## Cambridge International AS \& A Level

## MATHEMATICS

9709/11
Paper 1 Pure Mathematics 1
May/June 2022
MARK SCHEME
Maximum Mark: 75

## Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.
Cambridge International is publishing the mark schemes for the May/June 2022 series for most
Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

## GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.


## GENERIC MARKING PRINCIPLE 2:

Marks awarded are always whole marks (not half marks, or other fractions).

## GENERIC MARKING PRINCIPLE 3:

## Marks must be awarded positively:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous


## GENERIC MARKING PRINCIPLE 4

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

## GENERIC MARKING PRINCIPLE 5

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

## GENERIC MARKING PRINCIPLE 6

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

## Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5 Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

## PUBLISHED

## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

## Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the $M$ mark and in some cases an $M$ mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.
DM or DB When a part of a question has two or more 'method' steps, the $M$ marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO Correct Working Only
ISW Ignore Subsequent Working
Seen Or Implied
SC Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

WWW Without Wrong Working

AWRT Answer Which Rounds To

| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 1(a) | $x^{2}-8 x+11=(x-4)^{2} \ldots$ or $p=-4$ | B1 | If $p$ and $q$-values given after their completed square expression, mark the expression and ISW. |
|  | ... -5 or $q=-5$ | B1 |  |
|  |  | 2 |  |
| 1(b) | $(x-4)^{2}-5=1$ so $(x-4)^{2}=6$ so $x-4=[ \pm] \sqrt{6}$ | M1 | Using their $p$ and $q$ values or by quadratic formula |
|  | $x=4 \pm \sqrt{6} \text { or } \frac{8 \pm \sqrt{24}}{2}$ | A1 | Or exact equivalent. <br> No FT; must have $\pm$ for this mark. ISW decimals 1.55, 6.45 if exact answers seen. If M0, SC B1 possible for correct answers. |
|  |  | 2 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 2 | $a+12 d=12$ | B1 | For correct equation. |
|  | $\frac{30}{2}(2 a+(30-1) d)=-15$ | B1 | For correct equation in $a$ and $d$. If using $\frac{n}{2}(a+l)$, must replace $l$ with an expression involving $a$ and $d$. |
|  | $a=72, d=-5$ | B1 | Both values correct SOI. |
|  | $\mathrm{S}_{50}=\frac{50}{2}(2(\text { their } a)+49(\text { theird }))$ | M1 | Using sum formula with their $a$ and $d$ values obtained via a valid method. |
|  | $\mathrm{S}_{50}=-2525$ | A1 |  |
|  |  | 5 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 3(a) | $x^{4} \text { term is }[10 \times]\left(2 x^{2}\right)^{3}\left(\frac{k^{2}}{x}\right)^{2}$ | M1 | For selecting the term in $x^{4}$. |
|  | $80 k^{4} x^{4} \Rightarrow a=80 k^{4}$ <br> [ $x^{2}$ term is $\left.[6 \times](2 k x)^{2} \times 1=24 k^{2} x^{2} \Rightarrow\right] b=24 k^{2}$ | A1 <br> B1 | For correct value of $a$. Allow $80 k^{4} x^{4}$. <br> For correct value of $b$. Allow $24 k^{2} x^{2}$. |
|  |  | 3 |  |
| 3(b) | $80 k^{4}+24 k^{2}-216[=0] \quad\left[\Rightarrow 10 k^{4}+3 k^{2}-27=0\right]$ | M1 | Forming a 3 -term equation in $k$ (all terms on one side) with their $a$ and $b$ and no $x$ 's. |
|  | $\left(2 k^{2}-3\right)\left(5 k^{2}+9\right)[=0]\left[\Rightarrow k^{2}=\frac{3}{2}\right.$ or $\left.-\frac{9}{5}\right]$ | M1 | Attempt to solve 3-term quartic (or quadratic in another variable) by factorisation, formula or completing the square - see guidance. |
|  | $[k]= \pm \sqrt{\frac{3}{2}}$ | A1 | OE e.g. $\pm \frac{\sqrt{6}}{2}, \pm \sqrt{1.5}, \quad$ AWRT $\pm 1.22$ <br> Omission of $\pm \mathrm{A} 0$. <br> Additional answers A0. <br> If M1 M0, SC B1 can be awarded for correct final answer, max $2 / 3$. |
|  |  | 3 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 4(a) | $\begin{aligned} & \frac{\sin ^{3} \theta}{\sin \theta-1}-\frac{\sin ^{2} \theta}{1+\sin \theta}=\frac{\sin ^{3} \theta(1+\sin \theta)}{(\sin \theta-1)(1+\sin \theta)}-\frac{\sin ^{2} \theta(\sin \theta-1)}{(\sin \theta-1)(1+\sin \theta)} \\ & {\left[=\frac{\sin ^{3} \theta(1+\sin \theta)-\sin ^{2} \theta(\sin \theta-1)}{(\sin \theta-1)(1+\sin \theta)}\right]} \end{aligned}$ | *M1 | Using a common denominator. |
|  | $-\frac{\sin ^{2} \theta+\sin ^{4} \theta}{1-\sin ^{2} \theta}$ | DM1 | Reaching $\pm\left(1-\sin ^{2} \theta\right)$ in denominator. SOI by $\pm \cos ^{2} \theta$. |
|  | $-\frac{\sin ^{2} \theta\left(1+\sin ^{2} \theta\right)}{\cos ^{2} \theta}$ | DM1 | Using $\sin ^{2} \theta+\cos ^{2} \theta=1$ in denominator and isolating $\sin ^{2} \theta$ in numerator. |
|  | $-\tan ^{2} \theta\left(1+\sin ^{2} \theta\right)$ | A1 | AG - Using/stating $\tan \theta=\frac{\sin \theta}{\cos \theta}$ is sufficient for A1. May be working from both sides provided the argument is complete. <br> A0 if $\theta$ or brackets missing throughout, or sign errors. Allow recovery if AG follows from their working. |
|  | Alternative method for Q4(a) |  |  |
|  | $-\tan ^{2} \theta\left(1+\sin ^{2} \theta\right)=-\frac{\sin ^{2} \theta\left(1+\sin ^{2} \theta\right)}{1-\sin ^{2} \theta}$ | *M1 | Using $\tan \theta=\frac{\sin \theta}{\cos \theta}$ and $\sin ^{2} \theta+\cos ^{2} \theta=1$. |
|  | $\frac{-\sin ^{2} \theta-\sin ^{4} \theta}{(1-\sin \theta)(1+\sin \theta)}$ | DM1 | Factorising denominator. |
|  | $\frac{\sin ^{2} \theta+\sin ^{3} \theta-\sin ^{3} \theta+\sin ^{4} \theta}{(\sin \theta-1)(1+\sin \theta)}=\frac{\sin ^{3} \theta(1+\sin \theta)-\sin ^{2} \theta(\sin \theta-1)}{(\sin \theta-1)(1+\sin \theta)}$ | DM1 | Factorising numerator. |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 4(a) | $\frac{\sin ^{3} \theta}{\sin \theta-1}-\frac{\sin ^{2} \theta}{1+\sin \theta}$ | A1 | AG <br> A 0 if $\theta$ or brackets missing throughout, or sign errors. Allow recovery if AG follows from their working. |
|  |  | 4 |  |
| 4(b) | $-\tan ^{2} \theta\left(1+\sin ^{2} \theta\right)=\tan ^{2} \theta\left(1-\sin ^{2} \theta\right)$ leading to [2] $\tan ^{2} \theta=0$ | M1 | Obtaining a (trig function) ${ }^{2}=0$ WWW. |
|  | $\tan \theta=0$ leading to $[\theta=] \pi$ | A1 | Ignore extra solutions outside the interval ( $0,2 \pi$ ). |
|  | Alternative method for Q4(b) |  |  |
|  | $\begin{aligned} & -\frac{\sin ^{2} \theta}{\cos ^{2} \theta}\left(1+\sin ^{2} \theta\right)=\frac{\sin ^{2} \theta}{\cos ^{2} \theta}\left(1-\sin ^{2} \theta\right) \quad \text { leading to } \\ & -\sin ^{2} \theta-\sin ^{4} \theta=\sin ^{2} \theta-\sin ^{4} \theta \quad \text { leading to } \quad[2] \sin ^{2} \theta=0 \end{aligned}$ | M1 | Obtaining a (trig function) ${ }^{2}=0$ WWW. |
|  | $\sin \theta=0 \quad$ leading to $\quad[\theta=] \pi$ | A1 | Ignore extra solutions outside the interval ( $0,2 \pi$ ) . |
|  |  | 2 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 5(a) | $\text { Sector area }=\frac{1}{2} r^{2}\left(\frac{\pi}{6}\right)\left[=\frac{\pi}{12} r^{2}\right]$ | B1 | Using $\frac{1}{2} r^{2} \theta$ with $\theta$ in radians SOI. B 0 if using a value for $r$. |
|  | $\begin{aligned} & B D=\sin \frac{\pi}{6} r\left[=\frac{1}{2} r\right] \text { and } A D=\cos \frac{\pi}{6} \mathrm{r}\left[=\frac{\sqrt{3}}{2} r\right] \\ & \text { so triangle area }=\frac{1}{2}\left(\sin \frac{\pi}{6} r\right)\left(\cos \frac{\pi}{6} \mathrm{r}\right)\left[=\frac{1}{2} \times \frac{1}{2} r \times \frac{\sqrt{3}}{2} r\right] \\ & \text { or } \frac{1}{2} r\left(\cos \frac{\pi}{6} r\right)\left(\sin \frac{\pi}{6}\right)\left[=\frac{1}{2} r \times \frac{\sqrt{3}}{2} r \times \frac{1}{2}\right] \end{aligned}$ | B1 | SOI Finding triangle area. <br> Decimals B0 unless exact values seen in working. |
|  | Area of $B C D=\frac{1}{12} \pi r^{2}-\frac{\sqrt{3}}{8} r^{2}$ | B1 | OE e.g. $\frac{r^{2}}{4}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)$ with $\cos \frac{\pi}{6}$ and $\sin \frac{\pi}{6}$ evaluated. Must be exact, in terms of $r^{2}$. <br> ISW |
|  |  | 3 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 5(b) | Angle $B A C=\sin ^{-1}\left(\frac{\frac{\sqrt{3}}{2} r}{r}\right)\left[=\frac{\pi}{3}\right]$ | B1 | SOI by length of $A D, C D$ or arc, or by perimeter. |
|  | Length $A D=\cos \frac{\pi}{3} r\left[=\frac{1}{2} r\right] \quad$ [so length $C D=\frac{1}{2} r$ ] | M1 | SOI Finding length by Pythagoras, or by trigonometry with their angle $B A C$, provided $B A C \neq \frac{\pi}{6}$. |
|  | Length of arc $B C=r \times \frac{\pi}{3}$ | M1 | SOI Using $r \theta$ with $\theta$ in radians. Condone $\theta=\frac{\pi}{6}$. |
|  | Perimeter of $B C D=\frac{\sqrt{3}}{2} r+\frac{1}{2} r+\frac{\pi}{3} r$ | A1 | OE e.g. $r\left(\frac{\sqrt{3}+1}{2}+\frac{\pi}{3}\right)$ with e.g. $\cos \frac{\pi}{3}$ evaluated. <br> Must be exact, in terms of $r$. <br> ISW |
|  |  | 4 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 6(a) | $y=\frac{x^{2}-4}{x^{2}+4}$ leading to $\left(x^{2}+4\right) y=\left(x^{2}-4\right)$ leading to $x^{2} y+4 y=x^{2}-4$ | *M1 | For clearing denominator and expanding brackets. If swap variables first, look for $y^{2} x+4 x=y^{2}-4$. |
|  | $x^{2} y-x^{2}=-4 y-4$ leading to $x^{2}(1-y)=4 y+4$ leading to $x^{2}=\ldots$ | DM1 | For making $x^{2}$ the subject. If swap variables first, look for $y^{2}(1-x)=4 x+4 \Rightarrow y^{2}=\ldots$ |
|  | $x^{2}=\frac{4 y+4}{1-y} \quad$ leading to $x=\sqrt{\frac{4 y+4}{1-y}}$ leading to $\left[\mathrm{f}^{-1}(x)\right]=\sqrt{\frac{4 x+4}{1-x}}$ | A1 | OE e.g. $\sqrt{\frac{-4 x-4}{x-1}}$ without $\pm$ in final answer. |
|  | Alternative method for Q6(a) |  |  |
|  | $x=\frac{y^{2}-4}{y^{2}+4}$ leading to $\quad x=1-\frac{8}{y^{2}+4} \quad$ leading to $\quad x-1=\frac{-8}{y^{2}+4}$ | *M1 | For division and reaching $x-1=\ldots \quad($ or $y-1=\ldots)$ |
|  | $y^{2}+4=\frac{-8}{x-1}$ leading to $y^{2}=\frac{-8}{x-1}-4$ | DM1 | For making $y^{2}\left(\right.$ or $\left.x^{2}\right)$ the subject. |
|  | $[y=]\left[\mathrm{f}^{-1}(x)\right]=\sqrt{\frac{-8}{x-1}-4}$ | A1 | OE without $\pm$ in final answer. |
|  |  | 3 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 6(b) | $1-\frac{8}{x^{2}+4}=\frac{x^{2}+4}{x^{2}+4}-\frac{8}{x^{2}+4}\left[=\frac{x^{2}+4-8}{x^{2}+4}\right]=\frac{x^{2}-4}{x^{2}+4}$ | M1 A1 | Using common denominator or division to reach 1. Remainder -8. WWW |
|  | $0<\mathrm{f}(x)<1$ | B1 B1 | B1 for each correct inequality. B0 if contradictory statement seen. <br> Accept $\mathrm{f}(x)>0, \mathrm{f}(x)<1 ; 1>\mathrm{f}(x)>0 ;(0,1)$ <br> SC B1 for $0 \leqslant \mathrm{f}(x) \leqslant 1$. |
|  |  | 4 |  |
| 6(c) | Because the range of f does not include the whole of the domain of f (or any of it) | B1 | Accept an answer that includes an example outside the domain of f, e.g. $f(4)=\frac{12}{20}$. Must refer to the domain or $>$ 2. Need not explicitly use the term 'domain' but must not refer just to the range. |
|  |  | 1 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 7(a) | $(3 x-2)^{\frac{1}{2}}=\frac{1}{2} x+1 \Rightarrow 3 x-2=\left(\frac{1}{2} x+1\right)^{2}=\frac{1}{4} x^{2}+x+1$ | M1 | Equating curve and line, attempt to square; $\frac{1}{4} x^{2}+1 \mathrm{M} 0$ |
|  | $\Rightarrow \frac{1}{4} x^{2}-2 x+3[=0]\left[\Rightarrow x^{2}-8 x+12=0\right] \Rightarrow(x-6)(x-2)[=0]$ | M1 | Forming and solving a 3TQ by factorisation, formula or completing the square - see guidance. |
|  | $(2,2)$ and $(6,4)$ | A1 A1 | A1 for each point, or A1 A0 for two correct $x$-values. If M0 for solving, SC B2 possible: B1 for each point or B1 B0 for two correct $x$-values. |
|  |  | 4 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 7(b) | $\text { Area }= \pm \int_{[2]}^{[6]}\left((3 x-2)^{\frac{1}{2}}-\left(\frac{1}{2} x+1\right)\right)[\mathrm{d} x]$ | *M1 | For intention to integrate and subtract (M0 if squared). |
|  | $\pm\left[\frac{2}{9}(3 x-2)^{\frac{3}{2}}-\left(\frac{1}{4} x^{2}+x\right)\right]_{2}^{6}$ | B1 B1 | B1 for each bracket integrated correctly (in any form). |
|  | $\pm\left(\left[\frac{2}{9}(16)^{\frac{3}{2}}-\left(\frac{1}{4} \times 36+6\right)\right]-\left[\frac{2}{9}(4)^{\frac{3}{2}}-\left(\frac{1}{4} \times 4+2\right)\right]\right)$ | DM1 | $\pm(\mathrm{F}($ their 6$)-\mathrm{F}($ their 2$))$ with their integral. Allow 1 sign error. |
|  | $\frac{4}{9}$ | A1 | AWRT 0.444. <br> SC1 B1 for $\frac{4}{9}$ if *M1 B1 B1 DM0. <br> SC2 B1 for $\frac{4}{9}$ if *M1 B0 B0 DM0, provided limits stated. |
|  | Alternative method for question 7(b) |  |  |
|  | Area $= \pm \int_{[2]}^{[6]}(3 x-2)^{\frac{1}{2}} \quad[\mathrm{~d} x]-$ area of trapezium (or triangle + rectangle $)$ | *M1 | For intention to integrate and subtract (M0 if squared). |
|  | $\pm\left[\frac{2}{9}(3 x-2)^{\frac{3}{2}}\right]_{2}^{6}-4\left(\frac{2+4}{2}\right) \quad$ or $\quad \pm\left[\frac{2}{9}(3 x-2)^{\frac{3}{2}}\right]_{2}^{6}-\left(\frac{2+4}{2}+(2 \times 4)\right)$ | $\begin{array}{r} \text { B1 } \\ \text { B1 FT } \end{array}$ | B1 for bracket integrated correctly (in any form). B1 FT for using correct formula with their values. |
|  | $\pm\left(\left(\frac{2}{9}(16)^{\frac{3}{2}}-\frac{2}{9}(4)^{\frac{3}{2}}\right)-12\right)$ | DM1 | $\pm(\mathrm{F}($ their 6$)-\mathrm{F}($ their 2$))$ using their integral. Allow 1 sign error. |


| Question |  | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: | :---: |
| 7(b) | $\frac{4}{9}$ |  | A1 | AWRT 0.444. SC1 B1 for $\frac{4}{9}$ if *M1 B1 B1 DM0. SC2 B1 for $\frac{4}{9}$ if *M1 B0 B0 DM0, provided limits stated. |
|  |  |  | 5 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 8(a) | EITHER (1) $\{$ Translation $\}\binom{\left\{30^{\circ}\right\}}{\{0\}}$ OR (2) $\{$ Translation $\}\binom{\left\{60^{\circ}\right\}}{\{0\}}$ | B2,1,0 | B2 for fully correct, B1 with two elements correct. \{ \} indicates different elements. Accept angle in radians. |
|  | (3) $\{$ Stretch $\}\{$ factor 2$\}\{$ in $x$-direction $\}$ | B2,1,0 | B2 for fully correct, B1 with two elements correct. \{ \} indicates different elements. |
|  | (4) Stretch factor 4 in $y$-direction and correct order | B1 | Stretch, $y$-direction and factor and correct order. Correct order is either (1) then (3) or (3) then (2). (4) can be anywhere in the sequence. |
|  |  | 5 |  |
| 8(b) | $4 \sin \left(\frac{1}{2} x-30^{\circ}\right)=2 \sqrt{2} \Rightarrow \sin ^{-1}\left(\frac{\sqrt{2}}{2}\right)[=45]$ | M1 | SOI |
|  | $\frac{1}{2} x-30=45$ or $135 \Rightarrow x=2(45+30)$ or $x=2(135+30)$ | M1 | SOI. The M marks are independent. |
|  | $x=150^{\circ}, x=330^{\circ}$ | A1 | Both exact values, condone $\frac{5 \pi}{6}, \frac{11 \pi}{6}$. A 0 if extra solutions in the interval. Ignore other solutions outside $\left[0^{\circ}, 360^{\circ}\right]$. |
|  |  | 3 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 9(a) | Express as $(x+3)^{2}+(y-1)^{2}=26+9+1[=36]$ | M1 | Completing the square on $x$ and $y$ or using the form $x^{2}+y^{2}+2 g x+2 f y+c=0$, centre $(-g,-f)$ and radius $\sqrt{g^{2}+f^{2}-c}$. <br> SOI by correct answer. |
|  | Centre ( $-3,1$ ) | B1 |  |
|  | Radius 6 | B1 |  |
|  | So lowest point is ( $-3,-5$ ) | A1 FT | FT on their centre and their radius. |
|  |  | 4 |  |
| 9(b) | Intersects when $x^{2}+(k x-5)^{2}+6 x-2(k x-5)-26=0$ or $(x+3)^{2}+(k x-5-1)^{2}=36$ | *M1 | Substituting $y=k x-5$ into their circle equation or rearranging and equating $y$. |
|  | $x^{2}+k^{2} x^{2}-10 k x+25+6 x-2 k x+10-26=0$ <br> or $x^{2}+6 x+9+k^{2} x^{2}-12 k x+36=36$ <br> leading to $k^{2} x^{2}+x^{2}+6 x-12 k x+9[=0]$ or $\left(k^{2}+1\right) x^{2}+(6-12 k) x+9[=0]$ | DM1 A1 | Rearranging to 3-term quadratic (terms grouped, all on one side). Allow 1 error. <br> Correct quadratic (need to see 9 as constant term). |
|  | $\begin{aligned} & (6-12 k)^{2}-4\left(k^{2}+1\right) \times 9[>0] \\ & {\left[\text { leading to } 144 k^{2}-144 k+36-36 k^{2}-36>0\right]} \end{aligned}$ | DM1 | Using discriminant $b^{2}-4 a c[>0]$ with their values. Allow if in square root. |
|  | [ $108 k^{2}-144 k=0$ leading to $] k=0$ or $k=\frac{4}{3}$ | A1 | Need not see method for solving. |
|  | $k<0, k>\frac{4}{3}$ | A1 | Do not accept $\frac{4}{3}<k<0$. |
|  |  | 6 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 10(a) | $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}=6(-1)^{2}-\frac{4}{(-1)^{3}}>0 \therefore$ minimum or $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}=10 \therefore$ minimum | B1 | Sub $x=-1$ into $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}$, correct conclusion. WWW |
|  |  | 1 |  |
| 10(b) | $\frac{\mathrm{d} y}{\mathrm{~d} x}=2 x^{3}+\frac{2}{x^{2}}[+c]$ | *M1 | Integrating $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}$ (at least one term correct). |
|  | $0=-2+2+c$ leading to $c=[0]$ | DM1 | Substituting $x=-1, \frac{\mathrm{~d} y}{\mathrm{~d} x}=0$ (need to see) to evaluate $c$. DM0 if simply state $c=0$ or omit $+c$. |
|  | $y=\frac{1}{2} x^{4}-\frac{2}{x}+($ their $c) x+k$ | A1 FT | Integrated. FT their non-zero value of $c$ if DM1 awarded. |
|  | $\frac{9}{2}=\frac{1}{2}+2+k$ leading to $k=[2]$ | DM1 | Substituting $x=-1, y=\frac{9}{2}$ to evaluate $k(\operatorname{dep}$ on $* \mathrm{M} 1)$. |
|  | $y=\frac{1}{2} x^{4}-\frac{2}{x}+2$ | A1 | OE e.g. $2 x^{-1}$ or $\frac{4}{2}$. <br> A0 (wrong process) if $c$ not evaluated but correct answer obtained. |
|  |  | 5 |  |
| 10(c) | $\frac{\mathrm{d} y}{\mathrm{~d} x}=2 x^{3}+\frac{2}{x^{2}}=0$ | M1 | Their $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$. |
|  | Leading to $x^{5}=-1$ | M1 | Reaching equation of the form $x^{5}=a$. |
|  | So only stationary point is when $x=-1$ | A1 | $x=-1$ and stating e.g. 'only' or 'no other solutions. |
|  |  | 3 |  |


| Question | Answer | Marks | Guidance |
| :---: | :--- | :--- | :--- |
| $10(\mathrm{~d})$ | At $x=1, \frac{\mathrm{~d} y}{\mathrm{~d} x}=[4]$ | *M1 | Substituting $x=1$ into their $\frac{\mathrm{d} y}{\mathrm{~d} x}$. |
|  | $\frac{\mathrm{d} x}{\mathrm{~d} t}=\frac{\mathrm{d} x}{\mathrm{~d} y} \times \frac{\mathrm{d} y}{\mathrm{~d} t}=\frac{1}{4} \times 5$ | DM1 | OE Using chain rule correctly SOI. |
|  | $\frac{5}{4}$ | A1 | OE e.g. 1.25. |
|  |  | $\mathbf{3}$ |  |

