



Cambridge International AS & A Level

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MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

May/June 2022

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

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1 Find, in terms of a , the set of values of x satisfying the inequality

$$2|3x + a| < |2x + 3a|,$$

where a is a positive constant.

[4]

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2 Solve the equation $\cos(\theta - 60^\circ) = 3 \sin \theta$, for $0^\circ \leq \theta \leq 360^\circ$. [5]

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- 3 (a) Show that the equation $\log_3(2x + 1) = 1 + 2 \log_3(x - 1)$ can be written as a quadratic equation in x . [3]

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- (b) Hence solve the equation $\log_3(4y + 1) = 1 + 2 \log_3(2y - 1)$, giving your answer correct to 2 decimal places. [2]

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4 The curve $y = e^{-4x} \tan x$ has two stationary points in the interval $0 \leq x < \frac{1}{2}\pi$.

- (a) Obtain an expression for $\frac{dy}{dx}$ and show it can be written in the form $\sec^2 x(a + b \sin 2x)e^{-4x}$, where a and b are constants. [4]

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(b) Hence find the exact x -coordinates of the two stationary points. [3]

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5 The complex number $3 - i$ is denoted by u .

- (a) Show, on an Argand diagram with origin O , the points A , B and C representing the complex numbers u , u^* and $u^* - u$ respectively.

State the type of quadrilateral formed by the points O , A , B and C . [3]

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- (b) Express $\frac{u^*}{u}$ in the form $x + iy$, where x and y are real. [3]

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(b) Given also that $N = 625$ when $t = 50$, find the value of k . [2]

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(c) Obtain an expression for N in terms of t , and find the greatest value of N predicted by this model. [2]

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9 With respect to the origin O , the point A has position vector given by $\vec{OA} = \mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$. The line l has vector equation $\mathbf{r} = 4\mathbf{i} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$.

(a) Find in degrees the acute angle between the directions of OA and l . [3]

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(b) Find the position vector of the foot of the perpendicular from A to l . [4]

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(c) Hence find the position vector of the reflection of A in l . [2]

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10 The constant a is such that $\int_1^a x^2 \ln x \, dx = 4$.

(a) Show that $a = \left(\frac{35}{3 \ln a - 1}\right)^{\frac{1}{3}}$. [5]

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(b) Verify by calculation that a lies between 2.4 and 2.8. [2]

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(c) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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Additional Page

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