 marks]

7

Turn over for the next question

1 5 The performance of a CPU can be measured by the following formula:

$$T = \frac{I * CPI}{S}$$

Where:

T is the time it takes to run
I is the number of instructions executed
CPI is the average number of clock cycles per instruction
S is the speed of the CPU.

1 5 . 1 Looking at the above formula, state the relationship between T and CPI.

[1 mark]

1 5 . 2 Looking at the above formula, state the relationship between T and *S*.

[1 mark]

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In an experiment, the performance of two CPUs, **A** and **B**, is compared. **Table 5** gives the relevant values used. Both CPUs run the same number of instructions, ie the same software program.

Table 5

CPU	*S*	CPI
A	3	4
B	2	2

1 5 . 3 Determine which CPU, **A** or **B**, runs the instructions faster.
Use the information in **Table 5** and the performance formula on page 16.

[2 marks]

1 5 . 4 Calculate how much more quickly the faster CPU runs the instructions than the slower CPU. Give your answer as a percentage.

[1 mark]

5

Turn over for the next question

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Section B

Answer **all** questions in this section.

1 6 The designer of a CPU decides that each instruction word has the following parts:

x bits	y bits	z bits
Operation Code	**Operand 1**	**Operand 2**

x, **y** and **z** are the lengths of each individual part in number of binary bits.

Operation Code defines the instruction, eg add numbers.

Operand 1 and **Operand 2** contain data on which the **Operation Code** will work, eg the two numbers to add up.

This CPU has 28 unique operation codes, numbered 0–27. The largest number **Operand 1** can contain is 255 and the largest number **Operand 2** can contain is 7.

1 6 . 1 Complete **Table 6** giving the number of bits required to store:

Table 6

	Number of bits
an Operation Code up to a decimal value of 28 (ie the value of x)	
the largest number in Operand 1 (ie the value of y)	
the largest number in Operand 2 (ie the value of z)	

[3 marks]

1 6 . 2 What is the length of each instruction word:

[2 marks]

in bits? _____

in bytes? _____

1 6 . 3 In one of the instructions:

- The Operation Code is 0x1C
- Operand 1 is 160
- Operand 2 is 7.

Calculate the contents of this instruction in hexadecimal form.

[3 marks]

_____

_____

_____

_____

_____

_____

1 6 . 4 The contents of another instruction are 0x50F3

Determine the values of the Operation Code, Operand 1 and Operand 2.

[3 marks]

_____

_____

_____

_____

_____

_____

Question 16 continues on the next page

1 6 . 5 The designer decides that one of the instructions will combine Operand 1 and Operand 2 to allow for a bigger value.

How many values can be accommodated by this combination:

[2 marks]

in bytes? _____

in kilobytes (KB)? _____

1 6 . 6 The designer wants to increase the number of operation codes.

How many more operation codes can he have without having to change the instruction structure?

[1 mark]

_____

_____

_____

_____

_____

_____

14

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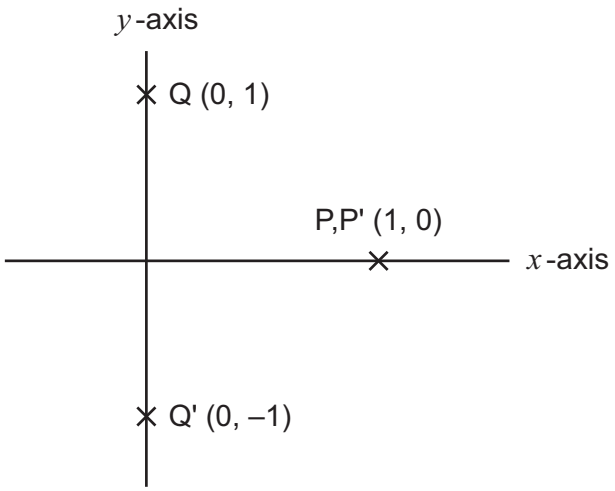
1 7 In computer graphics, operations on images such as reflection and rotation are frequently used to add realistic shapes and movements.

Consider **Figure 1**.

For reflection in the x-axis, we need to map:

point P $(1, 0)$ into point P' $(1, 0)$ and
point Q $(0, 1)$ into point Q' $(0, -1)$.

Figure 1



1 7 . 1 State the operator matrix required for reflecting any point in the x-axis.

[1 mark]

1 7 . 2 Show the use of the operator in Question **17.1** in reflecting R in the x-axis.

Assume a point R(x, y) is expressed as the matrix R = $\begin{pmatrix} x \\ y \end{pmatrix}$.

[2 marks]

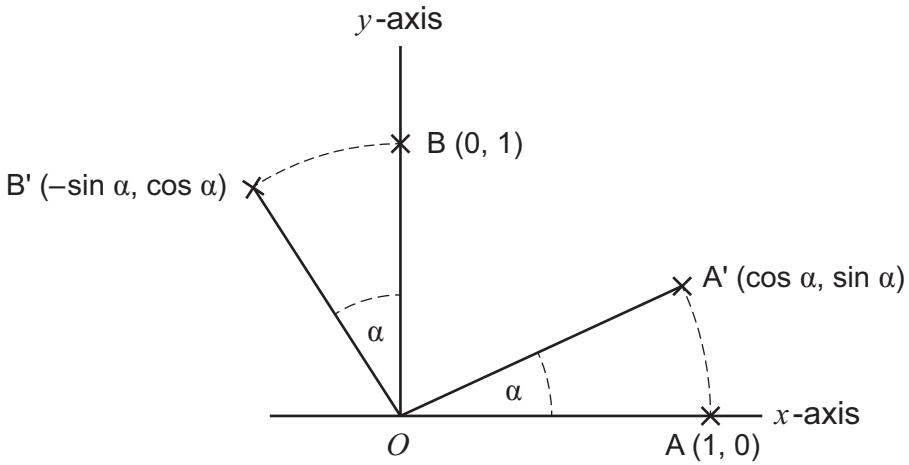
Question 17 continues on the next page

Consider **Figure 2**.

For rotation about the origin O through angle α, we need to map:

- point A $(1, 0)$ into point A' $(\cos \alpha, \sin \alpha)$ and
- point B $(0, 1)$ into point B' $(-\sin \alpha, \cos \alpha)$.

Figure 2



1 7 . 3 State the operator matrix required for rotating any point about the origin O.

[1 mark]

1 7 . 4 Show the use of the operator in Question **17.3** in rotating R about the origin O.

Assume a point R(x, y) is expressed as the matrix R = $\begin{pmatrix} x \\ y \end{pmatrix}$.

[2 marks]

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1 7 . 5 A point P is mapped into point P' by rotation through an angle α about the origin. The point P' is then mapped into point P" by reflection in the x-axis.

Determine the matrix which maps P into P".

[4 marks]

1 7 . 6 Show that P will not map into same P" if the operation is reversed, ie reflection followed by rotation.

[3 marks]

Question 17 continues on the next page

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1 7 . 7 A point P is mapped into the point P' by the shear $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$. The P' is then mapped into P" by reflection in the line $x = y$.

Determine the matrix equivalent to the double operation, ie shear followed by reflection.

[2 marks]

1 7 . 8 The coordinates of point P are $(1, 1)$. Using your answer from Question **17.7**, calculate the coordinates of point P".

[1 mark]

16

END OF QUESTIONS

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If needed, use the following pages to continue your answers. Write the question number beside your answer.

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