



# Cambridge International AS & A Level

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**MATHEMATICS**

**9709/32**

Paper 3 Pure Mathematics 3

**May/June 2022**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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2 Solve the equation  $3 \cos 2\theta = 3 \cos \theta + 2$ , for  $0^\circ \leq \theta \leq 360^\circ$ . [5]

Dotted lines for student response.

- 3 The polynomial  $ax^3 + x^2 + bx + 3$  is denoted by  $p(x)$ . It is given that  $p(x)$  is divisible by  $(2x - 1)$  and that when  $p(x)$  is divided by  $(x + 2)$  the remainder is 5.

Find the values of  $a$  and  $b$ .

[5]

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- 4 The equation of a curve is  $y = \cos^3 x \sqrt{\sin x}$ . It is given that the curve has one stationary point in the interval  $0 < x < \frac{1}{2}\pi$ .

Find the  $x$ -coordinate of this stationary point, giving your answer correct to 3 significant figures. [6]

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5 (a) By sketching a suitable pair of graphs, show that the equation  $\ln x = 3x - x^2$  has one real root. [2]

(b) Verify by calculation that the root lies between 2 and 2.8. [2]

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(c) Use the iterative formula  $x_{n+1} = \sqrt{3x_n - \ln x_n}$  to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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6 The variables  $x$  and  $y$  satisfy the differential equation

$$\frac{dy}{dx} = xe^{y-x},$$

and  $y = 0$  when  $x = 0$ .

(a) Solve the differential equation, obtaining an expression for  $y$  in terms of  $x$ .

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(b) Find the value of  $y$  when  $x = 1$ , giving your answer in the form  $a - \ln b$ , where  $a$  and  $b$  are integers. [1]

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7 The equation of a curve is  $x^3 + 3x^2y - y^3 = 3$ .

(a) Show that  $\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 - x^2}$ . [4]

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(b) Find the coordinates of the points on the curve where the tangent is parallel to the  $x$ -axis. [5]

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8 Let  $f(x) = \frac{x^2 + 9x}{(3x - 1)(x^2 + 3)}$ .

(a) Express  $f(x)$  in partial fractions.

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(b) Hence find  $\int_1^3 f(x) \, dx$ , giving your answer in a simplified exact form. [5]

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9 The lines  $l$  and  $m$  have vector equations

$$\mathbf{r} = -\mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} - \mathbf{k}) \quad \text{and} \quad \mathbf{r} = 5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} + \mathbf{k})$$

respectively, where  $a$  and  $b$  are constants.

(a) Given that  $l$  and  $m$  intersect, show that  $2b - a = 4$ .

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(b) Given also that  $l$  and  $m$  are perpendicular, find the values of  $a$  and  $b$ . [4]

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(c) When  $a$  and  $b$  have these values, find the position vector of the point of intersection of  $l$  and  $m$ . [2]

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10 The complex number  $-1 + \sqrt{7}i$  is denoted by  $u$ . It is given that  $u$  is a root of the equation

$$2x^3 + 3x^2 + 14x + k = 0,$$

where  $k$  is a real constant.

(a) Find the value of  $k$ .

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(b) Find the other two roots of the equation.

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- (c) On an Argand diagram, sketch the locus of points representing complex numbers  $z$  satisfying the equation  $|z - u| = 2$ . [2]

- (d) Determine the greatest value of  $\arg z$  for points on this locus, giving your answer in radians. [2]

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**Additional Page**

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