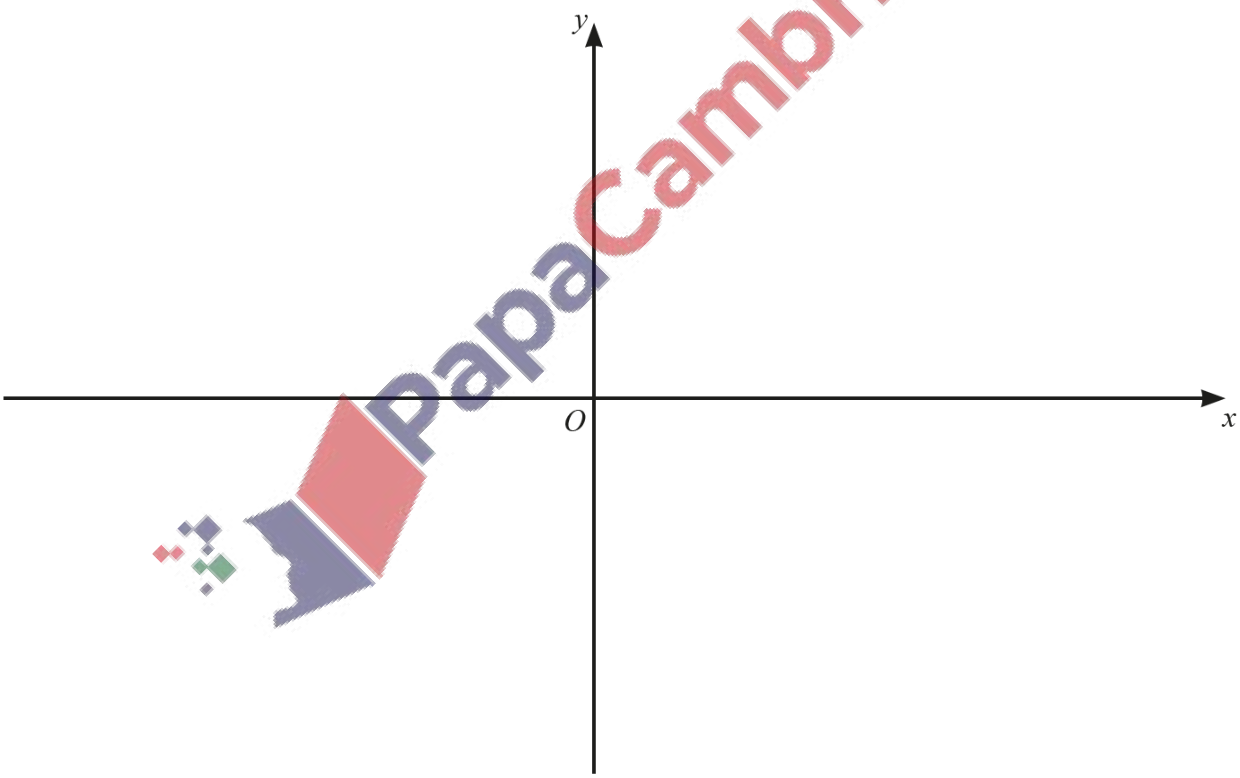
1. Nov/2023/Paper_0606/12/No.8

(a) It is given that $f : x \to (3x+1)^2 - 4$ for $x \geqslant a$, and that f^{-1} exists.

(i) Find the least possible value of a. [1]

(ii) Using this value of a, write down the range of f. [1]

(iii) Using this value of a, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the axes, stating the intercepts with the coordinate axes. [4]



(b) It is given that

$$g(x) = \ln(2x^2 + 5) \quad \text{for } x \geqslant 0,$$

$$h(x) = 3x - 2 \quad \text{for } x \geqslant 0.$$

Solve the equation $hg(x) = 4$ giving your answer in exact form. [3]

2. Nov/2023/Paper_0606/21/No.9

The functions f and g are defined as follows, for all real values of x.

$$f(x) = 2x^2 - 1$$
$$g(x) = e^x + 1$$

(a) Solve the equation $fg(x) = 8$. [3]

(b) For each of the functions f and g, either explain why the inverse function does not exist or find the inverse function, stating its domain. [4]

3. Nov/2023/Paper_0606/23/No.1

The functions f and g are defined as follows, for all real values of x.

$$f(x) = 2\sin x + 3\cos x$$

$$g(x) = e^{3x} - 1$$

(a) Find fg(0). [2]

(b) Find gg(x). [1]

(c) Solve the equation $g^{-1}(x) = \frac{1}{3}\ln 5$. [3]

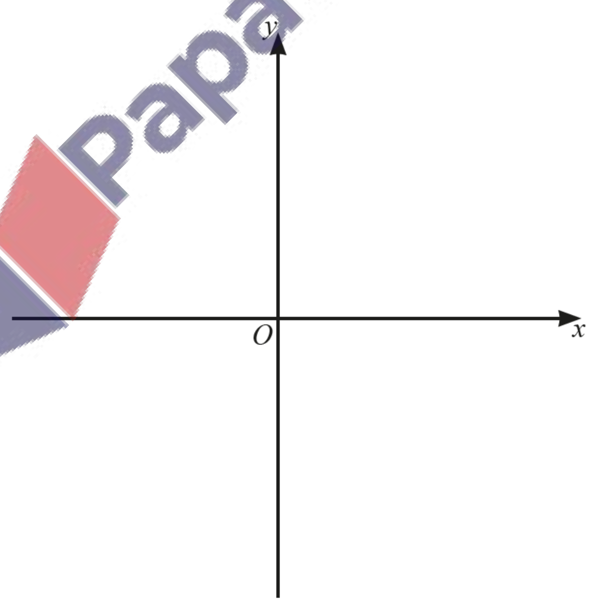
4. March/2023/Paper_0606/22/No.8

The function f is defined for $x \geqslant 0$ by $f(x) = 5 - 2e^{-x}$.

(a) **(i)** Find the domain of f^{-1}. [2]

(ii) Solve $ff^{-1}(x) = \sqrt{5x - 4}$. [3]

(iii) On the axes, sketch the graph of $y = f(x)$ and hence sketch the graph of $y = f^{-1}(x)$. Show clearly the positions of any points where your graphs meet the coordinate axes and the positions of any asymptotes. [4]



(b) The function g is defined for $0 \leqslant x \leqslant 0.2$ by $g(x) = \frac{3}{1-x}$.

Find and simplify an expression for $f^{-1}g(x)$. [4]

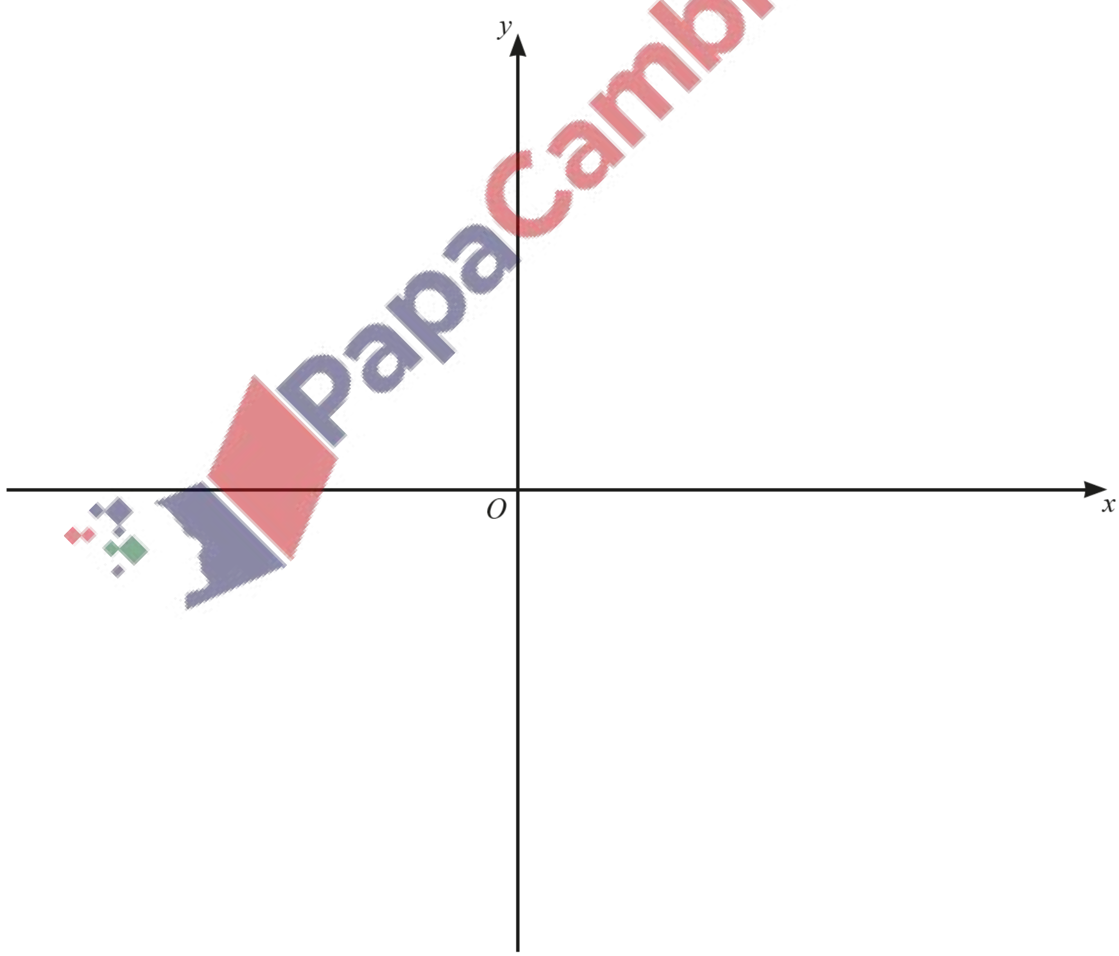
5. June/2023/Paper_0606/12/No.8

It is given that $f(x) = 2\ln(3x - 4)$ for $x > a$.

(a) Write down the least possible value of a. [1]

(b) Write down the range of f. [1]

(c) It is given that the equation $f(x) = f^{-1}(x)$ has two solutions. (You do not need to solve this equation). Using your answer to **part (a)**, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the axes below, stating the coordinates of the points where the graphs meet the axes. [4]



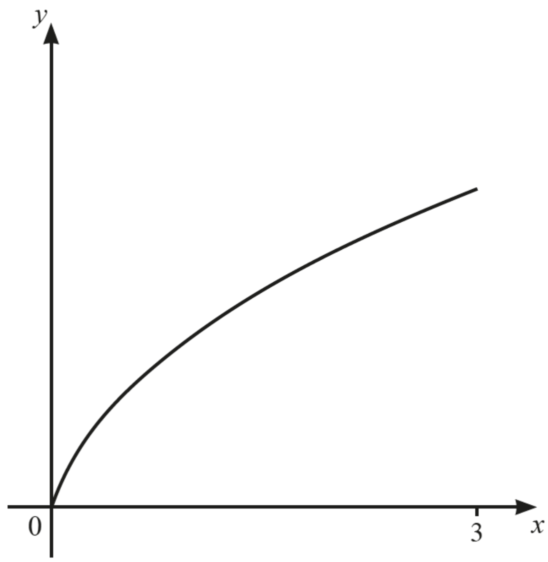
It is given that $g(x) = 2x - 3$ for $x \geqslant 3$.

(d) **(i)** Find an expression for $g(g(x))$. [1]

(ii) Hence solve the equation $fg(g(x)) = 4$ giving your answer in exact form. [3]

6. June/2023/Paper_0606/21/No.8

(a)



The diagram shows the graph of $y = \mathrm{f}(x)$ where f is defined by $\mathrm{f}(x) = \dfrac{3x}{\sqrt{5x+1}}$ for $0 \leqslant x \leqslant 3$.

(i) Given that f is a one-one function, find the domain and range of f^{-1}. [3]

(ii) Solve the equation $\mathrm{f}(x) = x$. [2]

(iii) On the diagram above, sketch the graph of $y = \mathrm{f}^{-1}(x)$. [2]

(b) The functions g and h are defined by

$$g(x) = \sqrt[3]{8x^3 + 3} \quad \text{for} \quad x \geqslant 1,$$

$$h(x) = e^{4x} \quad \text{for} \quad x \geqslant k.$$

(i) Find an expression for $g^{-1}(x)$. [2]

(ii) State the least value of the constant k such that $gh(x)$ can be formed. [1]

(iii) Find and simplify an expression for $gh(x)$. [1]

7. June/2023/Paper_0606/23/No.8

(a) The functions f and g are defined by

$$f(x) = \sec x \qquad \text{for } \frac{\pi}{2} < x < \frac{3\pi}{2}$$

$$g(x) = 3(x^2 - 1) \qquad \text{for all real } x.$$

(i) Find the range of f. [1]

(ii) Solve the equation $f^{-1}(x) = \frac{2\pi}{3}$. [3]

(iii) Given that gf exists, state the domain of gf. [1]

(iv) Solve the equation $gf(x) = 1$. [5]

(b) The function h is defined by $h(x) = \ln(4 - x)$ for $x < 4$. Sketch the graph of $y = h(x)$ and hence sketch the graph of $y = h^{-1}(x)$. Show the position of any asymptotes and any points of intersection with the coordinate axes. [4]