

1. Nov/2023/Paper_9702/41/No.4

A heavy metal sphere of mass 0.81 kg is suspended from a string. The sphere is undergoing small oscillations from side to side, as shown in Fig. 4.1.

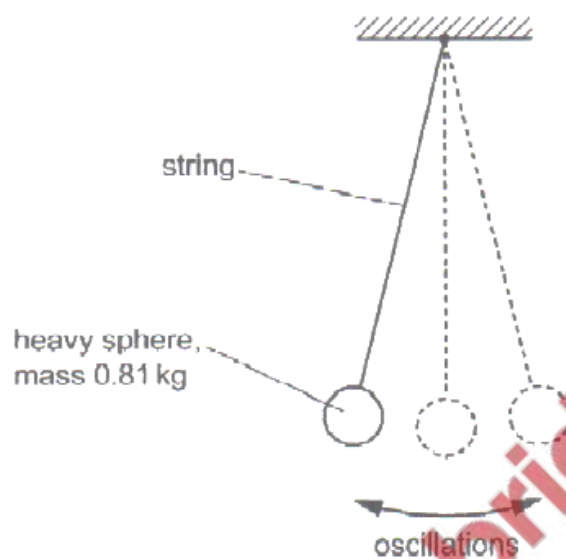


Fig. 4.1

The oscillations of the sphere may be considered to be simple harmonic with amplitude 0.036 m and period 3.0 s. $T = \frac{1}{f}$ \vec{x}_0

(a) State what is meant by simple harmonic motion.

- Motion in which acceleration is proportional to displacement and always directed towards a fixed point. [2]

(b) Calculate:

(i) the angular frequency of the oscillations

$$\omega = 2\pi f$$

$$= 2\pi \times \frac{1}{3.0}$$

$$= \frac{2\pi}{3.0}$$

$$= 2.1 \text{ rad s}^{-1} \text{ angular frequency} = \dots\dots\dots 2.1 \text{ rad s}^{-1} \text{ [2]}$$

(ii) the total energy of the oscillations.

$$E = \frac{1}{2} m \omega^2 x_0^2$$

$$= \frac{1}{2} \times 0.81 \times 2.1^2 \times 0.036^2$$

$$= \underline{2.3 \times 10^{-3} \text{ J}}$$

total energy = 2.3×10^{-3} J [2]

- (c) The suspended sphere is now lowered into water. The sphere is given a sideways displacement of +0.036 m from its equilibrium position and is then released at time $t = 0$. The water causes the motion of the sphere to be critically damped.

On Fig. 4.2, sketch the variation of the displacement x of the sphere from its equilibrium position with t from $t = 0$ to $t = 6.0$ s.

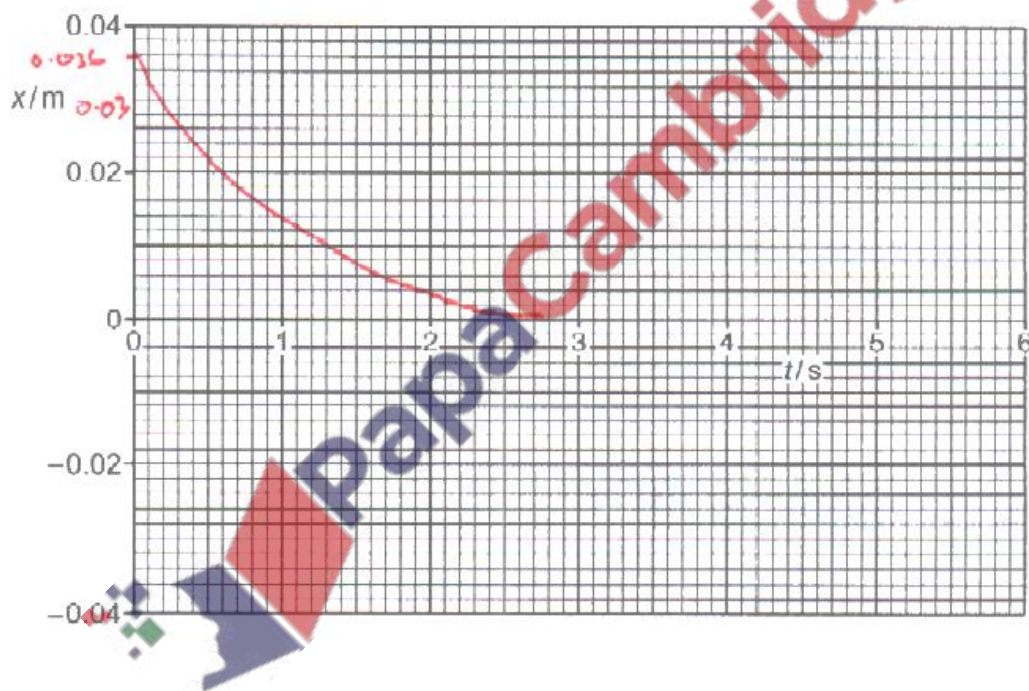


Fig. 4.2

[3]

[Total: 9]

A small steel sphere is oscillating vertically on the end of a spring, as shown in Fig. 4.1.

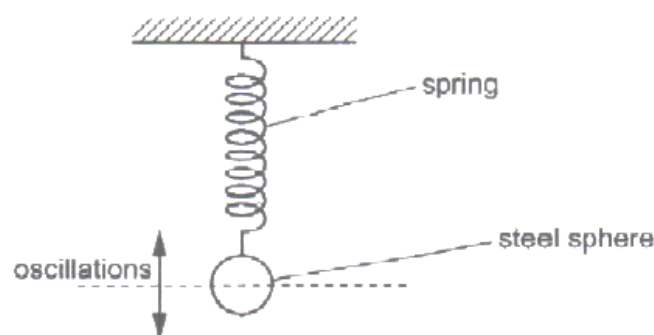


Fig. 4.1

The velocity v of the sphere varies with displacement x from its equilibrium position according to

$$v = \pm 9.7 \sqrt{(11.6 - x^2)}$$

where v is in cm s^{-1} and x is in cm.

- (a) (i) Calculate the frequency of the oscillations.

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi}$$

$$= \frac{9.7}{2\pi} = 1.547$$

frequency = 1.5 Hz [2]

- (ii) Show that the amplitude of the oscillations is 3.4 cm.

$$x_0 = \sqrt{11.6}$$

$$= 3.4 \text{ cm}$$

[1]

- (iii) Calculate the maximum acceleration a_0 of the sphere.

$$a = \omega^2 x_0$$

$$x_0 = 3.4 = 0.034 \text{ m}$$

$$a = 9.7^2 \times 0.034$$

$$= 3.199$$

$$\approx 3.2 \text{ m s}^{-2}$$

$a_0 = \dots\dots\dots 3.2 \dots\dots\dots \text{ m s}^{-2}$ [2]

(b) On Fig. 4.2, sketch the variation with x of the acceleration a of the sphere.

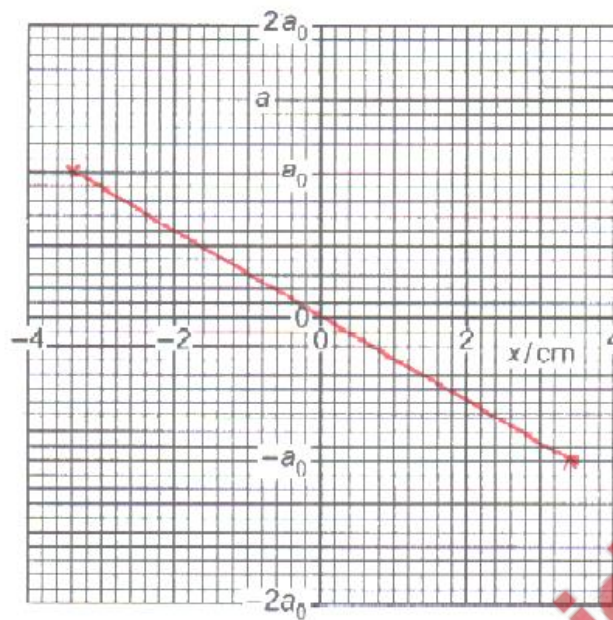


Fig. 4.2

[3]

(c) Describe, without calculation, the interchange between the potential energy and the kinetic energy of the oscillations.

- Sum of p.e and k.e is constant
- At maximum displacement, k.e is zero because v is zero, $k.e = \frac{1}{2}mv^2$
- At zero displacement k.e is maximum but p.e is minimum.

[3]

[Total: 11]

An object is suspended from a vertical spring as shown in Fig. 3.1.

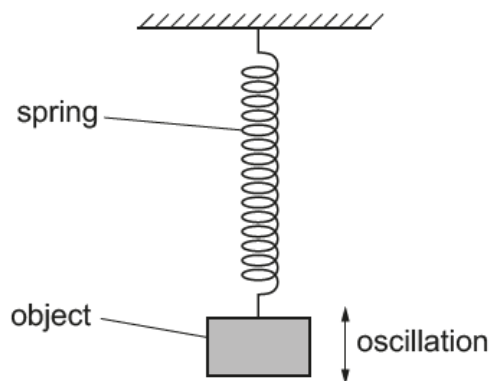


Fig. 3.1

The object is displaced vertically and then released so that it oscillates, undergoing simple harmonic motion.

Fig. 3.2 shows the variation with displacement x of the energy E of the oscillations.

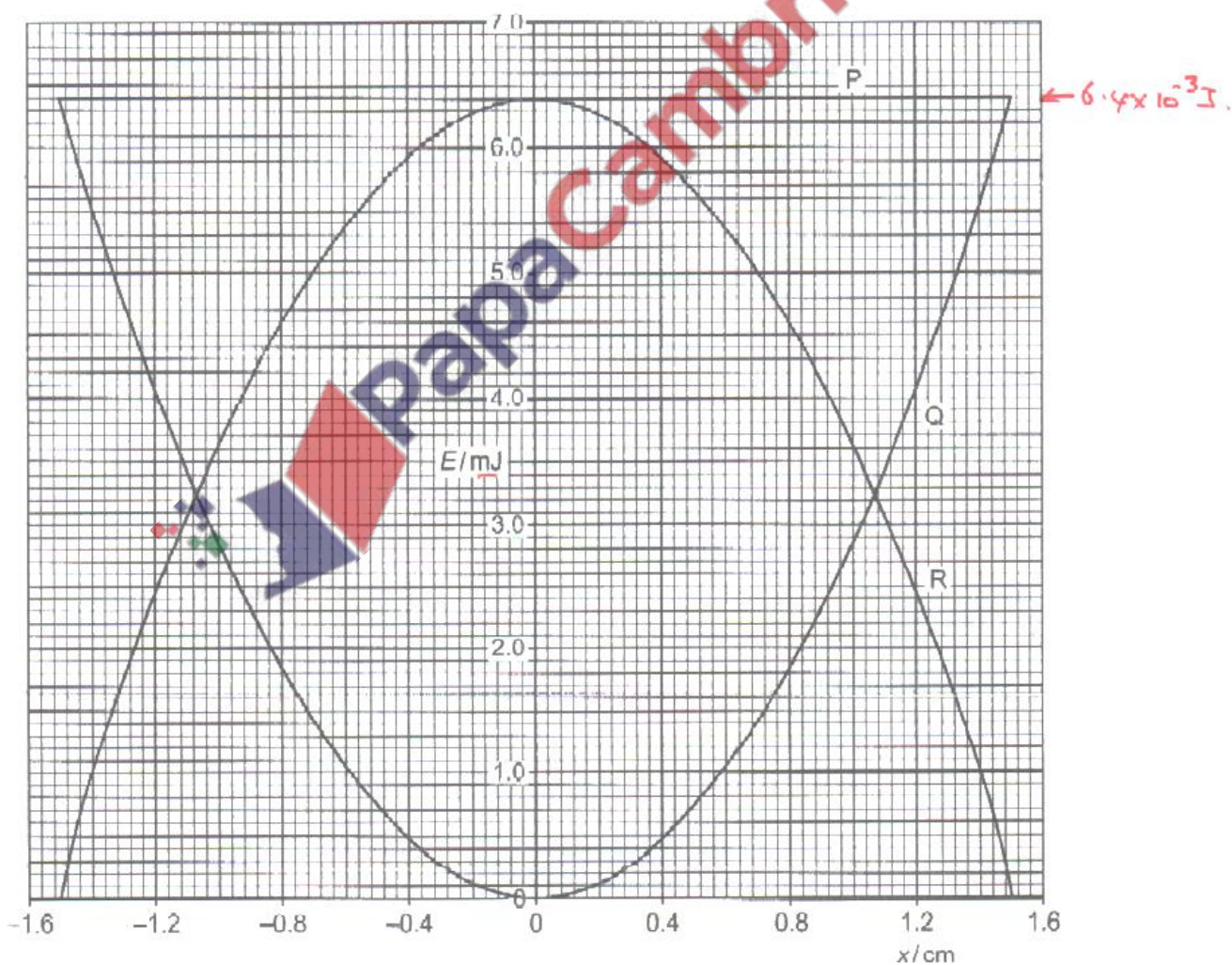


Fig. 3.2

The kinetic energy, the potential energy and the total energy of the oscillations are each represented by one of the lines P, Q and R.

- (a) State the energy that is represented by each of the lines P, Q and R.

P Total energy
 Q potential energy
 R Kinetic energy.

[2]

- (b) The object has a mass of 130 g. $\leftarrow 0.130 \text{ kg}$
 Determine the period of the oscillations.

$$E = \frac{1}{2} m \omega^2 x_0^2 \quad ; \quad E = \frac{1}{2} \times 0.13 \text{ kg} \times \omega^2 \times 0.015^2 \quad ; \quad T = \frac{2\pi}{\omega}$$

$$\omega = \frac{2\pi}{T} \quad ; \quad \text{but } E = 6.4 \times 10^{-3} \text{ J}$$

$$\therefore T = \frac{2\pi}{\omega} \quad ; \quad \therefore \omega = \sqrt{\frac{6.4 \times 10^{-3} \times 2}{0.13 \times 0.015^2}} = 20.9 \text{ rad s}^{-1}$$

period = 0.30 s [4]

- (c) (i) State the cause of damping.

..... Resistive forces, eg air resistance [1]

- (ii) A light card is attached to the object. The object is displaced with the same initial amplitude and then released. During each complete oscillation the total energy of the system decreases by 8.0% of the total energy at the start of that oscillation.

Determine the decrease in total energy, in mJ, of the system by the end of the first 6 complete oscillations.

After each oscillation

$$\text{energy remaining} = 100 - 8$$

$$= 92\%$$

$$= 0.92$$

for 6 oscillation

$$\text{energy decrease} = 6.4 - (6.4 \times 0.92^6)$$

$$= 2.5 \text{ mJ}$$

energy lost = 2.5 mJ [2]

(iii) State, with a reason, the type of damping that the card introduces into the system.

- Light damping since the system keeps on oscillating even up to 6 times. [1]

[Total: 10]

