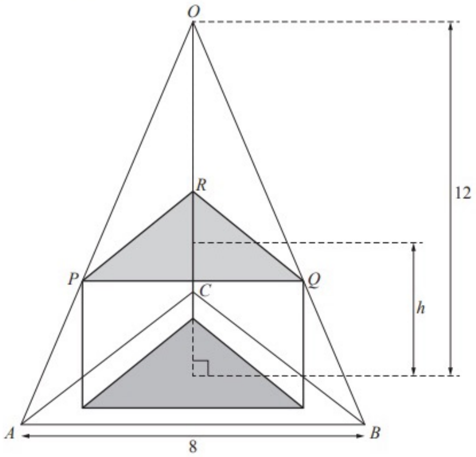
Difficult Questions Explained for Cambridge IGCSE

Additional Mathematics (0606)

1st edition, for examination until 2027

0606_m22_qp_22

12 In this question all lengths are in centimetres.



The diagram shows a right triangular prism of height h inside a right pyramid.
The pyramid has a height of 12 and a base that is an equilateral triangle, ABC, of side 8.
The base of the prism sits on the base of the pyramid.
Points P, Q and R lie on the edges OA, OB and OC, respectively, of the pyramid $OABC$.
Pyramids $OABC$ and $OPQR$ are similar.

(a) Show that the volume, V, of the triangular prism is given by $V = \frac{\sqrt{3}}{9}(ah^3 + bh^2 + ch)$ where a, b and c are integers to be found. [4]

$\frac{1}{2}(8^2) \times \sin 60° = $ Area of ABC

area of ABC $= 16\sqrt{3}$

$\frac{12-h}{12}$ - ratio of PQR & PQO length

$\left(\frac{12-h}{12}\right)^2$ - ratio of areas

Ratio of Areas $= \frac{144 - 24h + h^2}{144}$

Solve for PQR

$\frac{144-24h+h^2}{144} = \frac{\text{Area of PQR}}{\text{Area of ABC } (16\sqrt{3})}$

$\frac{144-24h+h^2}{144} = \frac{x}{16\sqrt{3}}$

ΔPQR

$= \underbrace{\frac{\sqrt{3}}{9}(h^2 - 24h + 144)}_{\text{cross-sectional area}}$

x height (h)

$\frac{\sqrt{3}}{9}(h^2 - 24h + 144)h$

$\frac{\sqrt{3}}{9}(h^3 - 24h^2 + 144h)$ #

Final Answer

(b) It is given that, as h varies, V has a maximum value. Find the value of h that gives this maximum value of V. [3]

#1: differentiate the volume equation with respect to h

$\frac{dV}{dh} = \frac{\sqrt{3}}{9}(3h^2 - 48h + 144)$

$\frac{\sqrt{3}}{9}(6h - 48)$

second derivative

$\boxed{\frac{\sqrt{3}}{9}(3h^2 - 48h + 144) = 0}$

solve this

$\left.\begin{array}{l} h = 12 \\ h = 4 \end{array}\right\}$ use 2nd derivative to find max

Maximum!

$\frac{\sqrt{3}}{9}(6(4) - 48) = \boxed{\frac{-8\sqrt{3}}{3}}$

$\frac{\sqrt{3}}{9}(6(12) - 48) = \frac{8\sqrt{3}}{3}$

$\boxed{h = 4 \text{ final answer}}$

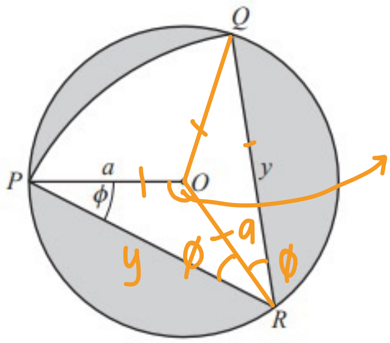
0606_m23_qp_22

9 In this question, all lengths are in centimetres and all angles are in radians.

(b)

Triangle POR is congruent to triangle QOR

PRQ = 2ϕ



$180 = \pi$

$P\hat{O}R = \pi - 2\phi$

The diagram shows a circle with centre O and radius a. Sector PQR is a sector of a different circle with centre R and radius y. Angle OPR is ϕ. Find, in terms of a and ϕ only, the total area of the three shaded regions. Simplify your answer. [4]

$\frac{1}{2}r^2\theta \rightarrow$ Area of Sector

$\pi r^2 \rightarrow$ Circle

$\pi(a)^2 - \frac{1}{2}(y)^2 2\phi$

Find y in terms of ϕ

- use cosine / sine rule

COSINE

$y^2 = a^2 + a^2 - 2a^2\cos(\pi - 2\phi)$

Substitute y^2 into $\pi(a)^2 - \frac{1}{2}(y)^2 2\phi$

$\pi(a)^2 - \frac{1}{\cancel{2}}\left(a^2 + a^2 - 2a^2\cos(\pi - 2\phi)\right)\cancel{2}\phi$

$\pi a^2 - (2a^2 - 2a^2\cos(\pi - 2\phi))$

Final answer

SINE

$\frac{a}{\sin\phi} = \frac{y}{\sin(\pi - 2\phi)}$

$\frac{a}{\sin\phi}(\sin(\pi - 2\phi)) = y$

$\frac{a\sin(\pi - 2\phi)}{\sin\phi} = y$

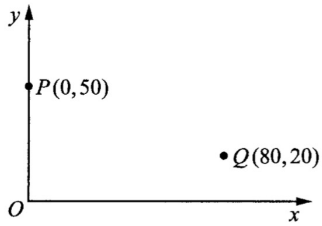
Substitute y into $\pi a^2 - \frac{1}{2}(y)^2 2\phi$

$\pi a^2 - \frac{1}{2}\left(\frac{a\sin(\pi - 2\phi)}{\sin\phi}\right)^2 2\phi$

$\pi a^2 - \frac{a^2\phi\sin^2(\pi - 2\phi)}{\sin^2\phi}$

0606_w02_qp_01

10



At 1200 hours, ship P is at the point with position vector $50\mathbf{j}$ km and ship Q is at the point with position vector $(80\mathbf{i} + 20\mathbf{j})$ km, as shown in the diagram. Ship P is travelling with velocity $(20\mathbf{i} + 10\mathbf{j})$ km h^{-1} and ship Q is travelling with velocity $(-10\mathbf{i} + 30\mathbf{j})$ km h^{-1}.

(i) Find an expression for the position vector of P and of Q at time t hours after 1200 hours. [3]

(ii) Use your answers to part **(i)** to determine the distance apart of P and Q at 1400 hours. [3]

(iii) Determine, with full working, whether or not P and Q will meet. [2]

i. $r_p = 50\hat{j} + t(20\hat{i} + 10\hat{j})$

$r_q = 80\hat{i} + 20\hat{j} + t(-10\hat{i} + 30\hat{j})$

$r = a + tv$

where r is the final position vector
a is original position vector
t is time
v is velocity vector

ii. Substitute 2 into t, 1200 → 1400 (2 hours)

$r_p = 50j + 2(20i + 10j)$

$= 40\hat{i} + 70\hat{j}$

$r_q = 80i + 20j + 2(-10i + 30j)$

$= 60\hat{i} + 80\hat{j}$

Distance between Q & P so $r_q - r_p$

$60\hat{i} + 80\hat{j} - (40\hat{i} + 70\hat{j})$

$= 20i + 10\hat{j}$

↳ Find magnitude of vector by using Pythagoras

$\sqrt{20^2 + 10^2}$

$= 10\sqrt{5}$ → Final Answer!

iii. When/if P & q meet, they will be equal to eachother

$50\hat{j} + t(20\hat{i} + 10\hat{j}) = 80\hat{i} + 20\hat{j} + t(-10\hat{i} + 30\hat{j})$

$50\hat{j} + 20t\hat{i} + 10t\hat{j} = 80\hat{i} + 20\hat{j} + -10t\hat{i} + 30t\hat{j}$

$20t\hat{i} + \hat{j}(50 + 10t) = \hat{i}(80 - 10t) + \hat{j}(20 + 30t)$

$\cancel{\hat{j}}(50 + 10t) = \cancel{\hat{j}}(20 + 30t)$

$50 + 10t = 20 + 30t$

$50 - 20 = 30t - 10t$

$30 = 20t$

$\frac{3}{2} = t$

$20t\cancel{\hat{i}} = \cancel{\hat{i}}(80 - 10t)$

$20t + 10t = 80$

$30t = 80$

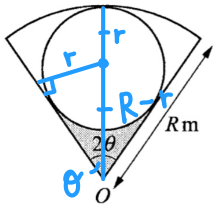
$t = \frac{8}{3}$

$\frac{3}{2} \neq \frac{8}{3}$ hence, P and q don't meet.

0606_w02_qp_01.

12 Answer only **one** of the following two alternatives.

EITHER



The diagram shows a garden in the form of a sector of a circle, centre O, radius R m and angle 2θ. Within this garden a circular plot of the largest possible size is to be planted with roses. Given that the radius of this plot is r m,

(i) show that $R = r\left(1 + \dfrac{1}{\sin\theta}\right)$. [4]

Given also that $\theta = 30°$,

(ii) calculate the fraction of the garden that is to be planted with roses. [4]

When the circular plot has been constructed, the remainder of the garden consists of three regions. Given further that $R = 15$,

(iii) calculate, to 1 decimal place, the length of fencing required to fence along the perimeter of the shaded region. [3]

i. $\sin\theta = \dfrac{r}{R-r}$

$R - r = \dfrac{r}{\sin\theta}$

$R = \dfrac{r}{\sin\theta} + r$

$R = r\left(\dfrac{1}{\sin\theta} + 1\right)$

ii. Convert $30° \rightarrow$ radians

$30° \times \dfrac{\pi}{180}$

$= \dfrac{\pi}{6}$

$R = r\left(\dfrac{1}{\sin\frac{\pi}{6}} + 1\right)$

$R = 3r$

Area of circle : πr^2

Area of Sector : $\frac{1}{\cancel{2}} R^2 \cancel{2}\theta$

Substitute $R = 3r$

: $R^2\theta$

: $(3r)^2\theta$

: $9r^2\theta$

Substitute $\dfrac{\pi}{6}$

Fraction : $\dfrac{\text{roses}}{\text{garden}}$

$= \dfrac{\cancel{\pi r^2}}{9\cancel{r^2}\left(\frac{\pi}{6}\right)}$

$= \dfrac{\cancel{6}\,^2}{\cancel{9}\,_3} = \dfrac{2}{3}$ #

Final Answer.

iii. $R = 3r$

$15 = 3r$

$\dfrac{\cancel{15}^{\,5}}{\cancel{3}_{\,1}} = r$

$5 = r$

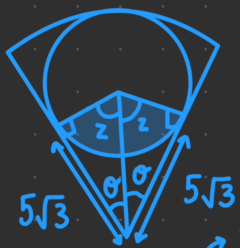
$\tan\theta = \dfrac{r}{x}$

$\tan\left(\dfrac{\pi}{6}\right) = \dfrac{5}{x}$

$\dfrac{5}{\tan\left(\frac{\pi}{6}\right)} = x$

$x = 5\sqrt{3}$

Finding y :



90° in radians

$z = \pi - \theta - \dfrac{\pi}{2}$

$= \pi - \dfrac{\pi}{6} - \dfrac{\pi}{2}$

$= \dfrac{\pi}{3}$

$2z = \dfrac{2}{3}\pi$

$y = 5 \times \dfrac{2\pi}{3}$

$y = \dfrac{10\pi}{3}$

Perimeter : Add all the sides

$5\sqrt{3} + 5\sqrt{3} + \dfrac{10\pi}{3}$

$= 27.79$

1 decimal place!

27.8 m

don't forget the unit

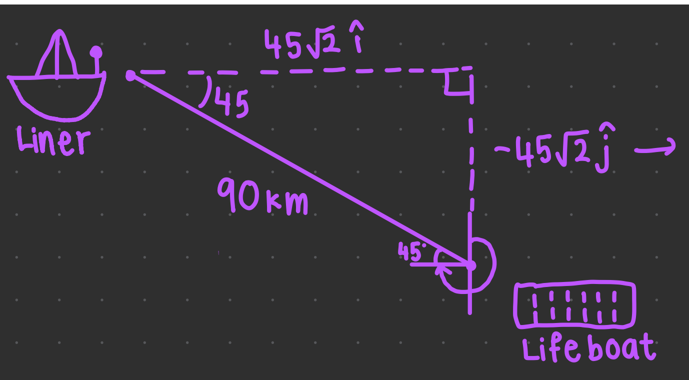
0606_s03_qp_01

4 An ocean liner is travelling at 36 km h^{-1} on a bearing of 090°. At 0600 hours the liner, which is 90 km from a lifeboat and on a bearing of 315° from the lifeboat, sends a message for assistance. The lifeboat sets off immediately and travels in a straight line at constant speed, intercepting the liner at 0730 hours. Find the speed at which the lifeboat travels. [5]

$t = 1.5$

$6:00 \rightarrow 7:30$

1.5 hours!



$\rightarrow$ use $C\frac{A}{H}$ for $\hat{i}$

use $S\frac{O}{H}$ for $\hat{j}$

Final position vector

$r = a + tv$

where r is the final position vector
a is original position vector
t is time
v is velocity vector

Let original Liner position = 0,0

$R_{Liner} = 36t\hat{i}$ $\quad t = 1.5$

$= 36(1.5)\,\hat{i}$

$= 54\hat{i} + 0\hat{j}$

Life boat original position: $45\sqrt{2}\,\hat{i} - 45\sqrt{2}\,\hat{j}$

$R_{Lifeboat} = (45\sqrt{2}\,\hat{i} - 45\sqrt{2}\,\hat{j}) + 1.5(x\hat{i} + y\hat{j})$

$= 45\sqrt{2}\,\hat{i} - 45\sqrt{2}\,\hat{j} + 1.5x\hat{i} + 1.5y\hat{j}$

$= \hat{i}(45\sqrt{2} + 1.5x) + \hat{j}(1.5y - 45\sqrt{2})$

For Liner & Lifeboat to meet the $R_{Liner} = R_{lifeboat}$

$$54\hat{i} + 0\hat{j} = \hat{i}(45\sqrt{2} + 1.5x) + \hat{j}(1.5y - 45\sqrt{2})$$

$54 = 45\sqrt{2} + 1.5x \qquad 0 = 1.5y - 45\sqrt{2}$

$54 - 45\sqrt{2} = 1.5x \qquad 45\sqrt{2} = 1.5y$

$\dfrac{54 - 45\sqrt{2}}{1.5} = x \qquad \dfrac{45\sqrt{2}}{1.5} = y$

$$\underbrace{x = 36 - 30\sqrt{2} \qquad y = 30\sqrt{2}}$$

This is velocity vector, to convert to speed, use Pythagoras theorum.

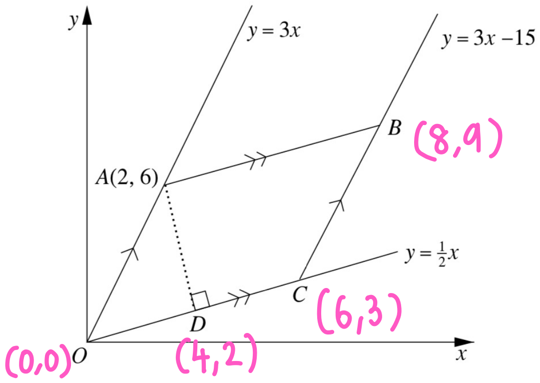
$$\sqrt{(36 - 30\sqrt{2})^2 + (30\sqrt{2})^2}$$

$= 42.9$ km/h

this is the final Answer!

0606_w03_qp_01

11 Solutions to this question by accurate drawing will not be accepted.



The diagram, which is not drawn to scale, shows a parallelogram $OABC$ where O is the origin and A is the point $(2, 6)$. The equations of OA, OC and CB are $y = 3x$, $y = \frac{1}{2}x$ and $y = 3x - 15$ respectively. The perpendicular from A to OC meets OC at the point D. Find

(i) the coordinates of C, B and D, [8]

(ii) the perimeter of the parallelogram $OABC$, correct to 1 decimal place. [3]

ii. $AO + AB + BC + CO$

$AO = BC$

$AB = CO$

$2AO + 2CO$

$AO = \sqrt{6^2 + 2^2}$ (pythagoras)

$AO = 2\sqrt{10}$

$CO = \sqrt{6^2 + 3^2}$

$CO = 3\sqrt{5}$

Perimeter $= 2(2\sqrt{10}) + 2(3\sqrt{5})$

$= 4\sqrt{10} + 6\sqrt{5}$

$= 26.1$ units

i. C : the point where line $y = 3x - 15$ & $y = \frac{1}{2}x$ intersect

C : $3x - 15 = \frac{1}{2}x$

$3x - \frac{1}{2}x = 15$

$\frac{5}{2}x = 15$

$x = 15 \div \frac{5}{2}$

$x = 6$

* After x is found substitute the value into either equations ($3x - 15 = y$ or $\frac{1}{2}x = y$) to find y coordinate.

$y = \frac{1}{2}x$

$= \frac{1}{2}(6)$

$y = 3$ #

C : $(6, 3)$

B. the line B is on is parallel to $y = \frac{1}{2}x$, hence they have same gradient ($\frac{1}{2}$)

Use A cords to find line equation.

$y = \frac{1}{2}x + c$

A : $(2, 6)$

$6 = \frac{1}{2}(2) + c$

$6 - \frac{1}{2}(2) = c$

$c = 5$

Line equation for AB =

$y = \frac{1}{2}x + 5$

Let AB line equation equal to $y = 3x - 15$

Because B is where $y = \frac{1}{2}x + 5$ & $y = 3x - 15$ intersect.

B : $\frac{1}{2}x + 5 = 3x - 15$

$5 + 15 = 3x - \frac{1}{2}x$

$20 = \frac{5}{2}x$

$20 \div \frac{5}{2} = x$

$x = 8$

$y = \frac{1}{2}(8) + 5$

$y = 9$

B : $(8, 9)$ #

D: Perpendicular to $y = \frac{1}{2}x$, Find line equation for AD.

$y = mx + c$

$m_2 = -\frac{1}{m_1} = -\frac{1}{\frac{1}{2}} = -2$

$y = 2x + c$

* Substitute A cords

$6 = -2(2) + c$

$6 + 4 = c$

$c = 10$

$y = -2x + 10$

let $-2x + 10$ equal to $\frac{1}{2}x$, intersection of those lines is point D.

$-2x + 10 = \frac{1}{2}x$

$10 = \frac{1}{2}x + 2x$

$10 = \frac{5}{2}x$

$4 = x$

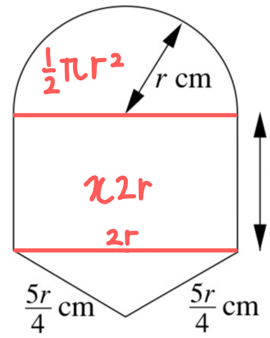
$y = \frac{1}{2}(4)$ $y = 2$ #

D: $(4, 2)$

0606_w03_qp_01

12 Answer only **one** of the following two alternatives.

EITHER



A piece of wire, 125 cm long, is bent to form the shape shown in the diagram. This shape encloses a plane region, of area A cm^2, consisting of a semi-circle of radius r cm, a rectangle of length x cm and an isosceles triangle having two equal sides of length $\frac{5r}{4}$ cm.

(i) Express x in terms of r and hence show that $A = 125r - \frac{\pi r^2}{2} - \frac{7r^2}{4}$. [6]

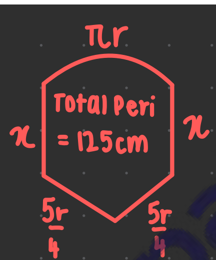
Given that r can vary,

(ii) calculate, to 1 decimal place, the value of r for which A has a maximum value. [4]

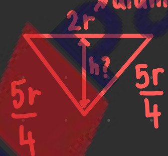
i. $\pi r + 2x + \frac{5r}{2} = 125$

$2x = 125 - \frac{5r}{2} - \pi r$

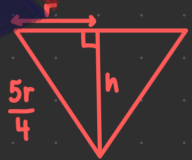
$x = \frac{125}{2} - \frac{5r}{4} - \frac{\pi}{2}$



$A = \underbrace{\frac{1}{2}\pi r^2}_{\text{semicircle}} + \overbrace{2rx}^{\text{rectangle}} + \underbrace{\frac{3}{4}r^2}_{\text{triangle}}$



To find h, you can split the triangle in half to use Pythagoras theorum.



$h = \sqrt{\left(\frac{5r}{4}\right)^2 - r^2}$

$h = \frac{3}{4}r$

Area of triangle: $\frac{1}{2}bh$

$= \frac{1}{\cancel{2}}(\cancel{2}r)(\frac{3}{4}r)$

$\boxed{= \frac{3}{4}r^2}$

Substitute $2x = 125 - \frac{5r}{2} - \pi r$ into rectangle area

$A = \frac{1}{2}\pi r^2 + (125 - \frac{5r}{2} - \pi r)r + \frac{3}{4}r^2$

$= \frac{1}{2}\pi r^2 + 125r - \frac{5r^2}{2} - \pi r^2 + \frac{3}{4}r^2$

$A = 125r - \frac{\pi r^2}{2} - \frac{7}{4}r^2$

Hence, Shown.

Final Answer!

ii. Diffrentiate the Area equation with respect to r

$\frac{dA}{dr} = 125 - \pi r - \frac{7}{2}r$

To find the maximum, let $\frac{dA}{dr} = 0$:

$125 - \pi r - \frac{7}{2}r = 0$

$125 = \pi r + \frac{7}{2}r$

$125 = r(\pi + \frac{7}{2})$

$\frac{125}{\pi + \frac{7}{2}} = r$

$\boxed{r = 18.8\text{ cm}}$

Final Answer!

0606_s04_qp_01

8 A curve has the equation $y = (ax + 3)\ln x$, where $x > 0$ and a is a positive constant. The normal to the curve at the point where the curve crosses the x-axis is parallel to the line $5y + x = 2$. Find the value of a. [7]

$5y + x = 2$

$y = -\frac{1}{5}x + \frac{2}{5}$

$m_1 = -\frac{1}{5}$

Line is perpendicular to normal

gradient of tangent $= -\frac{1}{m_1}$

$m_2 = 5$

$y = (ax+3)\ln x$

Crosses x-axis, $y = 0$

$0 = (ax+3)\ln x$

$\ln x = 0$

$x = e^0$

$x = 1$

to find gradient equation at $x=1$:

$y = (ax+3)\ln x$

u v

$u' = a$ $v' = \frac{1}{x}$

$\left.\frac{dy}{dx}\right|_{x=1} = (ax+3)\times\frac{1}{x} + a\times\ln x$

$\left.\frac{dy}{dx}\right|_{x=1} = \frac{ax+3}{x} + a\ln x$

$= \frac{a(1)+3}{(1)} + a\ln(1)$

$= a + 3 + a\cancel{\ln(1)}^{0}$

$= a + 3$

↳ gradient at $x=1$

gradient at tangent is parallel, so you can equate them to each other: $a + 3 = 5$

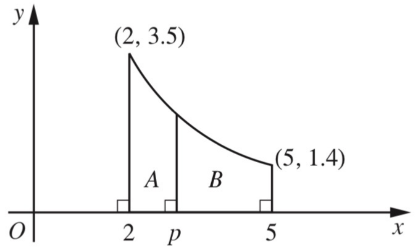
$a = 5 - 3$

$\boxed{a = 2 \#}$

↑Final answer!

0606_s04_qp_01

11



The diagram shows part of a curve, passing through the points (2, 3.5) and (5, 1.4). The gradient of the curve at any point (x, y) is $-\frac{a}{x^3}$, where a is a positive constant.

(i) Show that $a = 20$ and obtain the equation of the curve. [5]

The diagram also shows lines perpendicular to the x-axis at $x = 2$, $x = p$ and $x = 5$. Given that the areas of the regions A and B are equal,

(ii) find the value of p. [5]

i. gradient equal is diffrentiated version of y
So, integrate $-\frac{a}{x^3}$ to find what the equation of the line is

$$\frac{dy}{dx} = -\frac{a}{x^3}$$

$$y = \frac{a}{2x^2} + c$$

Substitute 2 points (2,3.5) & (5,1.4) to find C & a

$$-① \quad 3.5 = \frac{a}{2(2)^2} + c \qquad -② \quad 1.4 = \frac{a}{2(5)^2} + c$$

$$3.5 = \frac{a}{8} + c \qquad 1.4 = \frac{a}{50} + c$$

$$3.5(8) = a + 8c \qquad 1.4(50) = a + 50c$$

$$28 = a + 8c \qquad 70 = a + 50c$$

$$\cancel{a} + 8c = 28$$

$$-\ \cancel{a} + 50c = 70$$

$$-42c = -42$$

$$c = 1 \#$$

$$28 = a + 8(1)$$

$$28 - 8 = a$$

$$a = 20 \#$$

$$y = \frac{\cancel{20}^{10}}{\cancel{2}x^2} + 1 \qquad \boxed{y = \frac{10}{x^2} + 1}$$

Final Answer!

ii. Integrate $y = \frac{10}{x^2} + 1$ & then use $p-2$ & $5-p$ as limits to find the value of p

$$\int_2^p (10x^{-2} - 1)\,dx = \int_p^5 (10x^{-2} + 1)\,dx$$

$$\left[-\frac{10}{x} + x\right]_2^p = \left[-\frac{10}{x} + x\right]_p^5$$

$$-\frac{10}{p} + p - (-5 + 2) = -2 + 5 - \left(-\frac{10}{p} + p\right)$$

$$-\frac{10}{p} + p + 5 - 2 = -2 + 5 + \frac{10}{p} - p$$

$$\frac{p^2 - 10}{\cancel{p}} \cancel{+3} = \cancel{3} + \frac{10 - p^2}{\cancel{p}}$$

$$p^2 - 10 = 10 - p^2$$

$$p^2 + p^2 = 10 + 10$$

$$2p^2 = 20$$

$$p^2 = 10$$

$$\boxed{p = \sqrt{10} \#}$$

Final Answer!

0606_w04_qp_01

11 The line $4y = 3x + 1$ intersects the curve $xy = 28x - 27y$ at the point $P(1, 1)$ and at the point Q. The perpendicular bisector of PQ intersects the line $y = 4x$ at the point R. Calculate the area of triangle PQR. [9]

$4y = 3x + 1$

$x = \frac{4y-1}{3}$ → Substitute into $xy = 28x - 27y$

$\left(\frac{4y-1}{3}\right)y = 28\left(\frac{4y-1}{3}\right) - 27y$

$\frac{4y^2-y}{3} = \frac{112y-28}{3} - 27y$

$\frac{4y^2-y}{3} - \frac{112y-28}{3} = -27y$

$\frac{4y^2-y-112y-28}{3} = -27y$

$4y^2 - 113y - 28 = -27y(3)$

$4y^2 - 113y + 81y + 28 = 0$

$4y^2 - 32y + 28 = 0$

$y_1 = 7 \quad y_2 = 1$

$x = \frac{4y-1}{3}$

$x_1 = 9 \quad x_2 = 1$

$P(1,1) \quad Q = (9,7)$

To Find the perpendicular bisector

- Find gradient & midpoint

$m_1 = \frac{3}{4} \rightsquigarrow m_2 = -\frac{4}{3}$

Midpoint $= (5, 4)$

$y = mx + c$

$m = -\frac{4}{3}$

$(5, 4)$

$4 = \left(-\frac{4}{3}\right)(5) + c$

$4 = -\frac{20}{3} + c$

$4 + \frac{20}{3} = c$

$c = \frac{32}{3}$

$y = -\frac{4}{3}x + \frac{32}{3}$

$3y = -4x + 32$

$3y + 4x = 32$

$y = 4x$

$3(4x) + 4x = 32$

$12x + 4x = 32$

$16x = 32$

$x = 2$

$y = 4(2)$

$y = 8$

To Find the Area, Use the shoe lace method:

$P(1,1)\ Q(9,7)\ R(2,8)$

$\text{Area} = \frac{1}{2}\left|\begin{matrix} 1 & 9 & 2 & 1 \\ 1 & 7 & 8 & 1 \end{matrix}\right|$

$\text{Area} = \frac{1}{2}\left|7 + 72 + 2 - 9 - 14 - 8\right|$

$\text{Area} = \frac{1}{2}|50|$

$\text{Area} = 25 \text{ units}^2$

0606_w04_qp_01

12 Answer only **one** of the following two alternatives.

EITHER

(a) At the beginning of 1960, the number of animals of a certain species was estimated at 20 000. This number decreased so that, after a period of n years, the population was

$$20\,000e^{-0.05n}.$$

Estimate

(i) the population at the beginning of 1970, [1]

(ii) the year in which the population would be expected to have first decreased to 2000. [3]

(b) Solve the equation $3^{x+1} - 2 = 8 \times 3^{x-1}$. [6]

i. $20000e^{-0.05(10)}$ N = 10 because 1970 − 1960 = 10 year difference.

$= 12130$

ii. $20000e^{-0.05n} = 2000$

$e^{-0.05n} = \frac{200\cancel{0}}{2000\cancel{0}}$

$e^{-0.05n} = \frac{1}{10}$

$-0.05n = \ln\left(\frac{1}{10}\right)$

$n = \frac{\left(\ln\frac{1}{10}\right)}{-0.05}$

$n = 46.1$ years

$n = 47$ years – rounded up!

$1960 + 47 = \boxed{2007}$

Final answer!

(b) Let $3^x = y$

$(3^x)(3^1) - 2 = 8 \times \frac{(3^x)}{(3^1)}$

$3y - 2 = 8 \times \frac{1}{3}y$

$3y - 2 = \frac{8}{3}y$

$-2 = \frac{8}{3}y - 3y$

$+2 = +\frac{1}{3}y$

$2 = \frac{1}{3}y$

$2 \div \frac{1}{3} = y$

$y = 6$

$3^x = 6$ index → log form

$\log_3(6) = x$

0606_s05_qp_01

A piece of wire, of length 2 m, is divided into two pieces. One piece is bent to form a square of side x m and the other is bent to form a circle of radius r m.

(i) Express r in terms of x and show that the total area, A m^2, of the two shapes is given by

$$A = \frac{(\pi + 4)x^2 - 4x + 1}{\pi}.$$

[4]

Given that x can vary, find

(ii) the stationary value of A, [4]

(iii) the nature of this stationary value. [2]

Find the perimeter & equal it to the Length

$$\underbrace{2\pi r}_{\text{Circle}} + \underbrace{4x}_{\text{Square}} = 2$$

$$2\pi r = 2 - 4x$$

$$r = \frac{2-4x}{2\pi} \text{ (simplify)}$$

$$r = \frac{1-2x}{\pi}$$

Area Formula :

$$\underbrace{\pi r^2}_{\text{circle}} + \underbrace{x^2}_{\text{Square}}$$

- Substitute $\left(r = \frac{1-2x}{\pi}\right)$

$$\pi\left(\frac{1-2x}{\pi}\right)^2 + x^2$$

$$\pi\left(\frac{(1-2x)(1-2x)}{\pi^2}\right) + x^2$$

$$\cancel{\pi}\left(\frac{1+4x+4x^2}{\cancel{\pi^2}}\right) + x^2$$

$$\left[\frac{1+4x+4x^2}{\pi} + x^2\right] \text{ make denominator same}$$

$$\frac{1+4x+4x^2}{\pi} + \frac{x^2\pi}{\pi}$$

$$\frac{1+4x+4x^2+x^2\pi}{\pi} \rightarrow \frac{1+4x+(4+\pi)x^2}{\pi} \ \#$$

ii. Diffrentiate A and equal it to 0.

$$\frac{dA}{dx} = \frac{(\pi+4)x^2 - 4x + 1}{\pi}$$

$$\frac{dA}{dx} = \frac{2(\pi+4)x - 4}{\pi}$$

$$\frac{2(\pi+4)x - 4}{\pi} = 0$$

$$2(\pi+4)x - 4 = 0$$

$$2(\pi+4)x = 4$$

$$2(\pi+4)x = 4$$

$$x = \frac{\cancel{4}}{\cancel{2\pi+8}}$$

$$x = \frac{2}{\pi+4}$$

Substitute into A.

$$A = \frac{(x+4)x^2 - 4x + 1}{\pi}$$

$$A = \frac{\left(\left(\frac{2}{\pi+4}\right)+4\right)\left(\frac{2}{\pi+4}\right)^2 - 4\left(\frac{2}{\pi+4}\right) + 1}{\pi}$$

$$A = 0.14$$

iii) find $\frac{d^2A}{dx^2}$

$$\frac{2(\pi+4)x - 4}{\pi}$$

$$\frac{d^2A}{dx^2} = \frac{2\pi+8}{\pi}$$

$\frac{2\pi+8}{\pi} > 0$, hence minimum point

0606_w05_qp_01

5 The diagram, which is not drawn to scale, shows a horizontal rectangular surface. One corner of the surface is taken as the origin O and $\mathbf{i}$ and $\mathbf{j}$ are unit vectors along the edges of the surface.

A fly, F, starts at the point with position vector $(\mathbf{i} + 12\mathbf{j})$ cm and crawls across the surface with a velocity of $(3\mathbf{i} + 2\mathbf{j})$ cm s^{-1}. At the instant that the fly starts crawling, a spider, S, at the point with position vector $(85\mathbf{i} + 5\mathbf{j})$ cm, sets off across the surface with a velocity of $(-5\mathbf{i} + k\mathbf{j})$ cm s^{-1}, where k is a constant. Given that the spider catches the fly, calculate the value of k. [6]

Find Position Vector of F & S:

$R_F = (\hat{i} + 12\hat{j}) + t(3\hat{i} + 2\hat{j})$

$R_F = \hat{i} + 12\hat{j} + 3t\hat{i} + 2t\hat{j}$

$R_F = (3t+1)\hat{i} + (12+2t)\hat{j}$

$R_S = (85\hat{i} + 5\hat{j}) + t(-5\hat{i} + k\hat{j})$

$R_S = 85\hat{i} + 5\hat{j} + -5t\hat{i} + kt\hat{j}$

$R_S = (85-5t)\hat{i} + (5+kt)\hat{j}$

$R_F = R_S$, Solve for k & t

$3t + 1 = 85 - 5t$ $\qquad$ $5 + kt = 12 + 2t$

$3t + 5t = 85 - 1$ $\qquad$ $kt - 2t = 12 - 5$

$8t = 84$ $\qquad$ $k(10.5) - 2(10.5) = 7$

$t = \frac{84}{8}$ $\qquad$ $10.5k - 21 = 7$

$t = 10.5$ $\qquad$ $k = \frac{7+21}{10.5}$

$k = 2\frac{2}{3}$ # Final Answer!

9 **(a)** Find all the angles between 0° and 360° which satisfy the equation

$$3\cos x = 8\tan x.$$ [5]

(b) Given that $4 \leqslant y \leqslant 6$, find the value of y for which

$$2\cos\left(\frac{2y}{3}\right) + \sqrt{3} = 0.$$ [3]

(a) $3\cos x = 8\tan x$

$3\cos x = 8\left(\frac{\sin x}{\cos x}\right)$

$3\cos^2 x = 8\sin x$

↑ identities

$3(1-\sin^2 x) = 8\sin x$

$3\sin^2 x + 8\sin x - 3 = 0$

$\sin x = \frac{1}{3}$

$x = \sin^{-1}\left(\frac{1}{3}\right)$

$x_1 = 19.47$

$x_2 = 180-(19.47)$

$x_2 = 160.52$

$2\cos\left(\frac{2y}{3}\right) + \sqrt{3} = 0$

$2\cos\left(\frac{2y}{3}\right) = -\sqrt{3}$

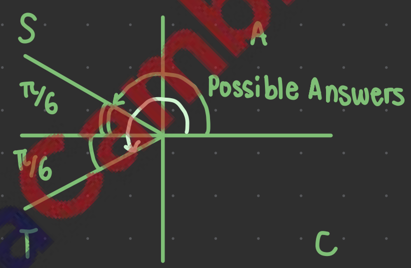
$\cos\left(\frac{2y}{3}\right) = \frac{-\sqrt{3}}{2}$

Basic angle:

$= \cos^{-1}\left(+\frac{\sqrt{3}}{2}\right)$

$= \frac{\pi}{6}$

$\cos\left(\frac{2y}{3}\right) = -\frac{\sqrt{3}}{2}$ ∴ S, T quadrants



$\frac{2y}{3} = 5\pi/6$

$y = \frac{5\pi}{6} \div \frac{2}{3}$

$y = \frac{5}{4}\pi$ [REJECT]

$4 \leqslant y \leqslant 6$

$\frac{2y}{3} = 7\pi/6$

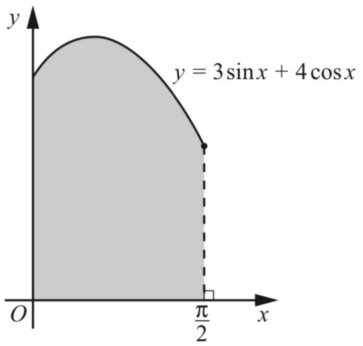
$y = \frac{7\pi}{6} \div \frac{2}{3}$

$y = \frac{7\pi}{4}$ or 5.5

0606_s07_qp_01

11 Answer only **one** of the following two alternatives.

EITHER



The graph shows part of the curve $y = 3\sin x + 4\cos x$ for $0 \leqslant x \leqslant \frac{\pi}{2}$ radians.

(i) Find the coordinates of the maximum point of the curve. [5]

(ii) Find the area of the shaded region. [5]

i. differentiate the y equation and let it equal to 0.

$$\frac{dy}{dx} = 3\cos x - 4\sin x$$

$$3\cos x - 4\sin x = 0$$

$$3\cos x = 4\sin x$$

$$\frac{3}{4} = \frac{\sin x}{\cos x}$$

$$\frac{3}{4} = \tan x$$

$$x = \tan^{-1}\left(\frac{3}{4}\right)$$

$$x = 0.644$$

$$y = 3\sin x + 4\cos x$$

$$y = 3\sin(0.644) + 4\cos(0.644)$$

$$y = 4.99$$

$$y = 5.00$$

maximum coordinate $= (0.644, 5.00)$

ii. Integrate y equation with limits 0 to $\frac{\pi}{2}$

$$\int_0^{\frac{\pi}{2}} 3\sin x + 4\cos x \, dx$$

$$= \left[-3\cos x + 4\sin x\right]_0^{\frac{\pi}{2}}$$

$$= \left(-3\cos\left(\frac{\pi}{2}\right) + 4\sin\left(\frac{\pi}{2}\right)\right) - \left(-3\cos(0) + 4\sin(0)\right)$$

$$= (0 + 4) - (-3 + 0)$$

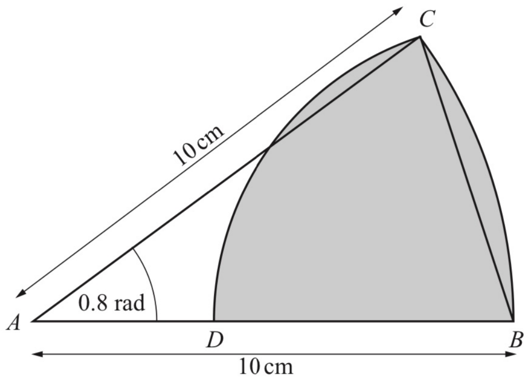
$$= 4 + 3$$

$$= 7 \text{ units}^2$$

Final Answer!

0606_w07_qp_01

10



The diagram shows a sector ABC of the circle, centre A and radius 10 cm, in which angle $BAC = 0.8$ radians. The arc CD of a circle has centre B and the point D lies on AB.

(i) Show that the length of the straight line BC is 7.79 cm, correct to 2 decimal places. [2]

(ii) Find the perimeter of the shaded region. [4]

(iii) Find the area of the shaded region. [4]

i. Use the Cosine rule to find BC.

$BC^2 = AB^2 + AC^2 - 2AB\,AC\cos(A)$

$BC = \sqrt{AB^2 + AC^2 - 2AB\,AC\cos(A)}$

$BC = \sqrt{(10)^2 + (10)^2 - 2(10\times10)\cos(0.8)}$

[Solve in calculator in RADIAN MODE]

$= \sqrt{100 + 100 - 200\cos(0.8)}$

$= 7.7883$

$= 7.79$

ii. Find arc length BC

$BC = r\theta$

$= 10(0.8)$

$= 8$

$BD = BC$ — radius of circle Centre point B

$BD = 7.79$

$DC = r\theta$

$r = 7.79$

to find θ use Sine rule!

$\frac{\sin A}{a} = \frac{\sin\theta}{b} \quad \frac{\sin 0.8}{7.79} = \frac{\sin\theta}{10}$

$10\left(\frac{\sin 0.8}{7.79}\right) = \sin\theta \quad \sin^{-1}\left(10\left(\frac{\sin 0.8}{7.79}\right)\right) = \theta$

$\theta = 1.17$ rad.

$DC = r\theta$

$= (7.79)(1.17)$

$= 9.11$

Perimeter $= BC + DB + DC$

$= 8 + 7.79 + 9.11$

$= 24.9$

iii. Area =

Sector DBC + (Sector CAB − triangle CAB)

Sector DBC $= \frac{1}{2}r^2\theta = \frac{1}{2}(7.79)^2(1.17)$

$= 35.50$

Sector CAB $= \frac{1}{2}r^2\theta = \frac{1}{2}(10)^2(0.8)$

$= 40$

Triangle CAB $= \frac{1}{2}r^2\sin\theta = \frac{1}{2}(10)^2\sin 0.8$

$= 35.87$

Area $= 35.50 + (40 - 35.87)$

$= 39.6$ unit^2

0606_s08_qp_01

10 In this question, **i** is a unit vector due east and **j** is a unit vector due north.

At 0900 hours a ship sails from the point P with position vector $(2\mathbf{i} + 3\mathbf{j})$ km relative to an origin O. The ship sails north-east with a speed of $15\sqrt{2}$ km h^{-1}.

(i) Find, in terms of **i** and **j**, the velocity of the ship. [2]

(ii) Show that the ship will be at the point with position vector $(24.5\mathbf{i} + 25.5\mathbf{j})$ km at 1030 hours. [1]

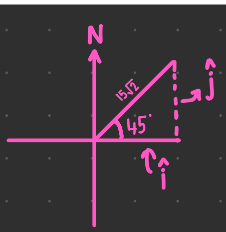
(iii) Find, in terms of **i**, **j** and t, the position of the ship t hours after leaving P. [2]

At the same time as the ship leaves P, a submarine leaves the point Q with position vector $(47\mathbf{i} - 27\mathbf{j})$ km. The submarine proceeds with a speed of 25 km h^{-1} due north to meet the ship.

(iv) Find, in terms of **i** and **j**, the velocity of the ship relative to the submarine. [2]

(v) Find the position vector of the point where the submarine meets the ship. [2]

i.



$\hat{i} = C\frac{A}{H}$

$\cos(45) = \frac{\hat{i}}{15\sqrt{2}}$ $\quad \hat{i} = 15$

$\hat{j} = S\frac{O}{H}$

$\sin(45) = \frac{\hat{j}}{15\sqrt{2}}$ $\quad \hat{j} = 15$

$\vec{V} = 15\hat{i} + 15\hat{j}$

ii. Position Vector $= a + tv$

$r = a + tv$

$r = (2\hat{i} + 3\hat{j}) + (1.5)(15\hat{i} + 15\hat{j})$

$r = (2\hat{i} + 3\hat{j}) + (22.5\hat{i} + 22.5\hat{j})$

$r = 24.5\hat{i} + 25.5\hat{j}$

iii. Position Vector $= a + tv$

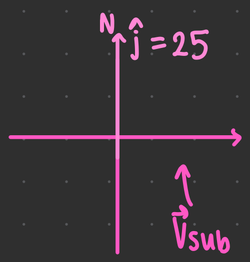
$r = a + tv$

$r = (2\hat{i} + 3\hat{j}) + (t)(15\hat{i} + 15\hat{j})$

iv.

$\vec{V}_{sub} = 25\hat{j}$

$\vec{V}_{ship} = 15\hat{i} + 15\hat{j}$



Relative velocity $= \vec{V}_{ship} - \vec{V}_{sub}$

$= 15\hat{i} + 15\hat{j} - 25\hat{j}$

$= 15\hat{i} - 10\hat{j}$

V. Let the position vector of Ship & Submarine equal to each other.

$r_{ship} = (2\hat{i} + 3\hat{j}) + (t)(15\hat{i} + 15\hat{j})$

$r_{sub} = a + tv$

$= (47\hat{i} - 27\hat{j}) + t(25\hat{j})$

$2\hat{i} + 3\hat{j} + 15\hat{i}t + 15\hat{j}t = 47\hat{i} - 27\hat{j} + 25\hat{j}t$

$(2 + 15t)\hat{i} + (3 + 15t)\hat{j} = 47\hat{i} + (25t - 27)\hat{j}$

$2 + 15t = 47$

$15t = 47 - 2$

$15t = 45$

$t = \frac{45}{15}$

$t = 3$

Substitute into position vector of either Ship or Submarine.

$R = (47\hat{i} - 27\hat{j}) + 3(25\hat{j})$

$= (47\hat{i} - 27\hat{j}) + 75\hat{j}$

$= 47\hat{i} + 48\hat{j}$

0606_s08_qp_01

12 OR

A particle moves in a straight line such that its displacement, s m, from a fixed point O at a time t s, is given by

$$s = \ln(t+1) \quad \text{for} \quad 0 \leqslant t \leqslant 3,$$

$$s = \tfrac{1}{2}\ln(t-2) - \ln(t+1) + \ln 16 \quad \text{for } t > 3.$$

Find

(i) the initial velocity of the particle, [2]

(ii) the velocity of the particle when $t = 4$, [2]

(iii) the acceleration of the particle when $t = 4$, [2]

(iv) the value of t when the particle is instantaneously at rest, [2]

(v) the distance travelled by the particle in the 4th second. [2]

i. Differentiate $S = \ln(t+1)$ and let $t = 0$

$$\frac{ds}{dt} = \frac{1}{t+1}$$

$$= \frac{1}{0+1}$$

$$= 1 \text{ m/s}$$

ii. Differentiate $S = \frac{1}{2}\ln(t-2) - \ln(t+1) + \ln 16$ and let $t = 4$

$$\frac{ds}{dt} = \frac{1}{2}\left(\frac{1}{t-2}\right) - \left(\frac{1}{t+1}\right)$$

$$\frac{ds}{dt} = \frac{1}{2t-4} - \frac{1}{t+1}$$

Substitute $t = 4$

$$\frac{1}{2(4)-4} - \frac{1}{4+1} = \frac{1}{20}$$

iii.

$$V = \frac{1}{2t-4} - \frac{1}{t+1}$$

$$\frac{dv}{dt} = -1(2t-4)^{-2} \times 2 - \left(-1(t+1)^{-2}\right)$$

$$\frac{dv}{dt} = -2(2t-4)^{-2} + 1(t+1)^{-2}$$

$\frac{dv}{dt}$ = acceleration

$$= -2(2(4)-4)^{-2} + 1(4+1)^{-2}$$

$$= -\frac{17}{200}$$

iv.

$$\frac{1}{2t-4} - \frac{1}{t+1} = 0$$

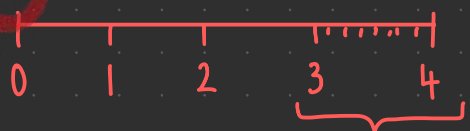
$$\frac{1}{2t-4} = \frac{1}{t+1}$$

$$t+1 = 2t-4$$

$$2t - t = 1 + 4$$

$$t = 5$$

v.



$$S_4 - S_3$$

$S_3 = \ln(3+1)$ $\quad S_3 = \ln(4)$

$$S_4 = \tfrac{1}{2}\ln(4-2) - \ln(4+1) + \ln 16$$

$$S_4 = \tfrac{1}{2}\ln(2) - \ln(5) + \ln 16$$

$$S_4 = \ln\sqrt{2} - \ln 5 + \ln 16$$

$$S_4 = \ln \frac{\sqrt{2}}{5} \times 16$$

$$S_4 = \ln \frac{16\sqrt{2}}{5}$$

$S_4 - S_3$, $\quad \ln \frac{16\sqrt{2}}{5} - \ln 4$

$$= \ln\left(\frac{16\sqrt{2}}{5} \div 4\right)$$

$$= \ln \frac{4\sqrt{2}}{5}$$

0606_w08_qp_01

8 A curve is such that $\frac{d^2y}{dx^2} = 4e^{-2x}$. Given that $\frac{dy}{dx} = 3$ when $x = 0$ and that the curve passes through the point $(2, e^{-4})$, find the equation of the curve. [6]

#1: integrate $\frac{d^2y}{dx^2}$ to find $\frac{dy}{dx}$

$\int 4e^{-2x}\, dx$

$= -2e^{-2x} + c \longrightarrow \frac{dy}{dx}$

$-2e^{-2(0)} + c = 3$

(Substitute $x=0$ & $\frac{dy}{dx}=3$)

$c = 3 + 2e^{-2(0)}$

$c = 5$

$\int -2e^{-2x} + 5\, dx$

$= e^{-2x} + 5x + c\ e^{-4}$

$y = e^{-2x} + 5x + c$

$y = e^{-2x} + 5x + c$

(Substitute $(2, e^{4})$)

$e^{-4} = e^{-2(2)} + 5(2) + c$

$e^{-4} - (e^{-2(2)} + 10) = c$

$-10 = c$

$\boxed{y = e^{-2x} + 5x - 10\ \#}$

Final Answer!!

12 Answer only **one** of the following two alternatives.

EITHER

A curve has equation $y = \frac{x^2}{x+1}$.

(i) Find the coordinates of the stationary points of the curve. [5]

The normal to the curve at the point where $x = 1$ meets the x-axis at M. The tangent to the curve at the point where $x = -2$ meets the y-axis at N.

(ii) Find the area of the triangle MNO, where O is the origin. [6]

(i) Stationary point : $\frac{dy}{dx} = 0$

$\frac{x^2}{x+1}$ $\quad u \cdot x^2 \quad v : x+1$

$\quad u' : 2x \quad v' : 1$

$\frac{vu' - uv'}{v^2}$ $\quad \frac{(x+1)(2x) - (x^2)(1)}{(x+1)(x+1)}$

$= \frac{2x^2 + 2x - x^2}{x^2 + 2x + 1}$

$\frac{dy}{dx} = \frac{x^2 + 2x}{x^2 + 2x + 1}$

$0 = \frac{x^2 + 2x}{x^2 + 2x + 1}$

$x^2 + 2x = 0$

$x = -2, x = 0 \quad y = 0 \quad y = -4$

Substitute into curve equation for y

$\boxed{(0,0) \quad (-2,-4)}$

Final Answer!!

(ii) Find coordinate M:

at $x = 1$, $y = \frac{1}{2}$

$\frac{dy}{dx}$ where $x = 1$

$= \frac{3}{4}$

gradient $= -\frac{4}{3}$

$y - \frac{1}{2} = -\frac{4}{3}(x - 1)$

$y = -\frac{4}{3}x + \frac{11}{6}$

at x axis so $y = 0$

$0 = -\frac{4}{3}x + \frac{11}{6} \quad x = \frac{11}{8} \quad M.(\frac{11}{8}, 0)$

Find coordinate of N:

$x = -2$, $y = -4$

$\frac{dy}{dx}$ where $x = -2$

gradient $= 0$, $y = -4$

at y axis so $x = 0 \quad N: (0, -4)$

Shoelace method:

ΔMNO

M: $(\frac{11}{8}, 0)$

N: $(0, -4)$

O: $(0, 0)$

M: $(\frac{11}{8}, 0)$

$A = \frac{1}{2} \left| \begin{matrix} \frac{11}{8} & 0 & 0 & \frac{11}{8} \\ 0 & -4 & 0 & 0 \end{matrix} \right|$

$= \frac{1}{2} \left| \left(\frac{11}{8} \times -4\right) \right|$

$= \frac{1}{2} \left| -\frac{11}{2} \right|$

$= \frac{1}{2}\left(\frac{11}{2}\right)$

$\boxed{\text{Area} = \frac{11}{4} \text{ unit}^2}$

0606_s09_qp_01

9 At 10 00 hours, a ship P leaves a point A with position vector $(-4\mathbf{i} + 8\mathbf{j})$ km relative to an origin O, where $\mathbf{i}$ is a unit vector due East and $\mathbf{j}$ is a unit vector due North. The ship sails north-east with a speed of $10\sqrt{2}$ km h^{-1}. Find

(i) the velocity vector of P, [2]

(ii) the position vector of P at 12 00 hours. [2]

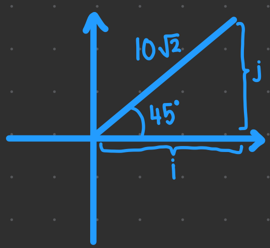
At 12 00 hours, a second ship Q leaves a point B with position vector $(19\mathbf{i} + 34\mathbf{j})$ km travelling with velocity vector $(8\mathbf{i} + 6\mathbf{j})$ km h^{-1}.

(iii) Find the velocity of P relative to Q. [REMOVED] [2]

(iv) Hence, or otherwise, find the time at which P and Q meet and the position vector of the point where this happens. [3]

(i) Speed: $10\sqrt{2}$
North East

Velocity vector:



$i = \cos(45) = \frac{i}{10\sqrt{2}}$

$i = 10$

$j = \sin(45) = \frac{j}{10\sqrt{2}}$

$j = 10$

Velocity vector $= 10i + 10j$

(ii) Position vector

$R = a + tv$

$a: -4i + 8j$

$t: 10.00 \rightarrow 12.00$
(2 hours)

$V: 10i + 10j$

$R = -4i + 8j + 2(10i + 10j)$

$= -4i + 8j + 20i + 20j$

$= 16i + 28j$

(iv) $r = a + tV$

P: at 12:00 $: 16i + 28j$ Q: at 12:00 $: 19i + 34j$

$a + tV = a + tV$

$16i + 28j + t(10i + 10j) = 19i + 34j + t(8i + 6j)$

$16i + 28j + 10ti + 10tj = 19i + 8ti + 34j + 6tj$

$(16 + 10t)i + (28 + 10t)j = (19 + 8t)i + (34 + 6t)j$

$16 + 10t = 19 + 8t$ $28 + 10t = 34 + 6t$

$10t - 8t = 19 - 16$ $10t - 6t = 34 - 28$

$t = \frac{3}{2}$ $t = \frac{6}{4}$ $t = \frac{3}{2}$

$\frac{3}{2} = 1.5h$ $12:00 + 1.5 = 13:30$

$r = a + tv$

$r = (19i + 34j) + \frac{3}{2}(8i + 6j)$

$r = 19i + 34j + 12i + 9j$

$r = 31i + 43j$

Position Vector $= 31i + 43j$

Time of meet $= 13:30$

Final Answer!!

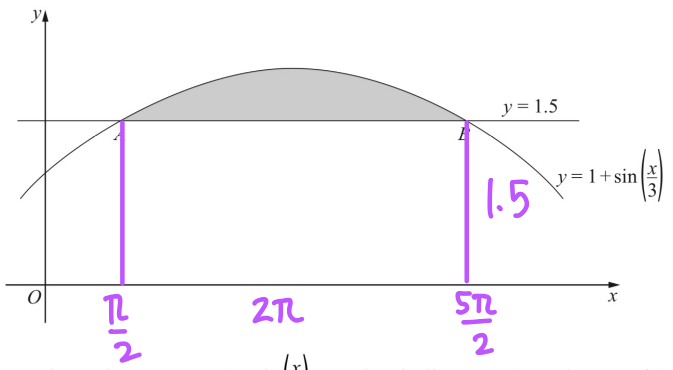
0606_s09_qp_01

12 Answer only **one** of the following two alternatives.

EITHER

(i) State the amplitude of $1 + \sin\left(\frac{x}{3}\right)$. [1]

(ii) State, in radians, the period of $1 + \sin\left(\frac{x}{3}\right)$. [1]



The diagram shows the curve $y = 1 + \sin\left(\frac{x}{3}\right)$ meeting the line $y = 1.5$ at points A and B. Find

(iii) the x-coordinate of A and of B, [3]

(iv) the area of the shaded region. [6]

i. $y = a \sin bx + c$

where a = amplitude

$\boxed{\text{So amplitude} = 1}$

ii. Period $= \frac{2\pi}{b} = \frac{2\pi}{\frac{1}{3}}$

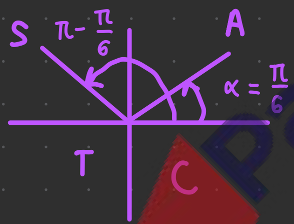
$\boxed{\text{Period} = 6\pi}$

ii Let $y = 1 + \sin\frac{x}{3}$ equal to $y = 1.5$

$1 + \sin\frac{x}{3} = 1.5$

$\sin\frac{x}{3} = 1.5 - 1$

$\frac{x}{3} = \sin^{-1}(0.5)$



$\frac{x}{3} = \frac{\pi}{6}$ $\quad \frac{x}{3} = \frac{5\pi}{6}$

$x = 3\left(\frac{\pi}{6}\right)$ $\quad x = 3\left(\frac{5\pi}{6}\right)$

$x = \frac{3\pi}{6}$ $\quad x = \frac{15\pi}{6}$

$x = \frac{\pi}{2}$ $\quad x = \frac{5\pi}{2}$

$\boxed{A = \frac{\pi}{2} \quad B = \frac{5\pi}{2}}$

Final Answer!!

iv) integrate $1 + \sin\left(\frac{x}{3}\right)$ with A & B as limits to find area under the graph:

$$\int_{\frac{\pi}{2}}^{\frac{5\pi}{2}} 1 + \sin\left(\frac{x}{3}\right)$$

$$\frac{x - \cos\left(\frac{x}{3}\right)}{\frac{1}{3}}$$

$$\left[x - 3\left(\cos\frac{x}{3}\right)\right]_{\frac{\pi}{2}}^{\frac{5\pi}{2}}$$

$$\frac{5\pi}{2} - 3\cos\frac{\frac{5\pi}{2}}{3} - \left(\frac{\pi}{2} - 3\cos\frac{\frac{\pi}{2}}{3}\right)$$

$= 2\pi + 3\sqrt{3}$

minus Area of rectangle $(2\pi \times 1.5)$

$= 2\pi + 3\sqrt{3} - 3\pi$

$\boxed{= 3\sqrt{3} - \pi}$

Final Answer!!

11 **(a)** Show that $\tan\theta + \cot\theta = \operatorname{cosec}\theta \sec\theta$. [3]

(b) Solve the equation

(i) $\tan x = 3\sin x$ for $0° < x < 360°$, [4]

(ii) $2\cot^2 y + 3\operatorname{cosec} y = 0$ for $0 < y < 2\pi$ radians. [5]

(a) $\tan\theta + \cot\theta$

$\tan\theta + \frac{1}{\tan\theta}$

$= \frac{\tan^2\theta + 1}{\tan\theta}$

$\frac{\sec^2\theta}{\tan\theta}$

$= \frac{1}{\cos^2\theta} \div \frac{\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta\sin\theta}$

$\operatorname{cosec}\theta \sec\theta = \text{RHS!}$

(b) $\tan x = 3\sin x$

$\frac{\sin x}{\cos x} = 3\sin x$

$\frac{\sin x}{\cos x} - 3\sin x = 0$

$\sin\left(\frac{1}{\cos x} - 3\right) = 0$

$\sin x = 0$ $\quad$ $\frac{1}{\cos x} - 3 = 0$

$x = \sin^{-1}(0)$ $\quad$ $\cos x = \frac{1}{3}$

$x = \cancel{0}, 180, \cancel{360}$ $\quad$ $x = \cos^{-1}\left(\frac{1}{3}\right)$

$x = 180°$ $\quad$ $x = 70.5$

$x = 360 - 70.5$

$x = 70.5, 289.5$

$\boxed{x = 70.5, 180, 289.5}$

↑ Final Answer!!

ii.) $2\cot^2 y + 3\operatorname{cosec} y = 0$

$2(\operatorname{cosec}^2 y - 1) + 3\operatorname{cosec} y = 0$

$2\operatorname{cosec}^2 y - 2 + 3\operatorname{cosec} y = 0$

$2\operatorname{cosec}^2 y + 3\operatorname{cosec} y - 2 = 0$

Let $\operatorname{cosec} y = x$

$2x^2 + 3x - 2 = 0$

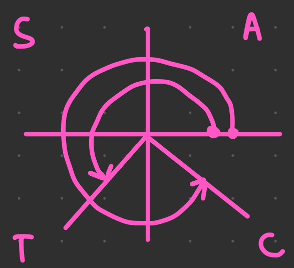
$x = \frac{1}{2}$ $\quad$ $x = -2$

$\operatorname{cosec} y = \frac{1}{2}$ $\quad$ $\operatorname{cosec} y = -2$

$\frac{1}{\sin y} = \frac{1}{2}$ $\quad$ $\frac{1}{\sin y} = -2$

$\sin y = 2$ $\quad$ $\sin y = -\frac{1}{2}$

[REJECT]



$y = \sin^{-1}\left(-\frac{1}{2}\right)$

$y = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$\boxed{y = \frac{7\pi}{6}, \frac{11\pi}{6}}$

↑ Final Answer!!

0606_s10_qp_11

11 Answer only **one** of the following two alternatives.

EITHER

A curve has equation $y = \dfrac{\ln x}{x^2}$, where $x > 0$.

(i) Find the exact coordinates of the stationary point of the curve. [6]

(ii) Show that $\dfrac{d^2y}{dx^2}$ can be written in the form $\dfrac{a\ln x + b}{x^4}$, where a and b are integers. [3]

(iii) Hence, or otherwise, determine the nature of the stationary point of the curve. [2]

i) $\dfrac{dy}{dx} = 0 \quad \dfrac{vu' - uv'}{v^2} = 0$

$\dfrac{\ln x}{x^2} \quad \dfrac{x^2\left(\frac{1}{x}\right) - \ln x(2x)}{(x^2)^2} = 0$

$u = \ln x$

$u' = \dfrac{1}{x} \quad x^2\left(\frac{1}{x}\right) - \ln x(2x) = 0$

$v = x^2 \quad x - 2x\ln x = 0$

$v' = 2x \quad -2x\ln x = -x$

$\ln x = \dfrac{-x}{-2x}$

$\ln x = \dfrac{1}{2}$

$x = e^{\frac{1}{2}}$

$y = \dfrac{\ln\left(e^{\frac{1}{2}}\right)}{\left(e^{\frac{1}{2}}\right)^2}$

$y = \dfrac{1}{2e}$

Coordinate : $\left(e^{\frac{1}{2}}, \dfrac{1}{2e}\right)$

Final Answer!!

---

ii.) $= \dfrac{x^2\left(\frac{1}{x}\right) - \ln x(2x)}{(x^2)^2} = \dfrac{x - 2x\ln x}{x^4} = \dfrac{x(1 - 2\ln x)}{x^4} = \dfrac{1 - 2\ln x}{x^3}$

$\dfrac{d}{dx}\left(\dfrac{1 - 2\ln x}{x^3}\right)$ $u = 1 - 2\ln x$, $u' = -\frac{2}{x}$, $v = x^3$, $v' = 3x^2$ $\quad \dfrac{vu' - uv'}{v^2} = \dfrac{x^3\left(-\frac{2}{x}\right) - 1 - 2\ln x\,(3x^2)}{(x^3)^2}$

$= \dfrac{-2x^2 - 3x^2(1 - 2\ln x)}{x^6} = \dfrac{\cancel{x^2}(-2 - 3(\ln - 2\ln x)}{x^{\cancel{6}}\,4} = \dfrac{2 + 3 + 6\ln x}{x^4} = \boxed{\dfrac{6\ln x + 5}{x^4}}$

Final Answer!!

---

iii. $\dfrac{d^2y}{dx^2} > 0$ minimum

$\dfrac{d^2y}{dx^2} < 0$ maximum

$\dfrac{6\ln x + 5}{x^4} = 0$

$6\ln x + 5 = 0$

$6\ln x = -5$

$\ln x = -\dfrac{5}{6}$

$x = e^{-\frac{5}{6}}$

$e^{-\frac{5}{6}} < 0$, maximum point!

OR

A curve is such that $\frac{dy}{dx} = 6\cos\left(2x + \frac{\pi}{2}\right)$ for $-\frac{\pi}{4} \leqslant x \leqslant \frac{5\pi}{4}$. The curve passes through the point $\left(\frac{\pi}{4}, 5\right)$.

Find

(i) the equation of the curve, [4]

(ii) the x-coordinates of the stationary points of the curve, [3]

(iii) the equation of the normal to the curve at the point on the curve where $x = \frac{3\pi}{4}$. [4]

[NEXT PAGE]

i.) integrate $\frac{dy}{dx}$ to find curve equation

$$\int 6\cos\left(2x + \frac{\pi}{2}\right) dx$$

$$y = \frac{6\sin\left(2x + \frac{\pi}{2}\right)}{2} + c$$

$$y = 3\sin\left(2x + \frac{\pi}{2}\right) + c$$

[Substitute $\left(\frac{\pi}{4}, 5\right)$]

$$5 = 3\sin\left(2\left(\frac{\pi}{4}\right) + \frac{\pi}{2}\right) + c$$

$$5 - 3\sin\left(2\left(\frac{\pi}{4}\right) + \frac{\pi}{2}\right) = c$$

$$c = 5$$

$$\boxed{y = 3\sin\left(2x + \frac{\pi}{2}\right) + 5 \ \#}$$

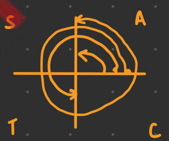
Final Answer!!

ii.) $\frac{dy}{dx} = 0$

$$6\cos\left(2x + \frac{\pi}{2}\right) = 0$$

$$\cos\left(2x + \frac{\pi}{2}\right) = 0$$

$$\left(2x + \frac{\pi}{2}\right) = \cos^{-1}(0)$$



$$\cos^{-1}(0) = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

$$2x + \frac{\pi}{2} = \frac{\pi}{2}$$

$$2x = 0$$

$$x = 0$$

$$2x + \frac{\pi}{2} = \frac{3\pi}{2}$$

$$2x = \frac{3\pi}{2} - \frac{\pi}{2}$$

$$x = \frac{\pi}{2}$$

$$2x + \frac{\pi}{2} = \frac{5\pi}{2}$$

$$2x = \frac{5\pi}{2} - \frac{\pi}{2}$$

$$x = \pi$$

0606_s10_qp_11

(iii) the equation of the normal to the curve at the point on the curve where $x = \frac{3\pi}{4}$. [4]

iii) Find gradient at $x = \frac{3\pi}{4}$ by substituting into $\frac{dy}{dx}$

$6\cos\left(2\left(\frac{3\pi}{4}\right)+\frac{\pi}{2}\right) = 6$

$m_1 = 6 \rightarrow$ Gradient of tangent

gradient of normal $= -\frac{1}{m_1}$

$m_2 = -\frac{1}{6}$

$y = -\frac{1}{6}x + c$

To get a set of coordinates, substitute $x = \frac{3\pi}{4}$ into curve equation.

$y = 3\sin\left(2\left(\frac{3\pi}{4}\right)+\frac{\pi}{2}\right) + 5$

$y = 5$

$y = -\frac{1}{6}x + c \quad \left(\frac{3\pi}{4}, 5\right)$

$5 = -\frac{1}{6}\left(\frac{3\pi}{4}\right) + c$

$5 + \frac{1}{6}\left(\frac{3\pi}{4}\right) = c$

$c = 5 + \frac{\pi}{8}$

$\boxed{y = -\frac{1}{6}x + 5 + \frac{\pi}{8}}$

Final Answer!!

0606_s10_qp_12

11 A particle moves in a straight line such that its displacement, x m, from a fixed point O on the line at time t seconds is given by $x = 12\{\ln(2t+3)\}$. Find

(i) the value of t when the displacement of the particle from O is 48 m, [3]

(ii) the velocity of the particle when $t = 1$, [3]

(iii) the acceleration of the particle when $t = 1$. [3]

i.) $48 = x$

Solve for t

$48 = 12(\ln(2t+3))$

$\frac{48}{12} = \ln(2t+3)$

$e^4 = 2t+3$

$e^4 - 3 = 2t$

$\boxed{t = \frac{e^4-3}{2}}$ → Final Answer!!

ii.) $\frac{dx}{dt}$ and let $t=1$

$12\left(\frac{2}{2t+3}\right)$ and let $t=1$

$12\left(\frac{2}{2+3}\right) = 12\left(\frac{2}{5}\right)$

$\boxed{= \frac{24}{5} \text{ m/s}}$ → Final Answer!!

ii.) $\frac{d^2x}{dt^2}$ and let $t=1$

$12\left(\frac{2}{2t+3}\right)$

$\frac{d}{dt}\left(\frac{24}{2t+3}\right)$ $\quad 24(2t+3)^{-1}$

$-24(2t+3)^{-2}(2)$

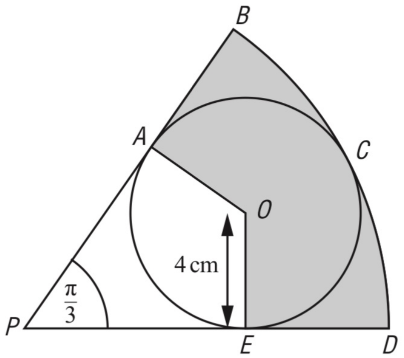
$= -48(2t+3)^{-2}$

$= -48(2+3)^{-2}$ $\boxed{= -\frac{48}{25} \text{ m/s}^2}$

Final Answer!!

0606_w10_qp_11

10



The diagram shows a circle, centre O, radius 4 cm, enclosed within a sector $PBCDP$ of a circle, centre P. The circle centre O touches the sector at points A, C and E. Angle BPD is $\frac{\pi}{3}$ radians.

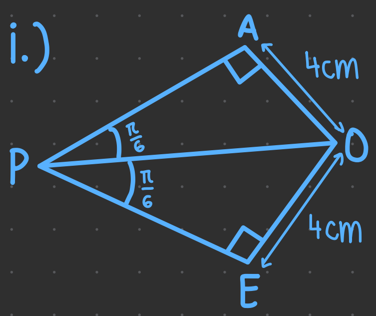
(i) Show that $PA = 4\sqrt{3}$ cm and $PB = 12$ cm. [2]

Find, to 1 decimal place,

(ii) the area of the shaded region, [4]

(iii) the perimeter of the shaded region. [4]

i.)



$\tan\left(\frac{\pi}{6}\right) = \frac{4}{PA}$

$PA = \frac{4}{\tan\left(\frac{\pi}{6}\right)}$

$PA = 4\sqrt{3}$

$PB = PC$

$PC = PO + \overset{\text{4, radius}}{OC}$

$\sin\left(\frac{\pi}{6}\right) = \frac{4}{PO}$

$PO = \frac{4}{\sin\left(\frac{\pi}{6}\right)}$

$PO = 8\text{cm}$

$PB = 4\text{cm} + 8\text{cm}$

$= 12\text{cm}$

ΔPOA & ΔPOE

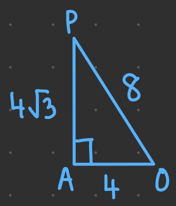
ii.) Area of Sector BPD − 2 triangles

Area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}(12)^2\left(\frac{\pi}{3}\right)$

$= 24\pi$

Area of triangle $= \frac{1}{2}bh$



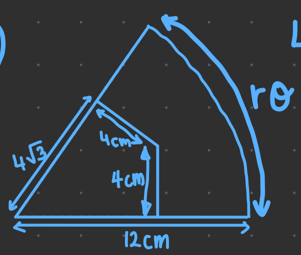
$= \frac{1}{2}(4\sqrt{3})(4)$

$= 8\sqrt{3}$

$24\pi - 2(8\sqrt{3}) = \boxed{47.69\text{cm}^2}$

Final Answer!!

iii)



$4 + 4 + 2(12 - 4\sqrt{3}) + 4\pi$

$8 + 24 - 8\sqrt{3} + 4\pi$

$\boxed{= 30.7\text{cm}}$

Final Answer!!

$12\left(\frac{\pi}{3}\right) = 4\pi$

0606_w10_qp_11

11 **(i)** Find $\int \frac{1}{\sqrt{1+x}}\,dx$. [2]

(ii) Given that $y = \frac{2x}{\sqrt{1+x}}$, show that $\frac{dy}{dx} = \frac{A}{\sqrt{1+x}} + \frac{Bx}{\left(\sqrt{1+x}\right)^3}$, where A and B are to be found. [4]

(iii) Hence find $\int \frac{x}{\left(\sqrt{1+x}\right)^3}\,dx$ and evaluate $\int_0^3 \frac{x}{\left(\sqrt{1+x}\right)^3}\,dx$. [4]

i.) $\int \frac{1}{\sqrt{1+x}}\,dx$

$\int 1(1+x)^{-\frac{1}{2}}\,dx$

$\boxed{= 2(1+x)^{\frac{1}{2}} + C}$ Final Answer!!

ii.) $\frac{d}{dx}\left(\frac{2x}{\sqrt{1+x}}\right)$

$2x(1+x)^{-\frac{1}{2}}$

$u = 2x$ $\quad uv' + vu'$

$u' = 2$

$v = (1+x)^{-\frac{1}{2}}$

$v' = -\frac{1}{2}(1+x)^{-\frac{3}{2}}$

$2x\left(-\frac{1}{2}(1+x)^{-\frac{3}{2}}\right) + (1+x)^{-\frac{1}{2}}(2)$

$-x(1+x)^{-\frac{3}{2}} + 2(1+x)^{-\frac{1}{2}}$

$= \frac{2}{(1+x)^{\frac{1}{2}}} - \frac{x}{(1+x)^{\frac{3}{2}}}$

$\boxed{\begin{array}{l} A = 2 \\ B = -1 \end{array}}$ Final Answer!!

iii.) $\int \frac{2}{(1+x)^{\frac{1}{2}}} - \frac{x}{(1+x)^{\frac{3}{2}}}\,dx = \frac{2x}{\sqrt{1+x}}$

$\int \frac{2}{(1+x)^{\frac{1}{2}}}\,dx - \int \frac{x}{(1+x)^{\frac{3}{2}}}\,dx = \frac{2x}{\sqrt{1+x}}$

$\int \frac{2}{(1+x)^{\frac{1}{2}}}\,dx - \frac{2x}{\sqrt{1+x}} = \int \frac{x}{(1+x)^{\frac{3}{2}}}\,dx$

$\int 2(1+x)^{-\frac{1}{2}} = 1(1+x)^{\frac{1}{2}}$

$\left[1(1+x)^{\frac{1}{2}} - \frac{2x}{\sqrt{1+x}}\right]_0^3$

insert limits:

$(1+3)^{\frac{1}{2}} - \frac{2(3)}{\sqrt{1+3}} - \left((0+1)^{\frac{1}{2}} - \frac{2(0)}{\sqrt{0+1}}\right)$

$\boxed{= 1}$

Final Answer!!

0606_w10_qp_13

12 **OR**

A curve has the equation $y = Ae^{2x} + Be^{-x}$ where $x \geqslant 0$. At the point where $x = 0$, $y = 50$ and $\frac{dy}{dx} = -20$.

(i) Show that $A = 10$ and find the value of B. [5]

(ii) Using the values of A and B found in part **(i)**, find the coordinates of the stationary point on the curve. [4]

(iii) Determine the nature of the stationary point, giving a reason for your answer. [2]

i.) Substitute $x=0$ & $y=50$ into $y=Ae^{2x}+Be^{-x}$

$50 = Ae^{2(0)} + Be^{-(0)}$

$A+B=50$ — ①

$\frac{dy}{dx} = -20$

$\frac{d}{dx}(Ae^{2x}+Be^{-x})$

$2Ae^{2x} - Be^{-x}$ when $x=0$, $\frac{dy}{dx}=-20$

$2Ae^{2(0)} - Be^{-(0)} = -20$

$2A - B = -20$ — ②

Solve ① & ② simultaneously

Elimination Method

$$\begin{array}{r} 2A - \cancel{B} = -20 \\ + \quad A + \cancel{B} = 50 \\ \hline 3A = 30 \end{array}$$

$A = 10$

Substitute $A=10$ into $A+B=50$

$10+B=50$

$B=40$

ii.) $\frac{dy}{dx}=0$

$2Ae^{2x} - Be^{-x} = 0$

$2(10)e^{2x} - 40e^{-x} = 0$

$20e^{2x} = 40e^{-x}$

$\frac{e^{2x}}{e^{-x}} = \frac{40}{20}$

$e^{3x} = 2$

$3x = \ln 2$

$x = \frac{\ln 2}{3}$

To find y, substitute x into Y equation

$10e^{2(\frac{\ln 2}{3})} + 40e^{-(\frac{\ln 2}{3})} = y$

$y = 47.6$

$\boxed{\left(\frac{\ln 2}{3}, 47.6\right)}$ → Final Answer!!

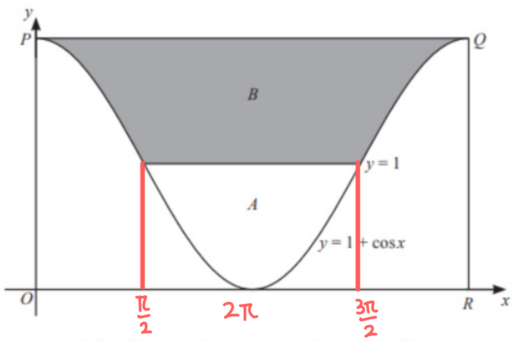
iii.) $\frac{d}{dx}(20e^{2x} - 40e^{-x})$

if $\frac{d^2y}{dx^2} > 0$ minimum & < 0, maximum

$40e^{2x} + 40e^{-x} > 0$ $\boxed{\text{minimum}}$

0606_S22_qp_22

11



The diagram shows part of the line $y = 1$ and one complete period of the curve $y = 1 + \cos x$, where x is in radians. The line PQ is a tangent to the curve at P and at Q. The line QR is parallel to the y-axis. Area A is enclosed by the line $y = 1$ and the curve. Area B is enclosed by the line $y = 1$, the line PQ and the curve.

Given that area A : area B is $1 : k$ find the exact value of k. [9]

(1) Find area PQOR $= 2 \times 2\pi = 4\pi$

(2) Area under the graph

$$= \int_0^{2\pi} 1 + \cos x \; dx$$

$$= \left[x + \sin x\right]_0^{2\pi}$$

$$= 2\pi + \sin(2\pi) - (0 + \sin(0))$$

$$= 2\pi$$

(3) Find A & B

$$A + B = 4\pi - 2\pi$$
$$= 2\pi$$

Let $1 + \cos x$ equal to 1 to find intersects ($\frac{3\pi}{2}$ & $\frac{\pi}{2}$)

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 1 + \cos x$$

$$\left[x + \sin x\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$\frac{3\pi}{2} + \sin\frac{3\pi}{2} - \left(\frac{\pi}{2} + \sin\frac{\pi}{2}\right) = \pi - 2$$

Area of A $= \pi - (\pi - 2)$ — Area of Rectangle

$A = 2$

(4) Find B

$$A + B = 2\pi$$
$$A = 2$$
$$B = 2\pi - 2$$

(5) Write A : B as ratio & simplify

$$(2 : 2\pi - 2) \div 2$$

$$1 : \underline{\pi - 1}$$

$$\boxed{k = \pi - 1}$$

Final Answer!!

6 The points $A(5, -4)$ and $C(11, 6)$ are such that AC is the diagonal of a square, $ABCD$.

(a) Find the length of the line AC. [2]

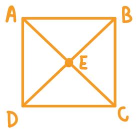
$AC = \sqrt{(5-11)^2 + (-4-6)^2}$

$AC = 2\sqrt{74}$ #

(b) (i) The coordinates of the centre, E, of the square are $(8, y)$. Find the value of y. [1]

Midpoint AC : $\left(\frac{5+11}{2}, \frac{-4+6}{2}\right)$

$= (8, 1)$ $y = 1$ #



(ii) Find the equation of the diagonal BD. [3]

ii.) BD = perpendicular bisector of AC

gradient AC : $\frac{-4-6}{5-11}$ midpoint AC $[8,1]$

$: \frac{5}{3}$

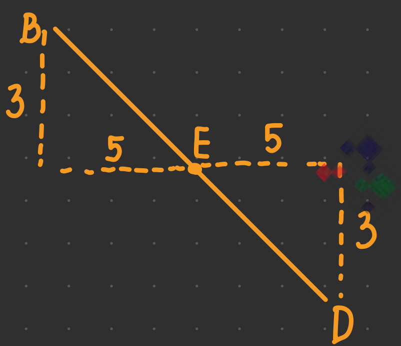
Perpendicular gradient $= -\frac{1}{\frac{5}{3}} = -\frac{3}{5}$

BD $\cdot$ $y - 1 = -\frac{3}{5}(x - 8)$ BD : $y = -\frac{3}{5}x + \frac{24}{5} + 1$

Final Answer!!

$\boxed{y = -\frac{3}{5}x + \frac{29}{5}}$

(iii) Given that the x-coordinate of B is less than the x-coordinate of D, write $\overrightarrow{EB}$ and $\overrightarrow{ED}$ as column vectors. [2]



use gradient $-\frac{3}{5}$
to see translation

$\overrightarrow{EB} : \begin{pmatrix} -5 \\ 3 \end{pmatrix}$

$\overrightarrow{ED} : \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

13 The functions f and g are defined, for $x > 0$, by

$$f(x) = \frac{2x^2 - 1}{3x},$$

$$g(x) = \frac{1}{x}.$$

(b) (i) Given that f^{-1} exists, write down the range of f^{-1}. [1]

(ii) Show that $f^{-1}(x) = \frac{px + \sqrt{qx^2 + r}}{4}$, where p, q and r are integers. [4]

b i.) range of $f^{-1}(x)$ = domain of $f(x)$

$\boxed{f^{-1}(x) > 0}$ → Final Answer!!

b ii.) $\frac{2x^2-1}{3x} = y$

$2x^2 - 1 = 3xy$

$2x^2 - 3xy - 1 = 0$

$a = 2, \ b = -3y, \ c = -1$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-3y) \pm \sqrt{(-3y)^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{3y \pm \sqrt{9y^2 + 8}}{4} \quad (f^{-1}(x) > 0)$$

$$\boxed{f^{-1}(x) = \frac{3x + \sqrt{9x^2 + 8}}{4}}$$

↑ Final Answer!!