CAMBRIDGE
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Cambridge International A Level

MATHEMATICS **9709/35**

Paper 3 Pure Mathematics 3 **May/June 2025**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1. Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2. Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3. Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5. Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
6. Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	**Meaning**
^	More information required
A0	Accuracy mark awarded zero
A1	Accuracy mark awarded one
B0	Independent accuracy mark awarded zero
B1	Independent accuracy mark awarded one
B2	Independent accuracy mark awarded two
BOD	Benefit of the doubt
BP	Blank Page
✗	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
FT	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
ISW	Ignore subsequent work
J	Judgement
JU	Judgement
M0	Method mark awarded zero
M1	Method mark awarded one
M2	Method mark awarded two
MR	Misread
O	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
PE	Judgment made by the PE
Pre	Premature approximation
SC	Special case
SEEN	Indicates that work/page has been seen

Annotation	**Meaning**
SF	Error in number of significant figures
✓	Correct
TE	Transcription error
XP	Correct answer from incorrect working

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

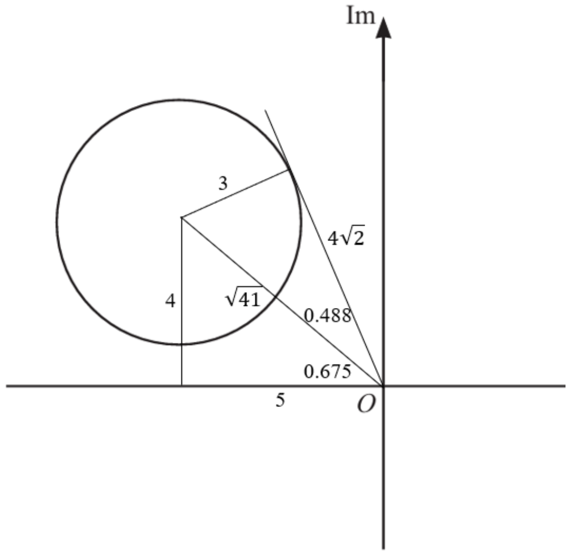
Question	Answer	Marks	Guidance
1	Use law of logarithm of a product or quotient	**M1**	$\ln 5 + \ln 6^{x-1}$ or $\ln 3^{4-2x} - \ln 5$ or $\ln 3^{4-2x} - \ln 6^{x-1}$ Allow logs to any base but must be consistent throughout the equation.
	Use law of logarithm of a power twice	**M1**	$(4-2x)\ln 3$ and $(x-1)\ln 6$. Omission of bracket(s) is an accuracy error if not corrected later. Can have M0M1, i.e. go wrong with product but powers dealt with correctly. E.g. $(4-2x)\ln 3 = \ln 5\times(x-1)\ln 6$, or $(4-2x)\ln 3 = (x-1)\ln 30$ or $3^{3x}\times 2^{x}$ written as $3x\ln 3\times x\ln 2$.
	Obtain a correct equation in any form, e.g. $(4-2x)\ln 3 = \ln 5 + (x-1)\ln 6$ or $1.10(4-2x) = 1.61 + 1.79(x-1)$	**A1**	May see $(4-2x)\ln 3$ written as $(2-x)\ln 9$.
	Obtain $x = 1.15$	**A1**	Must be 3 s.f. If no working seen, no marks available.
		4	

Question	Answer	Marks	Guidance
2	Reduce to an equation in a single trig function by use of correct trig formula(s)	**M1**	E.g. $\operatorname{cosec}^2\theta = \cot^2\theta + 1$.
	Obtain correct simplified solvable equation in one trig function with all terms on one side, any order, including *θ,* e.g. one of $4\cot^2\theta - 3\cot\theta - 1 = 0$ $\quad$ $\tan^2\theta + 3\tan\theta - 4 = 0$ $34\sin^4\theta - 49\sin^2\theta + 16 = 0$ $\quad$ $34\cos^4\theta - 19\cos^2\theta + 1 = 0$ $\frac{34}{9}\cos^2 2\theta + \frac{10}{3}\cos 2\theta = 0$ $\quad$ $*\sqrt{34}\sin\left(2\theta - \tan^{-1}\left(\frac{5}{3}\right)\right) = 3$	**A1**	Other equations may be possible. *For this equation, it is not necessary to have all terms on one side.
	Solve *their* equation correctly, by formula, factorisation or calculator, to obtain two values for a trig function, e.g. one of $\cot\theta = 1$ and $-\frac{1}{4}$ $\quad$ $0\ \tan\theta = 1$ and -4 $\sin\theta = \pm\frac{4}{\sqrt{17}}$ and $\pm\frac{1}{\sqrt{2}}$ $\quad$ $\cos\theta = \pm\frac{1}{\sqrt{17}}$ and $\pm\frac{1}{\sqrt{2}}$ $\cos 2\theta = 0$ and $-\frac{15}{17}$ $\quad$ $*\sin\left(2\theta - \tan^{-1}\left(\frac{5}{3}\right)\right) = \frac{3}{\sqrt{34}}$	**M1**	M0M1 is available if only sign slip(s) in trig formula(s). Incorrect solvable equation from use of correct trig formula(s) can score M1M1. *For this equation, only one value is needed.
	Obtain two of answers, $\frac{1}{4}\pi$, $-\frac{3}{4}\pi$, -1.33, 1.82 Second M1 can be implied by $\frac{1}{4}\pi$ or $-\frac{3}{4}\pi$, and -1.33 or 1.82	**A1**	ISW Accept 0.785 for $\frac{1}{4}\pi$ and –2.36 for $-\frac{3}{4}\pi$. AWRT –1.33, 1.82, 0.785, –2.36.
	Obtain all answers $\frac{1}{4}\pi$, $-\frac{3}{4}\pi$, -1.33, 1.82 and no others in the interval	**A1**	ISW Accept 0.785 for $\frac{1}{4}\pi$ and –2.36 for $-\frac{3}{4}\pi$. AWRT –1.33, 1.82, 0.785, –2.36. Ignore answers outside the given interval.
		5	

Question	Answer	Marks	Guidance
3(a)	Correct modulus = $\frac{5}{6}$ or 0.833 or $\frac{5}{6}e^{i\theta}$ or $r = \frac{5}{6}$ only	**B1**	Note 0.83 scores B0. Do not ISW other than converting fractions to decimals.
	Correct arg = –2.75 or $-\frac{11}{4}$ or $re^{-i2.75}$ or $\theta = -2.75$ only	**B1**	0.25 – 3 must be evaluated. B0 for $\theta = -2.75$i. Do not ISW other than converting fractions to decimals.
		2	
3(b)	Enlargement [scale] factor $\frac{1}{6}$ Allow expansion or stretch for enlargement Do not allow e.g. 'reduction' or 'shrink' or 'compression' for enlargement	**B1**	Need both parts. Not required to state centre (0, 0), but if incorrect then B0.
	Clockwise rotation of 3 [radians] or rotation of –3 [radians], or [anticlockwise] rotation of (2π – 3) [radians]	**B1**	Need both parts. Not required to state centre (0, 0), but if incorrect then B0. If B0B0 scored, allow **SC B1** for enlargement AND rotation stated.
		2	

Question	Answer	Marks	Guidance
4	Use the correct product rule (Note: may start with $12x^3\ln x$)	*M1	Attempt $\ln x^4 \frac{\mathrm{d}}{\mathrm{d}x}\left(3x^3\right)+3x^3\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln x^4\right)$ with *their* derivatives.
	Obtain the correct derivative in any form e.g. $9x^2\ln x^4+12x^2$ (If starting with $12x^3\ln x$, should get e.g. $36x^2\ln x+12x^2$)	A1	E.g. $9x^2\ln x^4+3x^3\times\frac{4x^3}{x^4}$. May see $\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln x^4\right)=\frac{4}{x}$.
	Equate to zero and eliminate ln, e.g. $x^4=\mathrm{e}^{-\frac{4}{3}}$	DM1	E.g. $ax^4=\mathrm{e}^b$, or $cx^3=\mathrm{e}^d$ or $x=\mathrm{e}^f$. Allow this mark even if in decimals. Allow **SCB1** if incorrect sign in product rule resulting in $x^4=\mathrm{e}^{\frac{4}{3}}$, OE.
	Obtain $x=\mathrm{e}^{-\frac{1}{3}}$ only or exact simplified equivalent	A1	ISW $x=\left(\mathrm{e}^{-\frac{4}{3}}\right)^{\frac{1}{4}}$ scores A0. $x=\pm\mathrm{e}^{-\frac{1}{3}}$ scores A0. Answers with no working score no marks.
	Obtain $y=-\frac{4}{\mathrm{e}}$ only or exact simplified equivalent	A1	ISW $y=3\mathrm{e}^{-1}\ln\mathrm{e}^{-\frac{4}{3}}$ scores A0. Answers with no working score no marks.
		5	

Question	Answer	Marks	Guidance
5(a)	Attempt at method for calculating the distance from the origin to the centre of the circle	**M1**	E.g. $\sqrt{4^2+5^2}$ or $\sqrt{41}$ may be seen on diagram.
	Obtain distance $\sqrt{41}-3$	**A1**	ISW AWRT 3.4(0). Need not identify as maximum and minimum, but if swapped, **M1 SCB1** max 2/3.
	Obtain distance $\sqrt{41}+3$	**A1**	ISW AWRT 9.4(0). Note: $\left(5+\frac{\sqrt{3}}{2}\right)^2+\left(4+\frac{\sqrt{3}}{2}\right)^2$ = 9.391 scores M0A0A0.
		3	

Question	Answer	Marks	Guidance
5(b)	Correct tangent and sight of correct trigonometry Identify or imply correct point on the circle and sight of $\tan^{-1}\frac{4}{5}$ or $0.674(7)$ or 38.7 or $\tan^{-1}\frac{5}{4}$ or 0.896 or 51.3 or $\sin^{-1}\frac{3}{\sqrt{41}}$ OE, or 0.487(6) or 27.9 Allow $\sin^{-1}\frac{5 \text{ or } 4}{\sqrt{41}}$ or $\cos^{-1}\frac{5 \text{ or } 4}{\sqrt{41}}$ instead of inverse tan If no correct tangent on diagram: $\tan^{-1}\frac{4}{5}$ or $\tan^{-1}\frac{5}{4}$ or $\tan^{-1}\frac{4}{-5}$ (or above alternative trigonometry) AND $\sin^{-1}\frac{3}{\sqrt{41}}$ OE B1FT $\tan^{-1}\frac{4}{5}-\sin^{-1}\frac{3}{\sqrt{41}}$ scores B1FT M0 A0 $\tan^{-1}\frac{4}{-5}-\sin^{-1}\frac{3}{\sqrt{41}}$ scores B1FT M1 1.98 A1 $-2\sin^{-1}\frac{3}{\sqrt{41}}-\tan^{-1}\frac{1}{5}$ scores B0 M0 A0	**B1FT**	 FT *their* $\sqrt{41}$ found from a correct method for B1 and M1 marks, e.g. by drawing correct tangent and sight of $\tan^{-1}\frac{4}{5}$ or $\tan^{-1}\frac{5}{4}$ or $\sin^{-1}\frac{3}{\sqrt{41}}$ OE. Note $\sin^{-1}\frac{3}{\sqrt{41}}$ may be $\cos^{-1}\frac{4\sqrt{2}}{\sqrt{41}}$ or $\tan^{-1}\frac{3}{4\sqrt{2}}$ or $\tan^{-1}\frac{3\sqrt{2}}{8}$. **SC** Thinks min arg z is actually max arg z: $\frac{\pi}{2}+\tan^{-1}\frac{5}{4}+\sin^{-1}\frac{3}{\sqrt{41}}$ B1FT 2.95447^c or 169.279° B1.

Question	Answer	Marks	Guidance
5(b)	Attempt at correct method to find the required argument, e.g. $\pi - \tan^{-1}\frac{4}{5} - \sin^{-1}\frac{3}{\sqrt{41}}$	**M1**	Or $\frac{\pi}{2} + \tan^{-1}\frac{5}{4} - \sin^{-1}\frac{3}{\sqrt{41}}$. FT *their* $\sqrt{41}$ found from a correct method for B1 and M1 marks. Previous B1 may be implied by a correct method here.
	Obtain 1.98 or 113.4	**A1**	AWRT E.g. 1.97923… or 113.401…
		3	

Question	Answer	Marks	Guidance
6(a)	Obtain $\frac{dy}{dt} = 3\sec^2 3t$	B1	OE
	Attempt chain rule on $2(\cos 3t)^{-1}$ for $\frac{dx}{dt}$ or attempt to differentiate $2\sec 3t$	M1	Attempt $\frac{dx}{dt} = -2(\cos 3t)^{-2}\frac{d}{dt}(\cos 3t)$ with *their* derivative.
	Obtain $\frac{dx}{dt} = 6\sec 3t \tan 3t$	A1	OE, e.g. $6(\cos 3t)^{-2}\sin 3t$. Allow unsimplified, e.g. $(-1) \times 2 \times (-1) \times 3$ for 6.
	Use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ with *their* $\frac{dy}{dt}$ and *their* $\frac{dt}{dx}$	M1	Expect, for example, $3\sec^2 3t \times \frac{1}{6\sec 3t\ \tan 3t}$ or $3\sec^2 3t \times \frac{1}{6(\cos 3t)^{-2}\sin 3t}$ if correct.
	Obtain $\frac{dy}{dx} = \frac{1}{2}\operatorname{cosec} 3t$	A1	WWW Must be in this form of answer given in the question. Not required to state $A = \frac{1}{2}$. Allow slips in notation for $\frac{dy}{dt}$, $\frac{dx}{dt}$ and $\frac{dy}{dx}$.

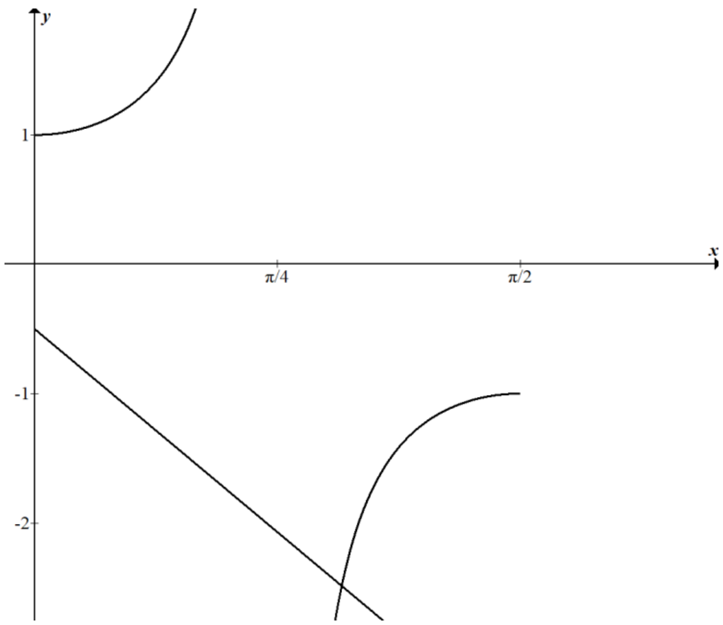
Question	Answer	Marks	Guidance
6(a)	**Alternative Method for Question 6(a)**		
	Convert to Cartesian form e.g. $1 + y^2 = \frac{x^2}{4}$	**B1**	
	Correct use of implicit differentiation e.g. $\frac{\mathrm{d}}{\mathrm{d}y} y^2 = 2y\frac{\mathrm{d}y}{\mathrm{d}x}$	**B1**	
	Obtain $2y\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{2}$	**B1**	OE
	Convert to parametric form e.g. $2\tan 3t\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\sec 3t}{2}$	**M1**	
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\operatorname{cosec} 3t$	**A1**	WWW Must be in this form of answer given in the question.
		5	

Question	Answer	Marks	Guidance
6(b)	Obtain $x = 2\sqrt{2}$ and $y = 1$ when $t = \frac{1}{12}\pi$	**B1**	Must be exact.
	Substitute $t = \frac{1}{12}\pi$ into $-1 \div$ *their* $\frac{dy}{dx}$ Allow a small slip, but not with *their* coefficient A	**M1**	Expect gradient of normal $= -\sqrt{2}$ if correct. Allow if in decimals. If *their* coefficient A is dealt with incorrectly M0, but allow second M1.
	Form equation of the normal with *their* (x, y), found using $x = \frac{2}{\cos 3t}$ and $y = \tan 3t$, and $-1 \div$ *their* $\frac{dy}{dx}$	**M1**	E.g. $y - 1 = -\sqrt{2}\left(x - 2\sqrt{2}\right)$ if correct or find c in equation of line. Allow M1 even if decimals. M0 if using gradient of tangent.
	Obtain equation of normal $y = -\sqrt{2}x + 5$ Require $y = mx + c$ and exact m and c	**A1**	CAO Accept $y = -\frac{2}{\sqrt{2}}x + 5$.
		4	

Question	Answer	Marks	Guidance
7(a)	Differentiate to obtain $\dfrac{k_1}{1+Bx^2}$ or $\dfrac{k_2}{x^2+\dfrac{1}{B}}$	M1	k_1 , $k_2 \neq 1$ and $B \neq 1$ or 4.
	Obtain $\dfrac{4}{1+16x^2}$	A1	OE, e.g. $\dfrac{1}{4}\times\dfrac{1}{x^2+\dfrac{1}{16}}$.
	Set $\dfrac{dy}{dx}=\dfrac{1}{4}$ and obtain $x=\pm\dfrac{\sqrt{15}}{4}$ or $\pm\dfrac{\sqrt{15}}{\sqrt{16}}$	A1	Or exact equivalent.
	Alternative Method for Question 7(a)		
	Rewrite as $\tan y = 4x$ and obtain $\sec^2 y = k\dfrac{dx}{dy}$	M1	
	Replace $\sec^2 y$ with $1+\tan^2 y$ and $\tan y$ with $4x$ to obtain $1+16x^2=4\dfrac{dx}{dy}$	A1	OE
	Set $\dfrac{dx}{dy}=4$ and obtain $x=\pm\dfrac{\sqrt{15}}{4}$ or $\pm\dfrac{\sqrt{15}}{\sqrt{16}}$	A1	Or exact equivalent.
		3	

Question	Answer	Marks	Guidance
7(b)	Commence integration by parts and reach $Ax\tan^{-1}(4x) \pm \int x\frac{B}{1+Cx^2}\,dx$	*M1	OE $A, B, C \neq 0$
	Obtain $x\tan^{-1}(4x) - \int \frac{4x}{1+16x^2}\,dx$	A1	OE
	Complete integration and obtain $x\tan^{-1}(4x) \pm \frac{B}{2C}\ln(1+16x^2)$	A1FT	OE SOI FT *their B* and *their C*. Expect $x\tan^{-1}(4x) - \frac{1}{8}\ln(1+16x^2)$ if correct.
	Substitute limits correctly in an expression of the form $Ax\tan^{-1}(4x) - D\ln(1+Cx^2)$	DM1	$D \neq 0$ Allow omission of substitution of $x = 0$.
	Obtain answer $\frac{1}{16}\pi - \frac{1}{8}\ln 2$ or exact equivalent	A1	ISW Allow, for example, $\frac{1}{16}\pi + \frac{1}{4}\ln\left(\frac{1}{\sqrt{2}}\right)$ or $\frac{1}{16}\pi + \frac{1}{4}\ln\left(\frac{\sqrt{2}}{2}\right)$ or $\frac{1}{16}\pi + \frac{1}{8}\ln\frac{1}{2}$.
	Alternative Method for Question 7(b)		
	Use substitution $4x = \tan y$ to find $\int Ay\sec^2 y\,dy$ and commence integration by parts to reach $Ay\tan y \pm B\int \tan y\,dy$	*M1	OE $A, B \neq 0$
	Obtain $\frac{1}{4}y\tan y - \frac{1}{4}\int \tan y\,dy$	A1	OE

Question	Answer	Marks	Guidance
7(b)	Complete integration and obtain $Ay\tan y \mp B\ln\cos y$	**A1FT**	OE FT *their B*. Expect $\frac{1}{4}y\tan y + \frac{1}{4}\ln\cos y$ if correct. Allow A1FT even if errors in substitution of limits into $\frac{1}{4}y\tan y$ if correct earlier.
	Substitute limits correctly in an expression of the form $Ay\tan y \pm B\ln\cos y$	**DM1**	Allow omission of substitution of $x = 0$.
	Obtain answer $\frac{1}{16}\pi - \frac{1}{8}\ln 2$ or exact equivalent	**A1**	Allow e.g. $\frac{1}{16}\pi + \frac{1}{4}\ln\left(\frac{\sqrt{2}}{2}\right)$ or $\frac{1}{16}\pi - \frac{1}{4}\ln\left(\frac{2}{\sqrt{2}}\right)$ or $\frac{1}{16}\pi + \frac{1}{8}\ln\frac{1}{2}$.
		5	

Question	Answer	Marks	Guidance
8(a)	Sketch $y = \sec 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$	M1	Need 1 or –1 and $\frac{1}{4}\pi$ or $\frac{1}{2}\pi$. Ignore regions outside $0 \leqslant x \leqslant \frac{1}{2}\pi$.
	Sketch $y = -2x - \frac{1}{2}$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ and justify the given statement	A1	Need a dot at the intersection of graphs, or dotted line parallel to the y-axis from where graphs cross to the x-axis, or state only one point of intersection OE. Do not allow, e.g. 'only one root'. Ignore regions outside $0 \leqslant x \leqslant \frac{1}{2}\pi$.
			Diagram for reference 
		2	

Question	Answer	Marks	Guidance
8(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.8$ and $x = 1.2$ Can use smaller interval provided it contains root M0 if working in degrees	**M1**	M1 two values attempted and at least one correct in $f(x) = \sec 2x + 2x + \frac{1}{2}$: $f(0.8) = -32.1 < 0$, $f(1.2) = 1.54 > 0$. M1 four values attempted and at least three correct when comparing $\sec 2x$ and $-2x - \frac{1}{2}$: Using 0.8: $\sec 2x = -34.2$ $-2x - \frac{1}{2} = -2.1$, so $-34.2 < -2.1$. Using 1.2: $\sec 2x = -1.36$ $-2x - \frac{1}{2} = -2.9$, so $-1.36 > -2.9$. M1 two values attempted and at least one correct in $f(x) = \frac{1}{2}\cos^{-1}\frac{-2}{4x+1} - x$: $f(0.8) = 0.234 > 0$, $f(1.2) = -0.239 < 0$. M1 four values attempted (must see 0.8 and 1.2 explicitly, not just embedded) and at least three correct when comparing x and $\frac{1}{2}\cos^{-1}\frac{-2}{4x+1}$: Using 0.8: $\frac{1}{2}\cos^{-1}\frac{-2}{4x+1} = 1.03$, so $0.8 < 1.03$. Using $x = 1.2$ $\frac{1}{2}\cos^{-1}\frac{-2}{4x+1} = 0.961$, so $1.2 > 0.961$
	Complete the argument correctly with correct calculated values < 0 and > 0 or change of sign is sufficient For A1, answers must be correct to at least 2 sf	**A1**	If accurate to only 1sf, M1A0.
		2	

Question	Answer	Marks	Guidance
8(c)	Express $x_{n+1} = \frac{1}{2}\cos^{-1}\frac{-2}{4x_n + 1}$ as $x = \frac{1}{2}\cos^{-1}\frac{-2}{4x+1}$	**M1**	Need consistent variable, could be x_n or x_{n+1}.
	Rearrange $x = \frac{1}{2}\cos^{-1}\frac{-2}{4x+1}$ to $\sec 2x = -2x - \frac{1}{2}$ with full and correct working, no slips allowed	**A1**	AG Full working should include $\cos 2x = \frac{-2}{4x+1}$ or $-2x - \frac{1}{2} = \frac{1}{\cos 2x}$ or $\cos 2x = \frac{1}{-2x - \frac{1}{2}}$.
	Alternative Method for Question 8(c)		
	Rearrange $\sec 2x = -2x - \frac{1}{2}$ to $x = \frac{1}{2}\cos^{-1}\frac{-2}{4x+1}$ after full and correct working, no slips allowed	**M1**	Full working should include $\frac{-2}{4x+1} = \frac{1}{\sec 2x}$ or $\frac{-2}{4x+1} = \cos 2x$ or $\cos 2x = \frac{1}{-2x - \frac{1}{2}}$.
	Express $x = \frac{1}{2}\cos^{-1}\frac{-2}{4x+1}$ as $x_{n+1} = \frac{1}{2}\cos^{-1}\frac{-2}{4x_n + 1}$	**A1**	AG
		2	

Question	Answer	Marks	Guidance
8(d)	Use the iterative formula correctly at least twice (consecutive) even if only 3 decimal places	**M1**	M0 if first value is not between 0.8 and 1.2 inclusive. M0 if working in degrees.
	Obtain final answer [x = or α =] 0.992	**A1**	A0 if state $x_n = 0.992$.
	Show sufficient iterations to at least 5 d.p. to justify 0.992 to 3 d.p. or show there is a sign change in the interval (0.9915, 0.9925)	**A1**	Iterations for each starting value: 0.8, 1.03356, 0.98547, 0.99373, 0.99226, 0.99252, 0.99247, 0.99248 0.9, 1.1030, 0.98938, 0.99303, 0.99238, 0.99250, 0.99248, 0.99248 1, 0.99116, 0.99271, 0.99244, 0.99249 1.1, 0.97510, 0.99560, 0.99193, 0.999258, 0.99246, 0.99248 1.2, 0.96143, 0.99813, 0.99148, 0.99265, 0.99245, 0.99248
		3	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $A+\frac{B}{3x-2}+\frac{C}{x+6}$ Values for *A*, *B* and *C* do not need to be substituted into this form to gain full marks	B1	$\frac{Dx+E}{3x-2}+\frac{F}{x+6}$ and $\frac{P}{3x-2}+\frac{Qx+R}{x+6}$ B0 However, can recover all marks from these $\frac{S}{3x-2}+\frac{T}{x+6}$ B0 and can only gain maximum M1A1
	Use a correct method for finding a constant	M1	
	Obtain one of $A = 4$, $B = 6$ and $C = -5$ or $F = -5$ or $P = 6$ or $S = 6$ or $T = -5$	A1	Allow maximum M1A1 for one or more 'correct' values after B0, even if from $\frac{S}{3x-2}+\frac{T}{x+6}$. $D = 12$ $E = -2$ $F = -5$ $P = 6$ $Q = 4$ $R = 19$ $S = 6$ $T = -5$
	Obtain a second value from $\frac{Dx+E}{3x-2} = A+\frac{B}{3x-2}$ or $\frac{Qx+R}{x+6} = A+\frac{C}{x+6}$	A1	
	Obtain a third value	A1	
	Alternative Method for Question 9(a)		
	Divide numerator by denominator and reach quotient of 4 and remainder of $Px+Q$	M1	Or by inspection P or $Q \neq 0$
	Obtain $4+\frac{-9x+46}{(3x-2)(x+6)}$ or $4+\frac{-9x+46}{3x^2+16x-12}$	A1	
	State or imply *their* remainder is of form $\frac{D}{3x-2}+\frac{E}{x+6}$ Values for *D* and *E* do not need to be substituted into this form to gain full marks	B1	

Question	Answer	Marks	Guidance
9(a)	Obtain one of $D = 6$, $E = -5$	**A1**	
	Obtain a second value	**A1**	
		5	
9(b)	Use the correct method to find the first two unsimplified terms of the expansion of $(3x-2)^{-1}$ or $(x+6)^{-1}$ or $\left(1-\frac{3}{2}x\right)^{-1}$ or $\left(1+\frac{1}{6}x\right)^{-1}$	**M1**	E.g. $-2^{-1}-2^{-2}(3x)$, or $6^{-1}-6^{-2}(x)$, or $1+\frac{3}{2}x$ or $1-\frac{1}{6}x$.
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction $B = 6$ $C = -5$ $A = 4$	**A1FT** **A1FT**	The FT is on B and C, e.g. $\frac{B}{-2}\left(1+\frac{3}{2}x+\left(\frac{3}{2}x\right)^2\right)+\frac{C}{6}\left(1-\frac{1}{6}x+\left(\frac{1}{6}x\right)^2\right)$ OE.
	$\frac{1}{6}-\frac{157}{36}x-\frac{1463}{216}x^2$	**A1**	OE Do not ISW, e.g. multiplication through by 216. Allow terms in any order.
		4	

Question	Answer	Marks	Guidance
10(a)	For reference $A = (2, -1, -6)$, $B = (b, -2, 3)$, $C = (-4, 5, -2)$		Allow column vectors throughout. Allow coordinates throughout except in 10(b). Allow *their* notation for vectors throughout
	Carry out a correct method for finding $\overrightarrow{AB}$ or $\overrightarrow{BC}$ Allow if use $\overrightarrow{BA}$ for $\overrightarrow{AB}$ or $\overrightarrow{CB}$ for $\overrightarrow{BC}$	**M1**	E.g. $\overrightarrow{AB} = (b-2)\mathbf{i} + (-2+1)\mathbf{j} + (3--6)\mathbf{k}$ or $\overrightarrow{BC} = (-4-b)\mathbf{i} + (5--2)\mathbf{j} + (-2-3)\mathbf{k}$.
	Correct method to form equation with *their* $\left\|\overrightarrow{AB}\right\| =$ *their* $\left\|\overrightarrow{BC}\right\|$	**M1**	May see $\sqrt{(b-2)^2 + 1^2 + 9^2} = \sqrt{(-4-b)^2 + 7^2 + 5^2}$ allow one further slip. Note that M0M1 is possible.
	Obtain $[b =]\ -\frac{1}{3}$	**A1**	
		3	

Question	Answer	Marks	Guidance
10(b)	Find $\overrightarrow{OA} + their\,\overrightarrow{BC}$ or $\overrightarrow{OC} - their\,\overrightarrow{AB}$ $\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{BA}$ **M1** $\overrightarrow{OD} = \overrightarrow{OC} - \overrightarrow{BA}$ **M0** $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC}$ **M1** $\overrightarrow{OD} = \overrightarrow{OA} - \overrightarrow{BC}$ **M0**	**M1**	E.g. $\left(2-\frac{11}{3}\right)\mathbf{i}+\left(-1+7\right)\mathbf{j}+\left(-6-5\right)\mathbf{k}$ or $\left(-4\mathbf{i}+5\mathbf{j}-2\mathbf{k}\right) - \left(\left(b-2\right)\mathbf{i}+\left(-2+1\right)\mathbf{j}+\left(3--6\right)\mathbf{k}\right)$ $= \left(-4+\frac{7}{3}\right)\mathbf{i}+\left(5+1\right)\mathbf{j}+\left(-2-9\right)\mathbf{k}$. May equate the midpoint of *AC* and *BD* to find *OD*: $\frac{1}{2}\left(\overrightarrow{OA}+\overrightarrow{OC}\right)=\frac{1}{2}\left(\overrightarrow{OB}+\overrightarrow{OD}\right)$. Incorrect order of vertices scores M0.
	Obtain $\left[\overrightarrow{OD} =\right] -\frac{5}{3}\mathbf{i}+6\mathbf{j}-11\mathbf{k}$	**A1**	Allow $\begin{pmatrix}-\frac{5}{3}\\6\\-11\end{pmatrix}$ but not $\begin{pmatrix}-\frac{5}{3}\mathbf{i}\\6\mathbf{j}\\-11\mathbf{k}\end{pmatrix}$. $\overrightarrow{OD} = +\frac{5}{3}\mathbf{i}-6\mathbf{j}+11\mathbf{k}$ scores **M1 A0**. $\overrightarrow{OD} = \left(-\frac{5}{3}, 6, -11\right)$ scores **M1 A0**.
		2	

Question	Answer	Marks	Guidance
10(c)	Carry out correct process for evaluating the scalar product of *their* $\pm\overrightarrow{BA}$ and *their* $\pm\overrightarrow{BC}$	M1	E.g. $\left(\frac{7}{3}\times-\frac{11}{3}\right)+(1\times7)+(-9\times-5)$, or $-\frac{77}{9}+7+45$ or $\frac{391}{9}$.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli for *their* pair of vectors and obtain cosine of angle (allow unsimplified form as in above scalar product)	M1	E.g. cosine of angle = $\dfrac{\frac{391}{9}}{\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{BC}\rvert}$ or $\dfrac{\frac{391}{9}}{\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{AB}\rvert}$ $=\dfrac{\left(\frac{7}{3}\times-\frac{11}{3}\right)+(1\times7)+(-9\times-5)}{\sqrt{\frac{49}{9}+1+81}\sqrt{\frac{121}{9}+49+25}}$ $\left(\dfrac{\frac{391}{9}}{\frac{787}{9}}=\dfrac{391}{787}=\ 0.4968\right)$ $\lvert\overrightarrow{AB}\rvert=\lvert\overrightarrow{BC}\rvert$, so may be expressed differently.
	Obtain answer 60.2 or 1.05	A1	
		3	

Question	Answer	Marks	Guidance
11	Separate variables correctly	**B1**	Sight of $\frac{1}{e^{3y}}$ sufficient for f(*y*) in f(*y*)d*y* = g(*x*)d*x* to obtain B1.
	Obtain term $-\frac{1}{3}e^{-3y}$	**B1**	
	Separate fractions and obtain term $\frac{1}{2}\ln\left(x^2+3\right)$	**B1**	
	Separate fractions and obtain term of the form $a\tan^{-1}bx$	**M1**	
	Obtain term $-\frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}$	**A1**	OE
	Use $x = 0$, $y = 0$ to evaluate a constant or as limits in a solution containing terms of the form $a\,e^{\pm 3y}$, $b\ln\left(x^2+3\right)$ and $c\tan^{-1}mx$	**M1**	
	Obtain correct solution in any form relating *x* and *y*	**A1**	E.g. $-\frac{1}{3}e^{-3y} = \frac{1}{2}\ln\left(x^2+3\right) - \frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} - \frac{1}{3} - \frac{1}{2}\ln 3$. Constant $-\frac{1}{3} - \frac{1}{2}\ln 3$ may be –0.883.
	Obtain –0.331	**A1**	OE AWRT. E.g. –0.33084… Note A0A1 is possible.
		8	