Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **February/March 2025**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

[Turn over

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1 Solve the equation

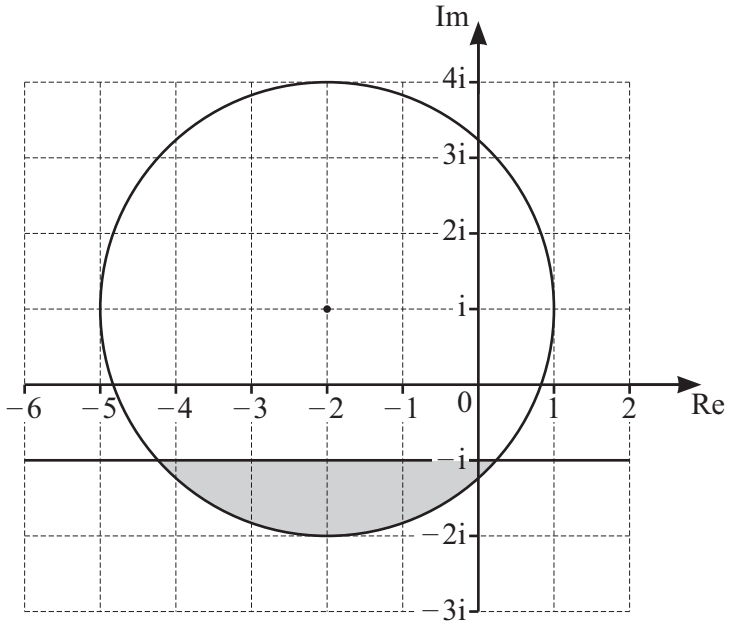
$$\ln\left(1-\mathrm{e}^{-2x}\right)+3=0.$$

Give your final answer correct to 4 decimal places. [3]

2 The equation of a curve is $xy^2 + \ln(x+2y) = 1$.

Find the gradient of the curve at the point where $x = 0$. [5]

3



The shaded region on the Argand diagram shows points representing complex numbers z defined by two inequalities. The shaded region is bounded by a circle and a line parallel to the real axis. The boundaries of the region are included in the shaded region.

(a) Find two inequalities in terms of z that define the shaded region. [3]

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(b) Find the greatest value of $|z|$ for points in this region. [3]

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4 By first expressing the equation $\tan(x-60°) = 2\cot x$ as a quadratic equation in $\tan x$, solve the equation for $0° \leqslant x \leqslant 180°$. [6]

5 The square roots of $-4+6\sqrt{5}\mathrm{i}$ can be expressed in the Cartesian form $x+\mathrm{i}y$, where x and y are real and exact.

By first forming a quartic equation in x or y, find the square roots of $-4+6\sqrt{5}\mathrm{i}$ in exact Cartesian form. [5]

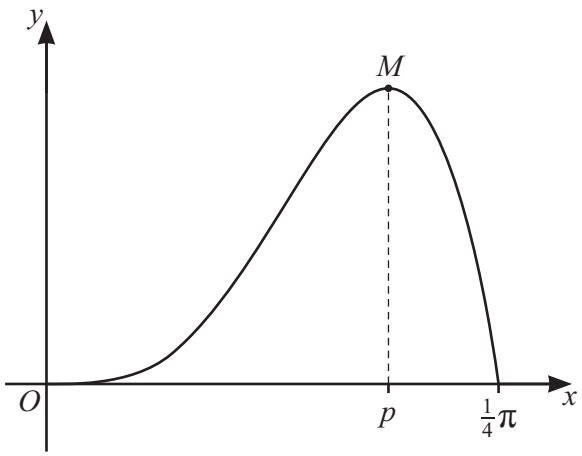
6 The variables x and θ satisfy the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \left(\frac{1}{5}x + 1\right)\sin^2 2\theta,$$

and $x = 5$ when $\theta = 0$.

Solve the differential equation and obtain an expression for x in terms of θ. [7]

7



The diagram shows the curve $y = x^3 \cos 2x$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$. The curve has a maximum point at M, where $x = p$.

(a) Show that p satisfies the equation $p = \frac{1}{2}\tan^{-1}\left(\frac{3}{2p}\right)$. [3]

(b) Show by calculation that $0.5 < p < 0.7$. [2]

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(c) Use an iterative formula based on the equation in part **(a)** to calculate p correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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8 Two lines have equations $\mathbf{r} = \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$.

(a) Show that the lines are skew. [5]

(b) Find the obtuse angle between the directions of the two lines. [3]

9 The polynomial $6x^3 + ax^2 + bx + 9$ is denoted by $\mathrm{p}(x)$, where a and b are constants. It is given that $(x-3)$ is a factor of $\mathrm{p}(x)$, and when the first derivative $\mathrm{p}'(x)$ is divided by $(x-3)$ the remainder is 72.

(a) Find the values of a and b. [5]

(b) When a and b have the values found in part **(a)**, factorise $\mathrm{p}(x)$ completely. [3]

(c) Hence solve the inequality $\mathrm{p}(x) < 0$. [2]

10 Let $f(x) = \dfrac{-7x^2 + 2x - 6}{(1+x)(4+x^2)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the exact value of $\int_0^2 \mathrm{f}(x)\,\mathrm{d}x$. Give your answer in the form $a\pi - \ln b$, where a and b are constants. [6]

11 Find the exact value of $\int_0^{\pi} x^2 \cos \frac{1}{3}x \, \mathrm{d}x$. [6]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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