1. Nov/2023/Paper_9709/61/No.6

A continuous random variable X takes values from 0 to 6 only and has a probability distribution that is symmetrical.

Two values, a and b, of X are such that $P(a < X < b) = p$ and $P(b < X < 3) = \frac{13}{10}p$, where p is a positive constant.

(a) Show that $p \leqslant \frac{5}{23}$. [1]

(b) Find $P(b < X < 6 - a)$ in terms of p. [2]

It is now given that the probability density function of X is f, where

$$f(x) = \begin{cases} \frac{1}{36}(6x - x^2) & 0 \leqslant x \leqslant 6, \\ 0 & \text{otherwise.} \end{cases}$$

(c) Given that $b = 2$ and $p = \frac{5}{27}$, find the value of a. [5]

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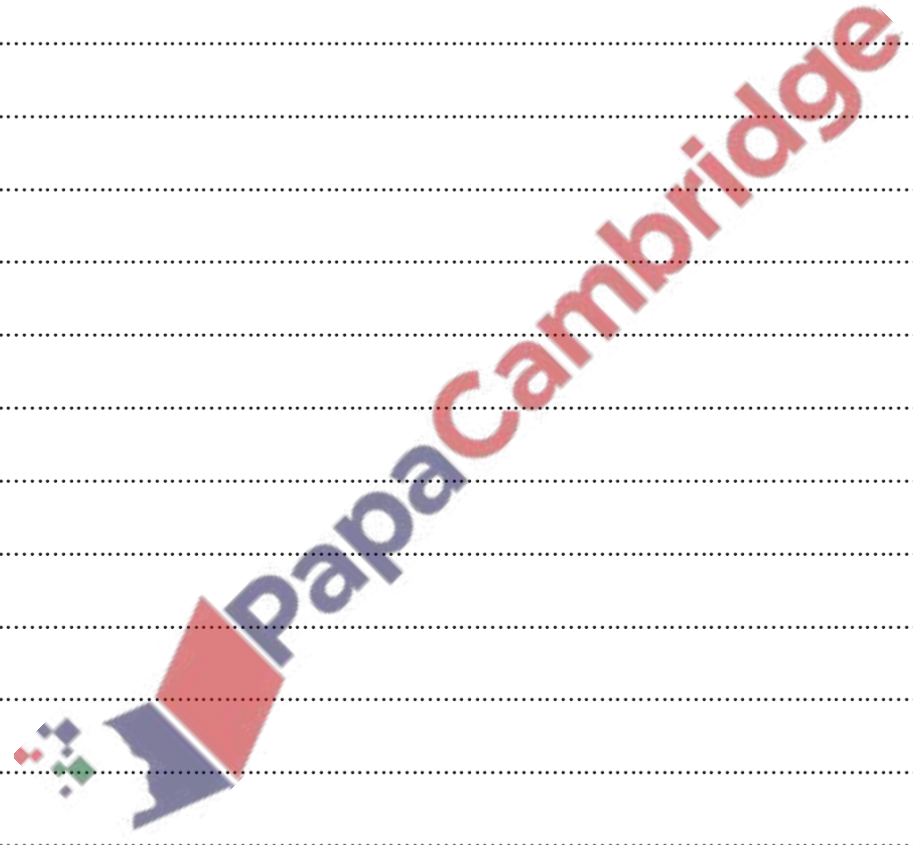...

The random variable X has probability density function, f, given by

$$f(x) = \begin{cases} \dfrac{1}{x^2} & a < x < b, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are positive constants.

(a) It is given that $E(X) = \ln 2$.

Show that $b = 2a$. [3]

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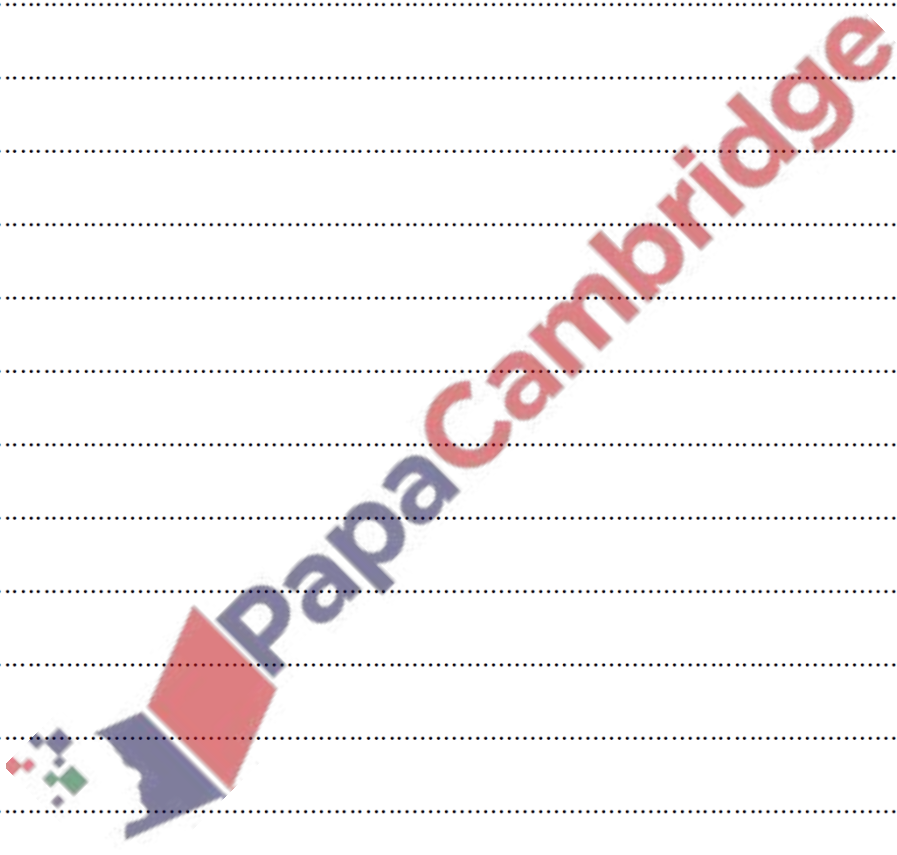...

(b) Show that $a = \frac{1}{2}$. [3]

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(c) Find the median of X. [3]

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