1. June/2023/Paper_9709/31/No.9

The constant a is such that $\int_0^a xe^{-2x}\,dx = \frac{1}{8}$.

(a) Show that $a = \frac{1}{2}\ln(4a+2)$. [5]

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(b) Verify by calculation that a lies between 0.5 and 1. [2]

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(c) Use an iterative formula based on the equation in **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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2. June/2023/Paper_9709/32/No.6

The equation $\cot \frac{1}{2}x = 3x$ has one root in the interval $0 < x < \pi$, denoted by α.

(a) Show by calculation that α lies between 0.5 and 1. [2]

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(b) Show that, if a sequence of positive values given by the iterative formula

$$x_{n+1} = \frac{1}{3}\left(x_n + 4\tan^{-1}\left(\frac{1}{3x_n}\right)\right)$$

converges, then it converges to α. [2]

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(c) Use this iterative formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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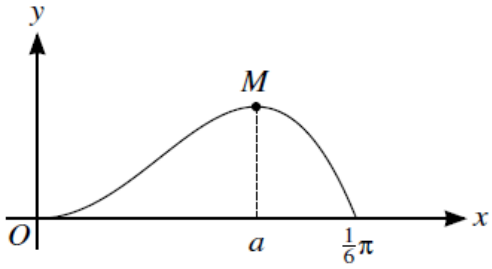
3. June/2023/Paper_9709/33/No.5



The diagram shows the part of the curve $y = x^2 \cos 3x$ for $0 \leqslant x \leqslant \frac{1}{6}\pi$, and its maximum point M, where $x = a$.

(a) Show that a satisfies the equation $a = \frac{1}{3}\tan^{-1}\left(\frac{2}{3a}\right)$. [3]

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(b) Use an iterative formula based on the equation in **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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