2023

Kinematics

[Unit 2]

Table of Contents

Equations of Motion

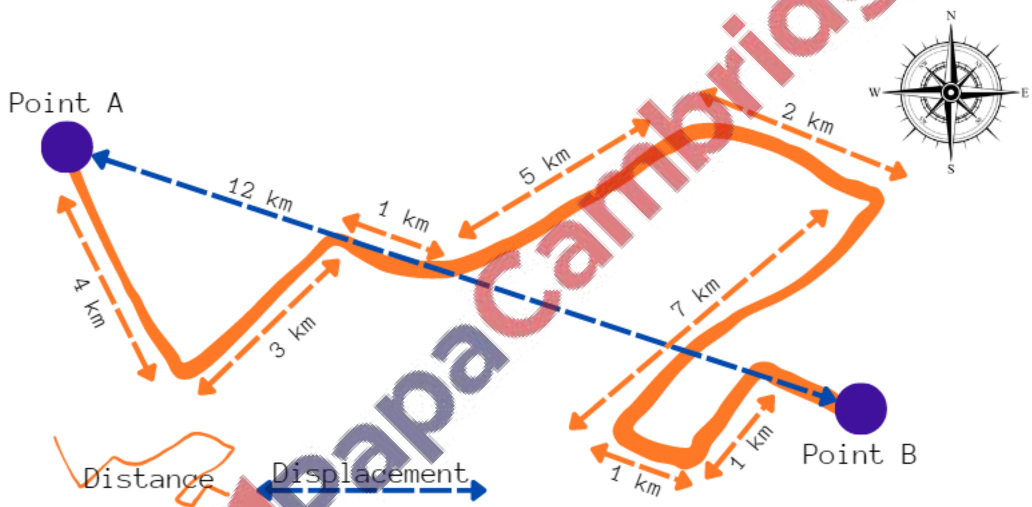
D D S V A

[aka. Distance, Displace, Speed, Velocity, and Acceleration]

Distance & Displacement

Distance is the length along the actual path travelled from the starting point to the finishing point.

Displacement is the change of position or the length travelled in a straight line in a specified direction from the starting to the finishing point.



The figure shows an **orange path** taken by the object, the **length of this path** is the **distance travelled**.

The **blue line** or arrow shows the **distance between the two points** and is the **displacement**.

Average Speed

Average speed is the distance moved along a path divided by the time taken:

$$avarage\ speed = \frac{distance\ moved\ along\ path}{time\ taken}$$

The unit of speed is meters per second or ms^{-1}.

The table below shows a bunch of values of speed which you should know:

Name	Value (ms^{-1})	Name	Value (ms^{-1})
Light	3×10^8	**Snail**	1×10^{-3}
Electrons around Nucleus	2.2×10^6	**Typical speed of car (80 kph)**	22
Jet airliner	2.5×10^2	**Walking Speed**	1.5
Earth around Sun	3×10^4	**Sprinter**	1×10^1

Note that speed only has meaning if it is quoted relative to a fixed reference and in most cases, this reference is the surface of the Earth.

Now, the Earth isn't fixed because it's orbiting the sun but we take it as fixed anyways.

Speed & Velocity

Normally, we'd say that "speed" and "velocity" are the same but that's not true in physics.

In physics, there is one BIG difference between the two is that velocity is a vector quantity and speed is a scalar quantity.

We already know that **speed uses distance** unlike **velocity which uses displacement.**

To put this into perspective, let's first look at the formulas for average speed and average velocity:

Average Speed

$$Average\ Speed = \frac{Distance}{Time\ Taken}$$

Average Velocity

$$Average\ Velocity = \frac{Displacement}{Time\ Taken}$$

As you can see, speed uses distance and velocity uses displacement.

The **average speed of an object might not be the same as the average velocity** due to the **different values of distance and displacement** (we talked about it **before**).

A Better Equation for Average Velocity

The previously stated word equation for average velocity was to sort of explain the difference between speed and velocity.

However, the formula we should memorise is:

$$\bar{v} = \frac{\Delta x}{\Delta t}\ or\ \bar{v} = \frac{(u+v)}{2}$$

Symbol	Quantity
v	Velocity
x	Displacement
t	Time

Note: The little "-" on top of v means "average" and "Δ" means "change in" so "Δx" means "change in displacement"

Quick Definitions for Types of Velocity

Constant Velocity

The speed of an object travelling at constant speed would be found using this formula and would be exact. (v=d/t)

Average Velocity

The speed calculated for an object travelling at different speeds would give us an average and is therefore the "average speed".

Instantaneous Velocity

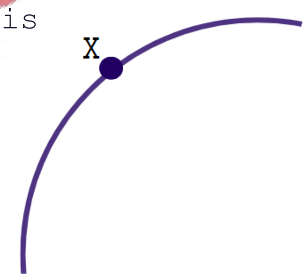
The speed calculated for an object at a specific moment in time is called the "instantaneous speed"; basically, it is the speed at a point.

Finding Instantaneous Speed

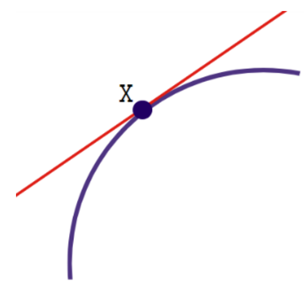
When we have a curve on the graph, it means the velocity is changing.

To get the instantaneous velocity for that, we need to use tangents.

So, let's say we have a graph and we want to get the instantaneous velocity at point X (check right).



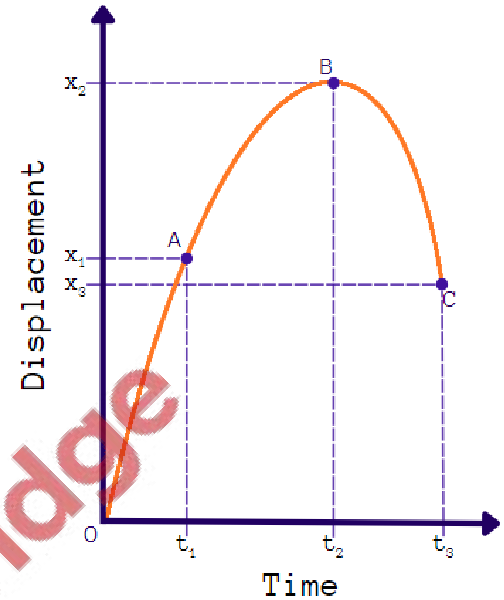
We can simply draw a tangent and then get the gradient like the figure on the right (under the question).



Describing Motion by Graphs

Position-time Graphs

Section	Description	Average Velocity
O	Starting Point of the particle.	N/A
O to A	Straight line meaning the particle is covering equal distances in equal periods (uniform velocity).	$\frac{x_1 - 0}{t_1 - 0}$
A to B	Particle is slowing down because the distances travelled in equal periods are decreasing.	$\frac{x_2 - x_1}{t_2 - t_1}$
B	Particle is at rest but only for a moment	0
B to C	Particle is moving back towards the origin (shown by negative values)	$\frac{x_3 - x_2}{t_3 - t_2}$



Displacement-time & Velocity-time Graphs

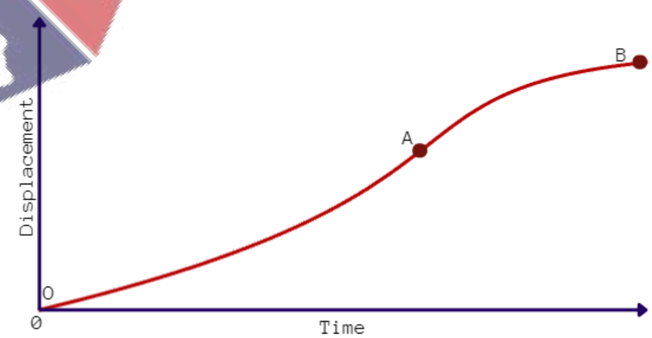
As the titles suggest, a displacement-time graph is a graph which has the displacement on the y-axis and the time on the x-axis.

Velocity-time graphs have velocity on the y-axis and time on the x-axis.

Now, the thing we need to know is how we can create a velocity-time graph using the displacement-time graph.

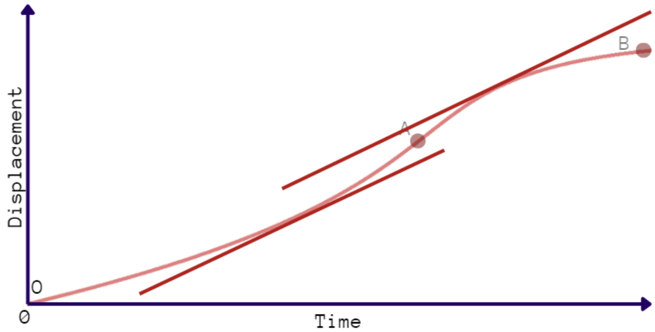
Using Displacement-time Graph to get Velocity-time Graph

Let's say we have a car which has the following displacement-time graph for a trip:



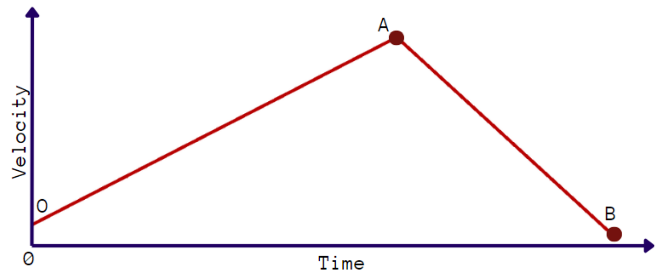
Section	Description
O to A	Distances travelled in equal distances are progressively increasing (acceleration).
A to B	Distances travelled in equal time intervals are progressively decreasing (deceleration).
B	No change in position (at rest).

To get the velocity-time graph, we first draw tangents to each curve present:



To get the specific values of the velocity, we can simply calculate the gradients of both tangents.

When we plot these tangents, we get the image shown on the right.



The reason we have the graph like this is because from the first tangent (OA), we accelerate which gives us a positive slope. However, the second tangent (AB) is when the car decelerates and therefore a negative slope.

Acceleration

Acceleration is a measure of the rate at which the velocity of an object/particle is changing.

The formula for average acceleration is:

$$Average\ Acceleration = \frac{Change\ in\ Velocity}{Time\ Taken}\ or\ \overline{a} = \frac{\Delta v}{\Delta t}$$

The unit of acceleration is ms^{-2} and is calculated by dividing the velocity (ms^{-1}) with the unit of time (s) which gives us ms^{-2}.

Instantaneous Acceleration

To get the instantaneous acceleration, we take extremely small time intervals.

We also need the direction along with the magnitude as acceleration is a vector quantity.

A particle moving with uniform velocity has no acceleration meaning that the magnitude and direction of the particle do not change with time.

We can find the acceleration of a particle using its velocity-time graph, this is done by drawing a tangent at the curve and finding the slope of it. (This **method**)

Examples of Acceleration Values

Name	Value (ms^{-2})	Name	Value (ms^{-2})
Due to circular motion of electrons around nucleus	9×10^{26}	**Due to circular motion of Earth around Sun**	6×10^{-5}
Car Crash	1×10^{3}	**Family Car**	2
At Equator, due to rotation of Earth	3×10^{-3}	**Free fall on Earth**	10
		Free fall on Moon	2

Uniformly Accelerated Motion

Using displacement, velocity, and acceleration, we can derive a few equations called the "**kinematic equations**".

Equation 1:	Equation 2:	Equation 3:
$v = u + at$	$s = ut + \frac{1}{2}at^2$	$s = vt - \frac{1}{2}at^2$

Equation 4:	Equation 5:
$v^2 = u^2 + 2as$	$s = \frac{(u+v)t}{2}$

Deriving the Equations:

Equation 1

For the first equation, imagine an object moving in a line with constant acceleration (a).

We use the formula $a = \frac{(v-u)}{t}$ which is rearranged to make:

$$v = u + at$$

Equation 2

For equation 2, we use the following equations:

$$s = \frac{(u+v)}{2} \times t$$
$$v = u + at$$

We simply substitute the second equation into the first one like so:

$$s = \frac{[u + (u + at)]}{2} \times t$$
$$s = \frac{2u + at}{2} \times t$$
$$s = t\left(u + {}^{1}/_{2}\, at\right)$$
$$s = ut + \frac{1}{2}at^2$$

Equation 3:

For equation 2, we use the following equations:

$$s = \frac{(u+v)}{2} \times t$$
$$v = u + at$$

We first make u the subject which gives us $u = v - at$.

Now we substitute the new equation into the first one:

$$s = \frac{[(v - at) + v]}{2} \times t$$
$$s = \frac{2v - at}{2} \times t$$
$$s = t\left(v - \frac{1}{2}at\right)$$
$$s = vt - \frac{1}{2}at^2$$

Equation 4:

For this equation we need the following equations:

$$v = u + at$$
$$s = \frac{(u+v)}{2} \times t$$

First, we make t the subject of equation 1 which give us $t = \frac{v-u}{a}$.

Now we can substitute the new equation into the second equation:

$$s = \frac{(u+v)}{2} \times \frac{v-u}{a}$$
$$s = \frac{(u+v)(v-u)}{2a}$$
$$s = \frac{uv - u^2 + v^2 - uv}{2a}$$
$$2as = -u^2 + v^2$$
$$v^2 = u^2 + 2as$$

Equation 5

For equation 5, we use the following equations:

$$\bar{v} = \frac{u+v}{2}$$
$$\bar{v} = \frac{s}{t}$$

We simply equate the two:

$$\frac{u+v}{2} = \frac{s}{t}$$
$$s = \frac{t(u+v)}{2}$$

Free fall Acceleration

It is the acceleration of any object attracted to the Earth as a result of being in its gravitational field.

When an object is released on Earth, it is attracted to its centre due to the force of gravity.

The object is attracted to the centre and therefore falls downwards given that no external forces act on it.

It is represented by "g" and has the value of 9.81 ms^{-2}.

Experiment for getting free fall acceleration

There are numerous ways of determining the acceleration of free fall.

The Method

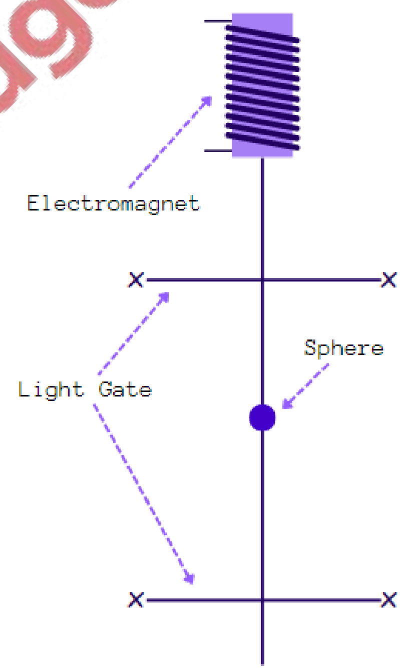
We will release a ball from the electromagnet (on top) this causes the ball to fall as it is affected by gravity.

When the ball passes through the first light gate, an electric timer is turned on and then turned off when the second is passed.

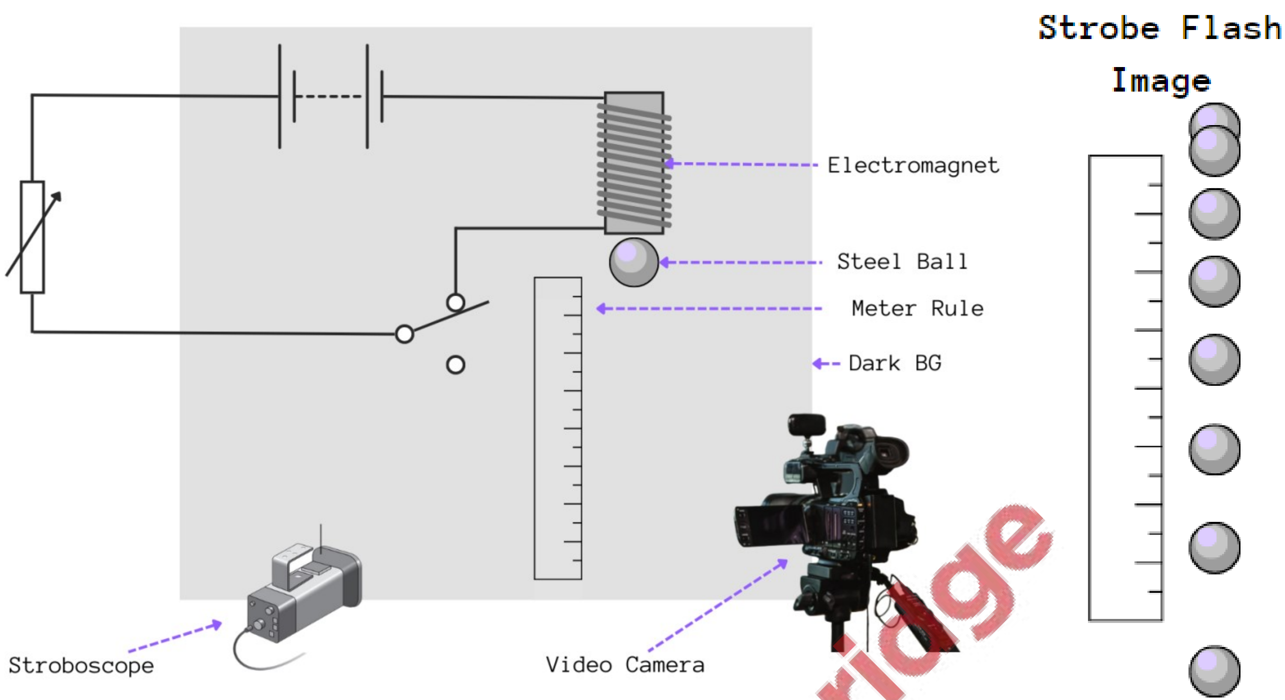
We can then use the formula for acceleration to calculate the acceleration due to free fall:

$$a = \frac{(v-u)}{t}$$

Obviously we don't have speed but we can get final velocity using $v = d/t$ and take initial velocity as 0.



Our Setup



[Stroboscope from the textbook]

The Process:

A steel ball is released from an electromagnet and falls under gravity.

The video camera is used to produce a film of the ball's fall and the stroboscope is used to flash light at a selected frequency.

The recording shows the position of the ball at regular time intervals along with the distance it has travelled (shown by meter rule).

Calculation:

First, we must decide the formula we use, in this case it will be:

$$s = \frac{1}{2}at^2$$

This is because we have values for time and displacement which leaves acceleration.

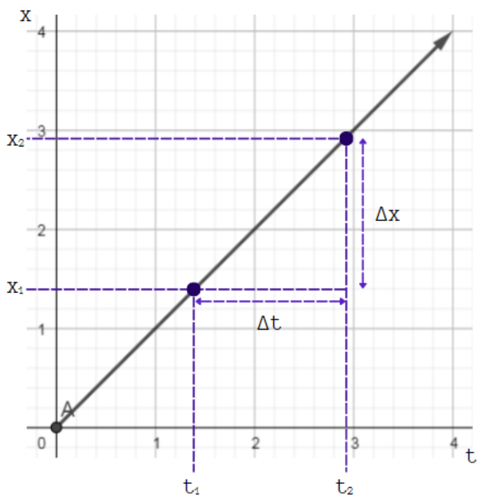
Given that the stroboscope was used at a frequency of 20 Hz and the time intervals were calculated using $T = \frac{1}{f}$, we can plot a displacement (s) against t^2.

The start time is going to be 0 as shown in the first image of the ball.

This graph gives us a gradient of 0.5a which we can use to get g.

Note: The values shown here are not the same as the figures

Graphs of Kinematic Equations

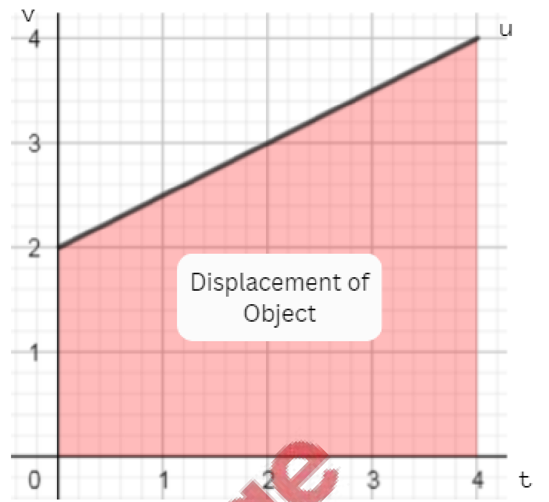


This is when an object moves in a straight line with constant velocity meaning that it covers equal distances in equal time intervals.

The displacement-time graph above shows the motion meaning that the gradient is the velocity.

In this graph, the gradient will be the average velocity and instantaneous velocity as the line is straight (not curved).

Here, the equation is $x = vt$.

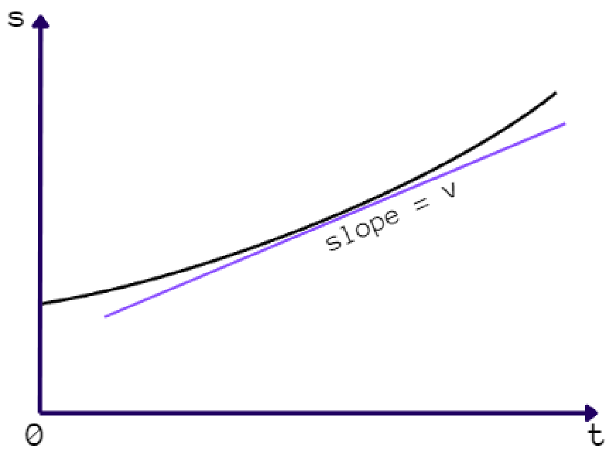


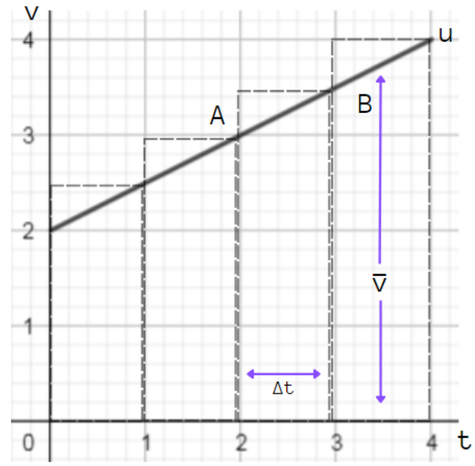
This is a velocity-time graph for an object moving in a straight line with constant acceleration, this means that the object's velocity increases by equal amounts in equal time intervals.

In this case, the object starts at a velocity of u at time t = 0.

Here, the gradient is acceleration and the area under the slope/gradient is the displacement of the object.

The equation here is $v = u + at$.





Using the velocity-time graph, we can make a displacement-time graph.

We do this by calculating the area between the graph and the t-axis succession values of t.

Look at the graph on the right.

We can split the graph and then get the areas of the individual sections, the displacement at a certain time is then just the sum of the areas until the time.

The equation which describes the velocity-time graph is:

$$s = ut + {}^{1}\!/_{2}\, at^2$$

Displacement for Non-Uniform Acceleration

When acceleration is uniform/constant, everything is easier because the graphs are also linear.

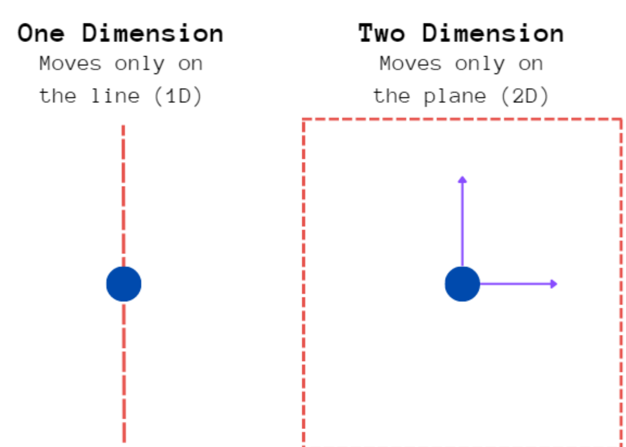
When the acceleration is changing, it gives us a curved velocity time graph.

The way to get the displacement for this is to simply make rectangles of same sizes and get their areas giving us a good estimate of the area.

2 Dimensional Motion under Constant Force

All of the graphs we've seen till now were of an object moving in a straight line, which is called "One Dimensional Motion".

2 dimensional motion is when the object moves on a plane rather than a single line.



Projectile Motion

Think of an object like a ball thrown at an angle by someone, the ball experiences a constant force which causes it to go in the direction it was thrown in:

Understanding Projectile Motion:

Let's think of an object which is sent in the horizontal direction and is subject to gravity (ignore air resistance).

Vertical Motion:

Let's first take a look at the vertical motion of the object.

We know that the object has an acceleration of g (gravity).

Using $v = u + at$, the quantity v_y, at time t is given by the equation:

$$v_y = gt$$

Note: v_y means the velocity on the y-axis

Also, at time t, the vertical displacement y is given by the formula:

$$y = \frac{1}{2}gt^2$$

Horizontal Motion:

Here, the acceleration is zero meaning that quantity v_h stays the same as the initial velocity (u).

Note: v_h means the velocity on the x-axis

At time t, the horizontal displacement x is given by the formula:

$$x = u_x t$$

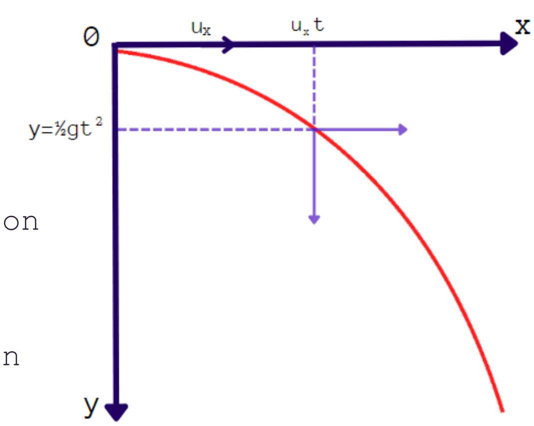
Note: u_x means the initial velocity on the x-axis

Finding the Velocity using the Components

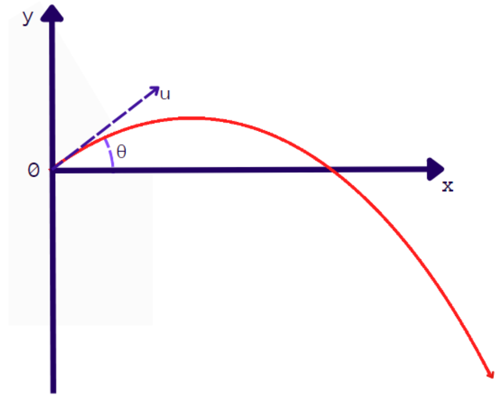
To find the velocity of the object at any time t, we must vectorially add the v_x and v_y components.

The resulting vector is the direction of motion of the object.

The curve which is traced out by an object subjected to a constant force in one direction is a parabola (right).



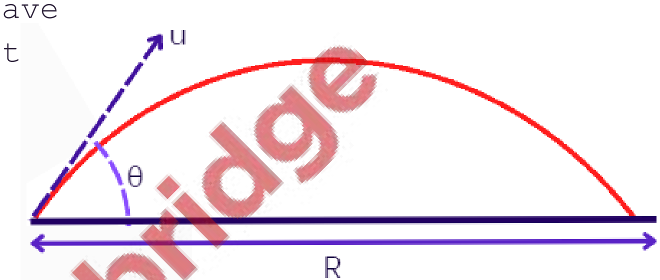
Other Cases for Projectile Motion:



If an object is sent with velocity (u) at an angle (θ) on the horizontal axis similar to the graph on the right.

The analysis of motion is similar to the example previously with the difference being that the initial y-component is:

$$y = u \sin \theta$$



Let's look at another figure; here we have an object projected with velocity (u) at an angle (θ) to the horizontal axis.

The range (R) is the distance from the point of projection to the point the object reaches the ground again.

The formula of R is given by:

$$R = \frac{(u^2 \sin 2\theta)}{g}$$

Sources (and Resources) Used

Most of the information has come from the **AS & A Level Physics Student Book by Hodder Education**.

Other resources/tools have also been used and are listed below:

Name	Link	Use
Save My Exams	**LINK**	Mainly understanding concepts to make them simpler
ZNotes	**LINK**	
Canva	**LINK**	Designing of figures and diagrams
GIMP 2.10.30	**LINK**	Diagrams & Figures
Geogebra	**LINK**	Vector diagrams
AS/A Level Syllabus	**LINK**	Checking syllabus
Word 2010	**LINK**	Creating the notes
Word 365	**LINK**	Exporting and stuff

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