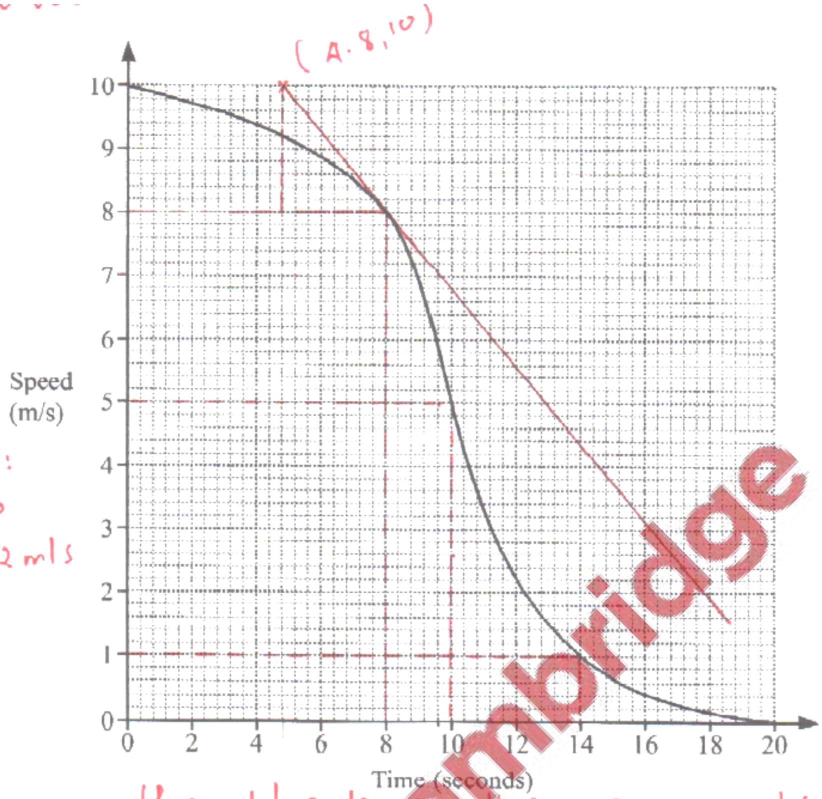
Functions – 2022 Nov IGCSE 0444

1. Nov/2022/Paper_23/No.15

15



Vertical scale:
1 small square represents 0.2 m/s

Horizontal scale: 1 small square represents 0.4 seconds

The graph shows the speed of a car as it slows down from a speed of 10 m/s until it stops at 20 seconds.

(a) Find the speed of the car at 14 seconds.

........1........ m/s [1]

(b) Find the average rate of change of the speed between 8 seconds and 10 seconds.

$\frac{\text{change in speed}}{\text{change in time}} = \frac{5-8}{10-8}$

$= -\frac{3}{2}$ m/s^2

........$\frac{-3}{2}$........ m/s^2 [2]

(c) By drawing a suitable tangent to the curve, find the rate of change of the speed at 8 seconds.

The points (4.8, 10) and (8, 8)

rate of change $= \frac{\Delta y}{\Delta x} = \frac{8-10}{8-4.8} = \frac{-2}{3.2}$

-0.625 m/s^2

........-0.625........ m/s^2 [2]

2. Nov/2022/Paper_23/No.19

$$f(x) = 5x - 3, \ x > 1$$

$$g(x) = \frac{10}{x-2}, \ x \neq 2$$

(a) Find $g(f(x))$.

Give your answer in its simplest form.

$$gf(x) = g(5x-3)$$

$$= \frac{10}{5x-3-2}$$

$$= \frac{10}{5x-5}$$

Factoring 5 in the denominator :

$$gf(x) = \frac{10}{5(x-1)} = \frac{2}{x-1}$$

$\frac{2}{x-1}$ .... [2]

(b) Find $g^{-1}(x)$.

Let $y = \frac{10}{x-2}$

Making x the subject :

$$y(x-2) = 10$$

$$y \times x + y \times -2 = 10$$

$$xy - 2y = 10$$

$$xy = 10 + 2y$$

$$x = \frac{10+2y}{y} = \frac{10}{y} + \frac{2y}{y}$$

$$x = \frac{10}{y} + 2$$

$$g^{-1}(x) = \frac{10}{x} + 2$$

$g^{-1}(x) =$ $\frac{10}{x} + 2$ .... [3]

(c) Find $f(f^{-1}(x-1))$.

$$f(x) = 5x - 3$$

Let $y = 5x - 3$

Making x the subject

$$\frac{5x}{5} = \frac{y+3}{5}$$

$$x = \frac{y+3}{5}$$

$$\therefore f^{-1}(x) = \frac{x+3}{5}$$

$$f^{-1}(x-1) = \frac{x-1+3}{5}$$

$$= \frac{x+2}{5}$$

$$f(f^{-1}(x-1)) = f\left(\frac{x+2}{5}\right)$$

$$= 5\left(\frac{x+2}{5}\right) - 3$$

$$= x + 2 - 3$$

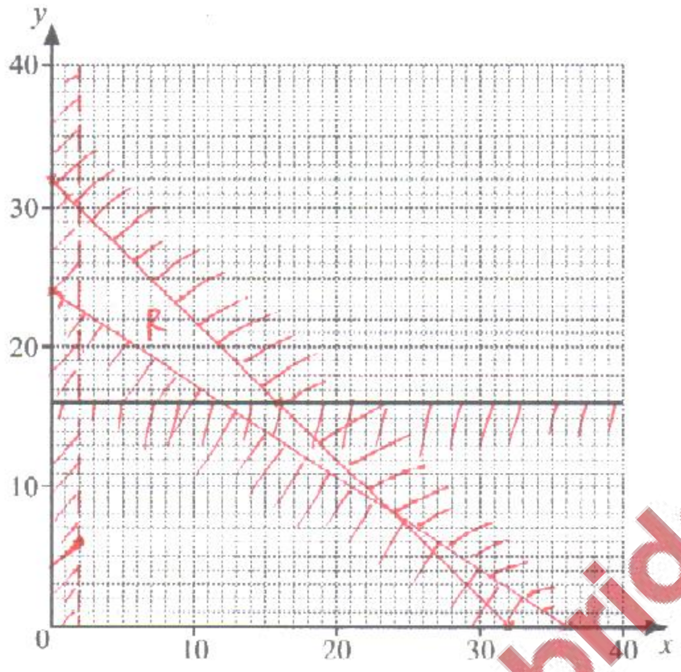
$$= x - 1$$

$$\therefore f(f^{-1}(x-1)) = x - 1$$

$x - 1$ .... [1]

3. Nov/2022/Paper_43/No.5(c)

(c) The line $y = 16$ is drawn on the grid.



The region R satisfies the following inequalities.

$y \geqslant 16 \qquad x > 2 \qquad 2x + 3y \geqslant 72 \qquad y \leqslant 32 - x$

(i) By drawing three more lines and shading the region **not required**, find and label region R. [6]

(ii) Find the integer coordinates (x, y) of the point in the region R that give the maximum value of $2x + y$.

Maximum value of $2x + y$

given the point $(16, 16)$

$2 \times 16 + 16$

$= 32 + 16 = 48$

given the point $(15, 17)$

$2 \times 15 + 17$

$= 30 + 17$

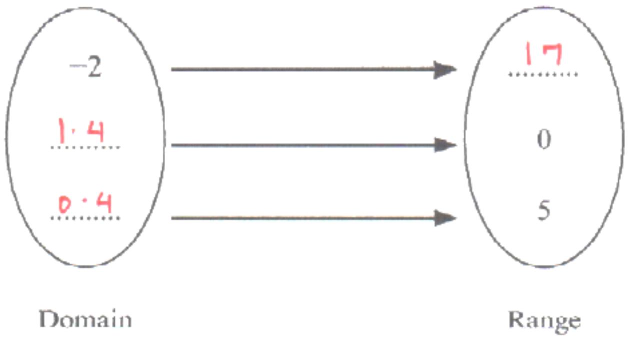
$= 47$

(16 , 16) [2]

4. Nov/2022/Paper_43/No.10

(a) $f(x) = 7 - 5x$

Complete the mapping diagram.



When $x = -2$
$f(-2) = 7 - 5(-2)$
$= 7 + 10$
$= 17$

When $f(x) = 0$
$0 = 7 - 5x$
$\Rightarrow 5x = 7$
$x = 1.4$

When $f(x) = 5$
$5 = 7 - 5x$
$5x = 7 - 5$
$5x = 2$
$x = 0.4$

[3]

(b) $T(x) = 50 + 30x$

A plumber charges $T(x)$ dollars for x hours of work.

(i) Find the charge for 4 hours of work.

$x = 4$
$T(4) = 50 + 30 \times 4$
$= 50 + 120$
$= \$170$

\$ ......170...... [1]

(ii) Find the number of hours of work when the charge is \$305.

$T(x) = 305$
$T(x) = 50 + 30x = 305$
$\Rightarrow 30x = 305 - 50$
$\frac{30x}{30} = \frac{255}{30}$
$x = 8.5$ hours

......8.5...... hours [2]

(iii) $C(x) = 20 + 50x$

Another plumber charges $C(x)$ dollars for x hours of work.

Find the number of hours of work when the charges of the two plumbers are the same.

$C(x) = T(x)$

$20 + 50x = 50 + 30x$

Collecting like terms

$50x - 30x = 50 - 20$
$\frac{20x}{20} = \frac{30}{20}$
$x = 1.5$ hours

......1.5...... hours [2]

(c) $j(x) = a \sin bx$

The amplitude of $j(x)$ is 5 and the period of $j(x)$ is $60°$.

Find the value of a and the value of b.

$60° = \frac{360°}{b}$

$\therefore b = \frac{360°}{60} = 6$

a - amplitude = 5

Period = $\frac{2\pi}{b}$

$a =$ 5

$b =$ 6 [2]

(d) (i) $\sin x° = 0.2$, for $0 \leqslant x \leqslant 360$

Find the values of x.

$\sin x° = 0.2$

$x° = \sin^{-1}(0.2)$

$x° = 11.5°$

or $x° = 180° - 11.5° = 168.5°$

$x = 11.5$ or $x = 168.5$ [2]

(ii) Complete the statement.

$\sin x = \cos($ $90 - x$ $)$ [1]

(e) $g(x) = 5^x - 2x$

Find the value of x when $g^{-1}(x) = 3$.

If $g^{-1}(x) = 3$, then $x = g(3)$

$x = g(3) = 5^3 - 2(3)$

$= 125 - 6$

$= 119$

$x =$ 119 [2]

(f) Describe fully the **single** transformation that maps the graph of $y = h(x)$ onto the graph of $y = 3h(x)$.

Vertical stretch of scale factor 3 along the x - axis.

[3]