1. Nov/2023/Paper_9709/11/No.4

The transformation R denotes a reflection in the x-axis and the transformation T denotes a translation of $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

(a) Find the equation, $y = g(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations R followed by T. [2]

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(b) Find the equation, $y = h(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations T followed by R. [2]

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(c) State fully the transformation that maps the curve $y = g(x)$ onto the curve $y = h(x)$. [2]

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(a) Express $4x^2 - 12x + 13$ in the form $(2x + a)^2 + b$, where a and b are constants. [2]

The function f is defined by $f(x) = 4x^2 - 12x + 13$ for $p < x < q$, where p and q are constants. The function g is defined by $g(x) = 3x + 1$ for $x < 8$.

(b) Given that it is possible to form the composite function gf, find the least possible value of p and the greatest possible value of q. [3]

(c) Find an expression for gf(x). [1]

The function h is defined by h(x) = $4x^2 - 12x + 13$ for $x < 0$.

(d) Find an expression for h^{-1}(x). [3]
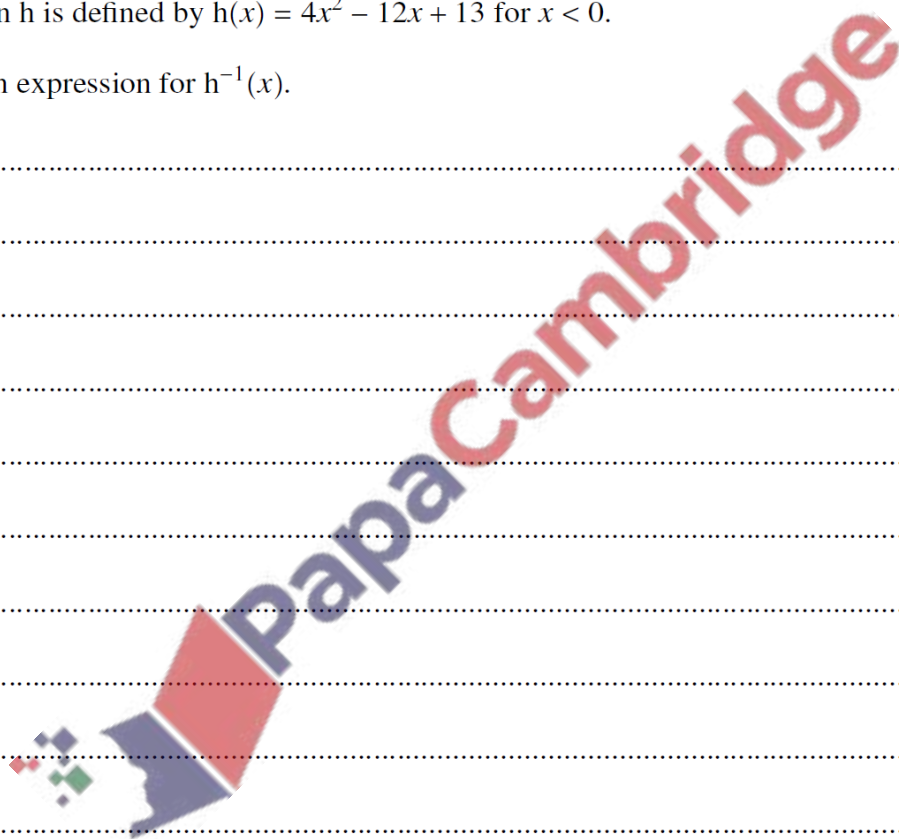
The equation of a curve is $y = x^2 - 8x + 5$.

(a) Find the coordinates of the minimum point of the curve.   [2]

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The curve is stretched by a factor of 2 parallel to the y-axis and then translated by $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$.

(b) Find the coordinates of the minimum point of the transformed curve.   [2]

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(c) Find the equation of the transformed curve. Give the answer in the form $y = ax^2 + bx + c$, where a, b and c are integers to be found. [4]

4.

Functions f and g are defined by

$$f(x) = (x + a)^2 - a \text{ for } x \leqslant -a,$$
$$g(x) = 2x - 1 \text{ for } x \in \mathbb{R},$$

where a is a positive constant.

(a) Find an expression for $f^{-1}(x)$. [3]

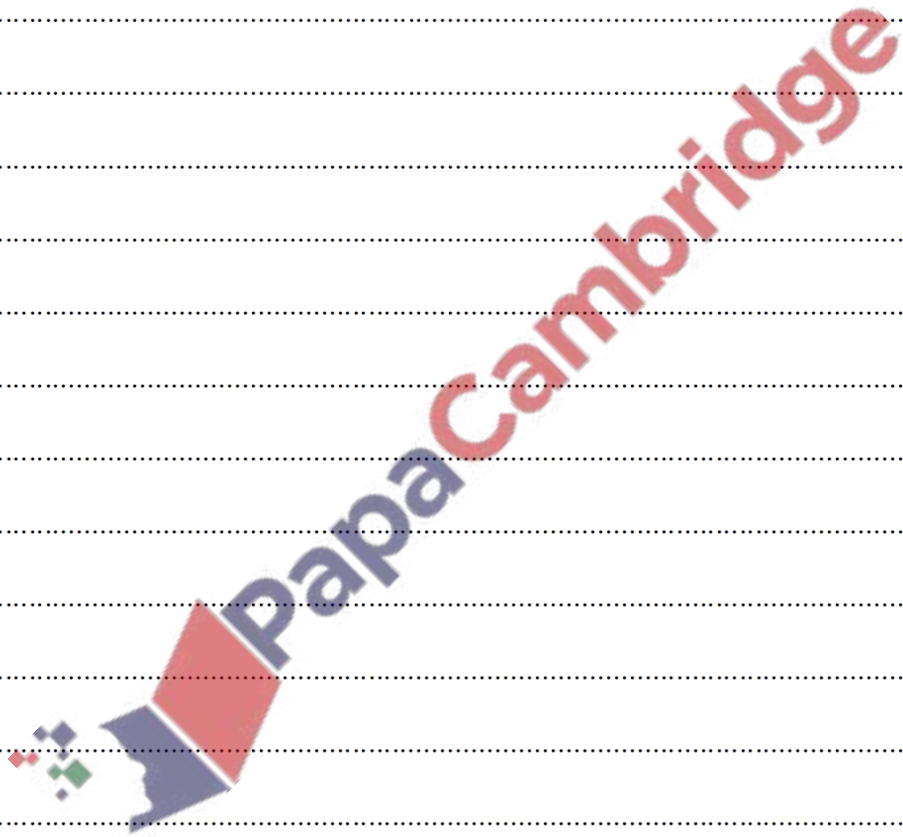..

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(b) **(i)** State the domain of the function f^{-1}. [1]

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(ii) State the range of the function f^{-1}. [1]

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(c) Given that $a = \frac{7}{2}$, solve the equation $gf(x) = 0$. [3]

The function f is defined by $f(x) = 1 + \dfrac{3}{x-2}$ for $x > 2$.

(a) State the range of f. [1]

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(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

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The function g is defined by $g(x) = 2x - 2$ for $x > 0$.

(c) Obtain a simplified expression for $gf(x)$. [2]

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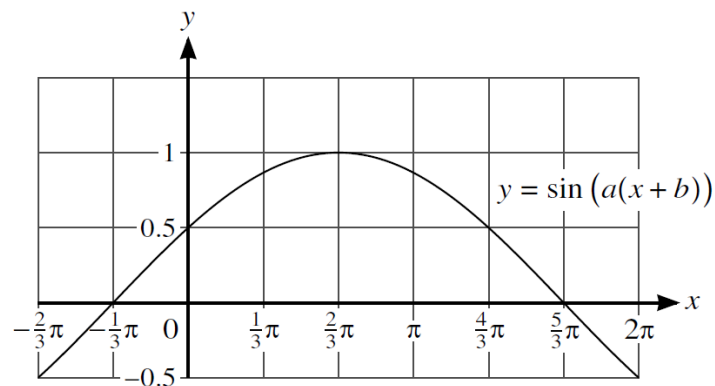..

The diagram shows part of the graph of $y = \sin(a(x + b))$, where a and b are positive constants.

(a) State the value of a and one possible value of b. [2]

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Another curve, with equation $y = f(x)$, has a single stationary point at the point (p, q), where p and q are constants. This curve is transformed to a curve with equation

$$y = -3f\left(\tfrac{1}{4}(x + 8)\right).$$

(b) For the transformed curve, find the coordinates of the stationary point, giving your answer in terms of p and q. [3]

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7.

A function f is defined by $f(x) = x^2 - 2x + 5$ for $x \in \mathbb{R}$. A sequence of transformations is applied in the following order to the graph of $y = f(x)$ to give the graph of $y = g(x)$.
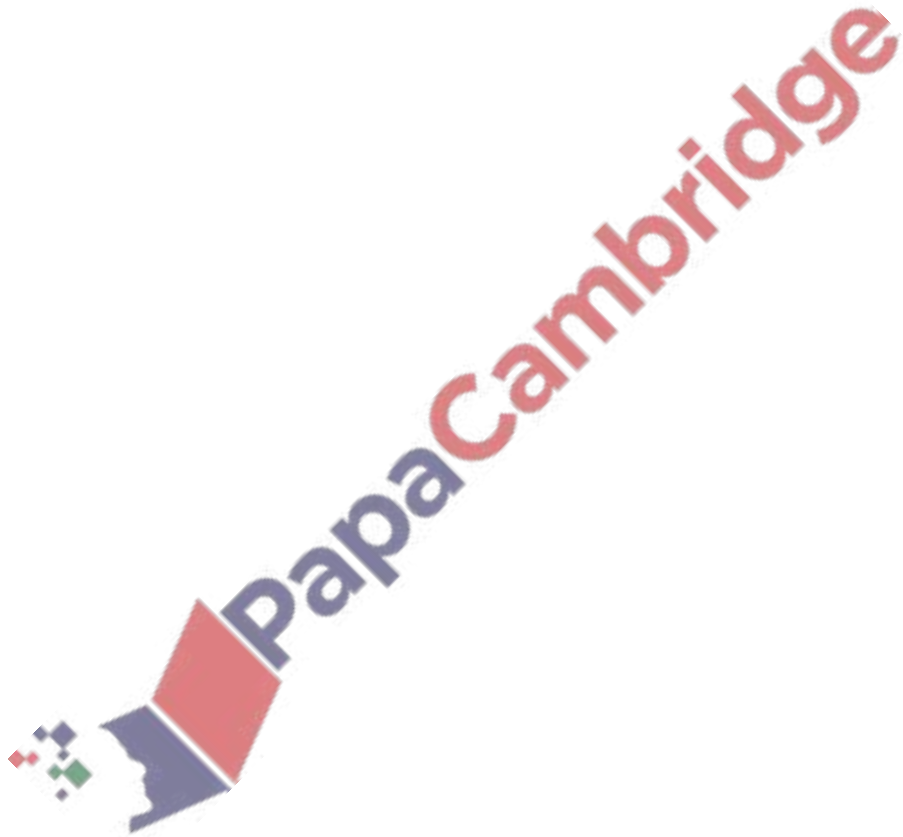
Stretch parallel to the x-axis with scale factor $\frac{1}{2}$

Reflection in the y-axis

Stretch parallel to the y-axis with scale factor 3

Find $g(x)$, giving your answer in the form $ax^2 + bx + c$, where a, b and c are constants. [4]

8.

The function f is defined by $f(x) = -3x^2 + 2$ for $x \leqslant -1$.

(a) State the range of f. [1]

(b) Find an expression for $f^{-1}(x)$. [3]

The function g is defined by $g(x) = -x^2 - 1$ for $x \leqslant -1$.

(c) Solve the equation $fg(x) - gf(x) + 8 = 0$. [5]