UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Ordinary Level

MARK SCHEME for the June 2004 question papers

4037 ADDITIONAL MATHEMATICS

4037/01	**Paper 1, maximum raw mark 80**
4037/02	**Paper 2, maximum raw mark 80**

These mark schemes are published as an aid to teachers and students, to indicate the requirements of the examination. They show the basis on which Examiners were initially instructed to award marks. They do not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the June 2004 question papers for most IGCSE and GCE Advanced Level syllabuses.

UNIVERSITY of CAMBRIDGE
International Examinations

Mark Scheme Notes

- Marks are of the following three types:

  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

- The following abbreviations may be used in a mark scheme or used on the scripts:

  AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

  BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

  CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

  ISW Ignore Subsequent Working

  MR Misread

  PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

  SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

- MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

- OW –1, 2 This is deducted from A or B marks when essential working is omitted.

- PA –1 This is deducted from A or B marks in the case of premature approximation.

- S –1 Occasionally used for persistent slackness.

- EX –1 Applied to A or B marks when extra solutions are offered to a particular equation.

JUNE 2004

GCE ORDINARY LEVEL

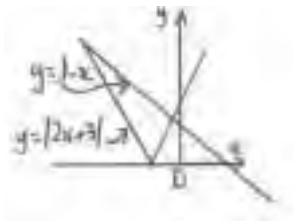
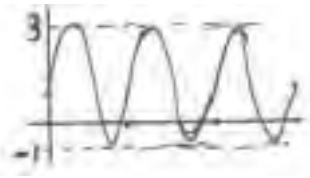
MARK SCHEME

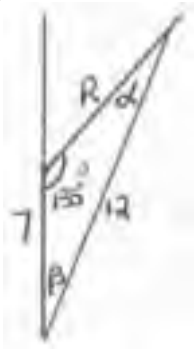
MAXIMUM MARK: 80

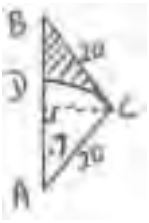
SYLLABUS/COMPONENT: 4037/01

ADDITIONAL MATHEMATICS
Paper 1

UNIVERSITY of CAMBRIDGE
International Examinations

1. (i) $y=(3x-2)\ (x^2+5)$ $dy/dx = \frac{(x^2+5)3 - (3x-2)2x}{(x^2+5)^2}$	M1 A1	Formula must be correct - allow unsimplified.
(ii) Num = $15 + 4x - 3x^2 = 0$ when $\rightarrow x = -5/3$ or $x = 3$	M1 A1 [4]	Setting to 0 + attempt to solve. Both correct.
2. $x^3 = 5x-2$ $x^3 - 5x + 2 = 0$ Tries to find a value $x = 2$ fits $(x-2) \rightarrow x^2 + 2x - 1 = 0$ Solution $\rightarrow x = -1 \pm\sqrt{2}$	M1 A1 M1 DM1 A1 [5]	Equating + attempt at a value by TI Co - allow for (x-2) or for f(2) Must be by (x-his value) As by quadratic scheme Co
3. (i)  $y = \lvert 2x+3 \rvert$ -ve then +ve slope Vertex at (-h,0) $y = 1 - x$ Line, -ve m, (k,0)	B1 DB1 B1 [3]	Must be 2 parts – ignore -2 to -1 V shape-Vertex on -ve x-axis + lines -ve slope, crosses axes at x,y +ve – allow if only in 1st or 2nd quadrants
(ii) $x + 2x + 3 = 1 \rightarrow x = -⅔$ (-0.65 to -0.70) $x - (2x+3) = 1 \rightarrow x = -4$ (-3.9 to -4.1)	B1 M1 Al [3]	From graph, or calculation or guess B2 if correct. M mark for any method. Squares both sides M1 quadratic A1 Answers A1
4. $x = a\sin(bx)+c$ (i) $a = 2$ and $b = 3$	B1 B1	Wrong way round - no marks. No labels - allow B1 if both correct.
(ii) $c = 1$	B1	Co
(iii) 3 cycles (0 to 360) -1 to 3  Period 120° + all correct.	B1 B1 DB1 [6]	Even if starting incorrectly. Needs to be marked - allow for any trig graph. Everything in relatively correct position - needs both B's

5. xy + 24 = 0 and 5y + 2x = 1 Makes x or y the subject and subs → $5y^2 = y + 48$ or $2x^2 - x = 120$ Solution of quadratic = 0 → (8,-3) and (-7.5,3.2) $d = \sqrt{(15.5^2+6.2^2)} = 16.7$	M1 A1 DM1 A1 M1 A1 √ [6]	x or y removed completely – condone poor algebra. A1 co. By scheme for quadratic = 0 Co M mark ind of anything before. A1√ on his 2 points.
6. $(300 \quad 240)\begin{pmatrix}.6 & .3 & .1\\ .5 & .4 & .1\end{pmatrix}\begin{pmatrix}4\\6\\8\end{pmatrix}$ $\left[or(4 \quad 6 \quad 8)\begin{pmatrix}.6 & .5\\ .3 & .4\\ .1 & .1\end{pmatrix}\begin{pmatrix}300\\240\end{pmatrix}\right]$ $(300 \quad 186 \quad 54)\begin{pmatrix}4\\6\\8\end{pmatrix} or (300 \quad 240)\begin{pmatrix}5\\5.2\end{pmatrix}$ Final answer → $2748	B2,1.0 M1 A1 M1 B1 [6]	For 3 correct matrices – independent of whether they are conformable – allow with or without the factor of 100. 1st product. Co. Matrices must be written in correct order – for M mark, the 2x3 or 3x2 must be used. 2nd product. By any method, inc numerical. Omission of 100 loses last B1 only.
7.  $\frac{\sin\alpha}{7} = \frac{\sin 135}{12}$ → α = 24.4° = 20.6°. Bearing is 020.6°	B1 M2 A1 A1 [5]	Correct triangle of velocities - must be 7,12 and 135° opposite 12. Sine rule used in his triangle. If 45° or 135° between 7 and 12, allow M1 for cos rule, M1 for sine rule Co. Co. Allow 21°.

8. $y = (ax+3)\ln x$ On x-axis, $y = 0$ $ax + 3 = 0 \rightarrow x$ is -ve $\rightarrow$no soln But $\ln x = 0 \rightarrow x = 1$ $dy/dx = a\ln x + (ax+3).(1/x)$ Use of $m_1m_2 = -1$ Gradient of tangent = -1 (-1/5) $\rightarrow a = 2$	M1 A1 M1 B1 M1 A1 A1 [7]	Needs an attempt at solution. Ignore other solutions at this stage. Correct use of "uv" formula. For d/dx(lnx), even if M0 given above. Could equate m with -1 (dy/dx) Co. Co.
9. (a) $\left(x - \frac{1}{2x^5}\right)^{18}$ $_{18}C_{15}\,(x)^{15}\,(1/2x^5)^3$ $\rightarrow 18.17.16(-⅛) \div 6$ $\rightarrow -102$ (b) $(1 + kx)^n$ Coeff of $x^2 = {}_nC_2k^2$ Coeff of $x^3 = {}_nC_3k^3$ Equating and changing to factorials $\rightarrow k = 3/(n-2)$ or equivalent without factorials	B1 B1 B1 [3] B1 B1 M1 A1 [4]	For $_{18}C_3$ or $_{18}C_{15}$ For $(\pm½)^3$ – even if in $(1/2x)^3$ Co Co. Co. Needs attempt at nCr Co
10. (i) Area = Δ – sector BCA = π – 1.4 or height = 20sin0.7  $\Delta = ½.20^2\sin(\pi - 1.4)$ or ½bh = 197.1 Sector = $½20^20.7 = 140$ $\rightarrow$ Area = 57.1 (ii) DC = 20 x 0.7 (=14) AB = 2 x 20cos0.7 or cos rule BD = AB – 20 = 10.6 $\rightarrow$ Perimeter = 44.6 Could be [5] + [3] if AB used in part (i)	M1 M1 M1 A1 [4] M1 M1 M1 A1 [4]	Award for either of these. Correct method for area of Δ Use of $½r^2\theta$ Co Use of $s = r\theta$ Correct trig – could gain this in (i) Co

11. (i) $m = -a/x^3 \rightarrow y = \frac{1}{2}ax^{-2}$ (+c) Puts in (2, 3.5) → $28 = a + 8c$ Puts in (5, 1.4) → $70 = a + 50c$ Solution → $a = 20, c = -1$	M1 A1 DM1 M1 A1 [5]	Any attempt to integrate. Co. Substitutes one of his points – even if +c missing Correct method of soln. Both co. (beware fortuitous ans. a = 20 given) N.B: assumes a = 20 without checking that both points work (M1A0DM1M0A1)
(ii) $\int(10x^{-2} + 1)dx = -10x^{-1} + x$ $A = [\ ]^p - [\ ]^2 = -10/p + p + 3$ $B = [\ ]^5 - [\ ]^p = 10/p - p + 3$ $P = \sqrt{10}$ or 3.16	M1 A1√ M1 M1 A1 [5]	Integrates his "curve" Use of limits correctly in either A or B or in A+B (2 to 5). Award M1 for each. (Can get these if only one integration) co
12 EITHER 12 questions – 3 trig, 4 alg, 5 calc Answer 8 from 12. (a) (i) ${}_{12}C_8 = 495$ (ii) T and A → 0 T and C → 1 A and C → 9 Total = 10 8 dresses, A → H (b) (i) ${}_8P_5 = 6720$ (ii) ⅛ of (i) = 840 or ${}_7P_4$ (iii) ⅝ of (i) = 4200 or 5 x (ii) or ${}_8P_5 - {}_7P_5$	 M1 A1 M1 A1 [4] M1 A1 M1 A1√ M1 A1√ [6]	 ${}_{12}C_8$ gets M1. Answer only gets both marks. Needs to have considered 2 of the possibilities. Must be ${}_8P_5$ for M1 – co for A1. Any method ok. √ on (i) if appropriate Any method ok. √ on (i) or (ii)

12 OR

x	2	4	6	8	10
y	9.8	19.4	37.4	74.0	144.4
lgy	0.99	1.29	1.57	1.87	2.16

(i) Finds values of lgy	M1	Knows what to do. Don't penalise incorrect scale.
Draws graph accurately.	A1 [2]	Points correct to ½ small square.
(ii) lgy = lgA + xlgb	B1	Anywhere – even if no graph
m = lgb → b = 1.4 (± 0.05)	M1 A1	Gradient measured + equated to lgb.
c = lgA → A = 5.0 (± 0.2)	M1 A1 [5]	Intercept measured + equated to lgA.
(iii) lgy = xlg2	B1	Even if no line – give if line correct.
i.e Straight line Y = 0.301x	M1	Must be a line.
x = 4.5 (± 0.2)	A1 [3]	To this accuracy.

Use of simultaneous eqns in part (ii) gets B1 only, unless both points used are on his line, in which case allow marks if to correct accuracy.

DM 1 for quadratic equation. Equation must be set to 0 if using formula or factors.

<u>Formula</u>
Must be correct
– ignore arithmetic and algebraic slips.

<u>Factors</u>
Must attempt to put quadratic into 2 factors.
Each factor then equated to 0.

JUNE 2004

GCE ORDINARY LEVEL

MARK SCHEME

MAXIMUM MARK: 80

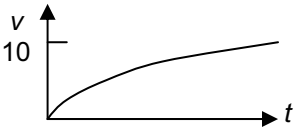
SYLLABUS/COMPONENT: 4037/02

ADDITIONAL MATHEMATICS
Paper 2

UNIVERSITY of CAMBRIDGE
International Examinations

1	[4]	$(\mathbf{i} - 7\mathbf{j}) + \lambda(0.6\mathbf{i} + 0.8\mathbf{j}) = 4\mathbf{i} + k\,\mathbf{j}$ $1 + 0.6\lambda = 4 \Rightarrow \lambda = 5$ $-7 + 0.8\lambda \Rightarrow -7 + 0.8 \times 5 = -3 = k$	M1 A1 M1 A1
2	[4]	Attempt at $\cos^{-1} 0.3 \Rightarrow$ [72.5° A0] = 1.266 [5.017, 7.549] accept 1.3 $x + 1 = 2.532, 10\,034, 15.098 \Rightarrow x = 14.1$ or better	M1 A1 M1 A1
3	[4]	**(i)** Some vegetarians in the college are over 180 cm tall [or equivalent] **(ii)** No cyclists in the college are over 180 cm tall [or equivalent] **(iii)** $B \cap C \subset A'$ [or equivalent]	B1 B1 B1 B1
4	[4]	$\left(1 + \frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right) \Rightarrow \frac{1 - \cos^2\theta}{\cos\theta \sin\theta}$ $1 - \cos^2\theta \equiv \sin^2\theta$ $\frac{\sin^2\theta}{\cos\theta \sin\theta} \rightarrow \tan\theta$ Must be useful use of Pythagoras	M1 M1 B1 A1
5	[5]	$x = \frac{\sqrt{20} \pm \sqrt{20 - (4 \times 2)}}{2} = \sqrt{5} \pm \sqrt{3}$ or $\frac{\sqrt{20} \pm \sqrt{12}}{2}$ $\frac{1}{\sqrt{5} + \sqrt{3}} + \frac{1}{\sqrt{5} - \sqrt{3}}$ [or $\frac{2}{\sqrt{20} + \sqrt{12}} + \frac{2}{\sqrt{20} - \sqrt{12}}$] rationalising each fraction or bringing to common denominator Denominator = 2 [or 8] $\Rightarrow \frac{1}{c} + \frac{1}{d} = \sqrt{5}$	M1 A1 M1 A1 A1
6	[6]	**(a)** $2x^2 - 3x - 14 = 0 \Rightarrow (2x - 7)(x + 2) = 0 \Rightarrow x = -2, 3.5$ $\{x : x < -2\} \cup \{x : x > 3.5\}$ **(b)** Eliminate $y \Rightarrow x^2 + 4(8 - kx) = 20$ [or $x \Rightarrow \left(\frac{8 - y}{k}\right)^2 + 4y = 20$] $x^2 - 4kx + 12 = 0$ [or $y^2 + (4k^2 - 16)y + (64 - 20k^2) = 0$] Apply " $b^2 = 4ac$ " $16k^2 = 48$ [or $16k^4 = 48k^2$] $\Rightarrow k = \pm\sqrt{3}$	M1 A1 A1 M1 M1 A1

7 [6]	**(i)** e^{2x-3} (= 7) $\Rightarrow$ x = ½ (3 + ln 7) ≈ 2.47 ~ 2.48 (not 2.5)	M1 A1
	(ii) h = $2e^x - 3$ (x, y or) h > – 3 accept ≥	B1 B1
	(iii) h^{-1} (or y) = ln {½ (x + 3)} or ln(x + 3) – ln2 or lg{½(x + 3)}/lge but ln{½(y + 3)} M1 A0 lg (or log) {½(x + 3)} M1 A0	M1 A1 (M1 for logs taken in valid way
8 [8]	**(i)** $\log_3(2x+1) - \log_3(3x-11) = \log_3 \dfrac{2x+1}{3x-11}$ [Or, later, give M1 for log + log=log(product)	M1
	$\log_3(\ ) = 2 \Rightarrow (\ ) = 3^2$	B1
	$2x + 1 = 9(3x - 11) \Rightarrow x = 4$	DM1 A1
	(ii) $\log_4 y = \dfrac{\log_2 y}{\log_2 4} = \dfrac{1}{2}\log_2 y$ [or $\log_2 y = \dfrac{\log_4 y}{\log_4 2} = 2\log_4 y$]	M1 A1
	½ $\log_2 y + \log_2 y = 9$ [or $\log_4 y + 2\log_4 y = 9$] $\Rightarrow$ $y = 2^6$ or $4^3 = 64$	DM1 A1
9 [8]	$6 + 4x - x^2 \equiv 10 - (x-2)^2$	M1 A1
	(i) x = 2 y = 10 Maximum	B1√B1√B1
	(ii) f(0) = 6, f(2) = 10, f(5) = 1 $\Rightarrow$ 1 ≤ f ≤10 [alternatively 1 ≤ B1, ≤ 10 B1]	M1 A1
	(iii) f has no inverse; it is not 1:1	B1
10 [10]	**(i)** m_{BC} = 3/5 Equation of AD is $y - 4 = 3/5(x + 2)$	B1 M1 A1
	m_{AC} = – ¼ Equation of CD is $y - 2 = 4(x - 6)$	B1 M1 A1
	(ii) Solve $x = 8, y = 10$	M1 A1
	(iii) Length of AC = Length of CD = $\sqrt{68}$	M1 A1
11 [10]	**(i)** d/dx $(2x-3)^{3/2}$ = $(2x-3)^{1/2}$ × 3/2 × 2	M1 A1
	dy/dx = 1 × $(2x-3)^{3/2}$ + (x + 1) × { candidate's d/dx $(2x-3)^{3/2}$ }	M1
	= $\sqrt{2x-3}\{(2x-3)+3(x+1)\} = 5x\sqrt{2x-3} \Rightarrow k = 5$	A1
	(ii) $\delta y \approx$ dy/dx × δx = $(dy/dx)_{x=6} \times p$ = 90p	M1 A1
	$(y)_{x=6+p} = (y)_{x=6} + \delta y = 189 + 90p$	A1√
	(iii) $\int x\sqrt{2x-3}\,dx$ = 1/5 (x + 1)($2x-3)^{3/2}$	M1
	$[\ ]_2^6$ = 1/5 (189 – 3) =37.2	DM1 A1

12 [11] **EITHER**	**(i)** $a = dv/dt = 5e^{-1/2\,t}$ $v = 8 = 10(1 - e^{-1/2\,t}) \Rightarrow e^{-1/2\,t} = 0.2 \Rightarrow a = 1$ **(ii)** $s = \int v\,dt = \int\left(10 - 10\,e^{-t/2}\right)dt = 10t + 20e^{-t/2}$ $[\ ]_0^6 = (60 + 20e^{-3}) - (20) \approx 41$ **(iii)** 10 **(iv)** 	M1 A1 M1 A1 M1 A1 DM1 A1 B1 B2,1,0
12 [11] **OR**	**(i)** $d/d\theta\{(\cos\theta)^{-1}\} = -(\cos\theta)^{-2}(-\sin\theta) = \sin\theta/\cos^2\theta$ **(ii)** $AX = 2\sec\theta$ $PX = 2\tan\theta$ $T = \frac{2\sec\theta}{3} + \frac{10 - 2\tan\theta}{5}$ **(iii)** $\frac{dT}{d\theta} = \frac{2}{3}\frac{\sin\theta}{\cos^2\theta} - \frac{2}{5}\sec^2\theta$ $= 0$ when $5\sin\theta = 3 \Rightarrow \sin\theta = 3/5$ $PX = 2\tan\theta = 2 \times ¾ = 1.5$	M1 A1 B1 B1 M1 A1 B1 B1√ M1 A1 A1