

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the October/November 2012 series

4037 ADDITIONAL MATHEMATICS

4037/23

Paper 2, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2012 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Page 2	Mark Scheme	Syllabus	Paper
	GCE O LEVEL – October/November 2012	4037	23

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol $\checkmark$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2, 1, 0 means that the candidate can earn anything from 0 to 2.

Page 3	Mark Scheme	Syllabus	Paper
	GCE O LEVEL – October/November 2012	4037	23

The following abbreviations may be used in a mark scheme or used on the scripts:

AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through $\sqrt{}$ ” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.
OW –1,2	This is deducted from A or B marks when essential working is omitted.
PA –1	This is deducted from A or B marks in the case of premature approximation.
S –1	Occasionally used for persistent slackness – usually discussed at a meeting.
EX –1	Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

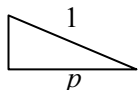
Page 4	Mark Scheme	Syllabus	Paper
	GCE O LEVEL – October/November 2012	4037	23

1 1.2 $5x + 7 = -13$ or $25x^2 + 70x + 49 = 169$ $5(5x - 6)(x + 4) = 0$ -4	B1 M1 A1 [3]	correct positive value correct method to find second value correct final answer
2 (i) $\frac{1}{6 \times 7 - 8 \times 4} \begin{pmatrix} 6 & -8 \\ -4 & 7 \end{pmatrix}$ (ii) $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{10} \begin{pmatrix} 6 & -8 \\ -4 & 7 \end{pmatrix} \begin{pmatrix} 39 \\ 23 \end{pmatrix}$ $= \begin{pmatrix} 5 \\ 0.5 \end{pmatrix}$	B1B1 [2] M1 A1 [2]	B1 for each part of the inverse pre-multiply $\begin{pmatrix} 39 \\ 23 \end{pmatrix}$ by their inverse correct answers, correctly associated
3 $(3\sqrt{3} - 1)^2 = 27 - 6\sqrt{3} + 1$ or $(3\sqrt{3} - 1)(2\sqrt{3} + 3) = 18 + 7\sqrt{3} - 3$ $\times \frac{2\sqrt{3} + 3}{2\sqrt{3} + 3}$ or $28 - 6\sqrt{3} = \frac{a\sqrt{3} + b}{3}(2 - 3)$ $\frac{38\sqrt{3} + 48}{3}$ or $a = 38, b = 48$	M1 A1 M1 A1 [4]	multiplication, including $a\sqrt{3} \times b\sqrt{3} = 3ab$ a correct expansion valid method to obtain a value for a or b correct answers

Page 5	Mark Scheme	Syllabus	Paper
	GCE O LEVEL – October/November 2012	4037	23

4 $\overrightarrow{XZ} = \begin{pmatrix} 16 \\ 20 \end{pmatrix}$ $\overrightarrow{OY} = \begin{pmatrix} 4 \\ -27 \end{pmatrix} + \frac{3}{4} \begin{pmatrix} 16 \\ 20 \end{pmatrix} \text{ or } \begin{pmatrix} 20 \\ -7 \end{pmatrix} + \frac{1}{4} \begin{pmatrix} -16 \\ -20 \end{pmatrix}$ $= \begin{pmatrix} 16 \\ -12 \end{pmatrix}$ $ \overrightarrow{OY} = \sqrt{16^2 + (-12)^2} \text{ oe}$ unit vector in direction of $\overrightarrow{OY} = \begin{pmatrix} 0.8 \\ -0.6 \end{pmatrix} \text{ oe}$ OR $\overrightarrow{OY} - \overrightarrow{OX} = 3\overrightarrow{OZ} = 3\overrightarrow{OY}$ $4\overrightarrow{OY} = \begin{pmatrix} 4 \\ -27 \end{pmatrix} + 3 \begin{pmatrix} 20 \\ -7 \end{pmatrix} = \begin{pmatrix} 64 \\ -48 \end{pmatrix}$ $\overrightarrow{OY} = \begin{pmatrix} 16 \\ -12 \end{pmatrix} \text{ etc.}$ OR $\overrightarrow{OY} = \frac{\overrightarrow{OX} + 3\overrightarrow{OZ}}{4}$ $= \frac{\begin{pmatrix} 4 \\ -27 \end{pmatrix} + 3 \begin{pmatrix} 20 \\ -7 \end{pmatrix}}{4}$ $= \begin{pmatrix} 16 \\ -12 \end{pmatrix}^4 \text{ etc.}$	B1 M1 A1 M1 A1 [5] B1 M1 A1 B1 M1 A1	correct vector for $\overrightarrow{XZ}$ valid method for $\overrightarrow{OY}$ correct vector for OY uses Pythagoras to find length of $\overrightarrow{OY}$ correct vector expression correct vector equation collect $\overrightarrow{OY}$ s and substitute for $\overrightarrow{OX}$ and $\overrightarrow{OZ}$ correct vector for OY correct use of intercept theorem substitute for $\overrightarrow{OX}$ and $\overrightarrow{OZ}$ and divide correct vector for OY
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Page 6	Mark Scheme	Syllabus	Paper
	GCE O LEVEL – October/November 2012	4037	23

5 $mx + 2 = mx^2 + 7x + 11$ $mx^2 + 7x - mx + 9 = 0$ $(7 - m)^2 - 4 \times m \times 9 \sim 0$ $m^2 - 50m + 49 \sim 0$ $(m - 1)(m - 49), m = 1, 49$ $1 < m < 49$	M1* A1 M1dep* A1 M1 A1 [6]	eliminates either y or x correct equation compares discriminant with 0 correct quadratic solves 3-term quadratic for m correct answer
6 (a) $\sec^2 x = \frac{1}{p^2}$ $\tan^2 x = \sec^2 x - 1 = \frac{1}{p^2} - 1$ OR $\sin^2 x = 1 - p^2$ $\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - p^2}{p^2}$ OR $\sqrt{1 - p^2}$  $\tan x = \frac{\sqrt{1 - p^2}}{p}$ $\tan^2 x = \frac{1 - p^2}{p^2}$ (b) $\cot^2 \theta + 2(\cot \theta \tan \theta) + \tan^2 \theta$ $\cot^2 \theta = \operatorname{cosec}^2 \theta - 1$ or $\tan^2 \theta = \sec^2 \theta - 1$ completion “AG”	B1 M1 A1 [3] B1 M1 A1 B1 M1 A1 B1 B1 B1 [3]	correct expression for $\sec^2 x$ in terms of p substitution in correct formula (ps only) correct answer, oe correct expression for $\sin^2 x$ in terms of p substitution in correct formula (ps only) correct answer, oe ‘opposite’ is $\sqrt{1 - p^2}$ $\tan x = \text{their opposite} \div \text{their adjacent}$ correct answer, oe correct squaring of bracket use of a correct relevant formula correct completion

Page 7	Mark Scheme	Syllabus	Paper
	GCE O LEVEL – October/November 2012	4037	23

7 (a) $\int \left(x^{\frac{3}{2}} + 3x^{\frac{1}{2}} \right) dx$ $\frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} (+c)$ (b) $\frac{k}{2x+5} \text{ oe}$ $\frac{-10}{2x+5} \text{ oe}$ $\frac{k}{2 \times 10 + 5} - \frac{k}{5}$ 1.6	B1 M1 A1 [3] M1 A1 M1 A1√ [4]	correct expression in terms of indices increase fractional power by 1 correct answer, ISW integral of correct form, k a constant correct integral, ignore '+ c' their integral with $x=10$ subtract their integral with $x=0$ correct answer, ft their $k \left(= \frac{-4}{25}k \right)$
8 gradient $\frac{9-3}{1-(-2)} (=2)$ (AD) $y-5=2(x-4)$ or $y=2x-3$ (CD) $y-9=-\frac{1}{2}(x-1)$ or $x+2y=19$ solves equation for AD with equation for CD D is (5, 7) $\text{area} = \frac{1}{2} \begin{vmatrix} 4 & -2 & 1 & 5 & 4 \\ 5 & 3 & 9 & 7 & 5 \end{vmatrix} = \frac{1}{2} 26 - 66 $ or $= \frac{1}{2} (\sqrt{5} + \sqrt{45}) \sqrt{20}$ $= 20$ OR (X on BC, $AX \parallel DC$) gradient $= \frac{9-3}{1-(-2)} (=2)$ (BC) $y-9=2(x-1)$ or $y=2x+7$ (AX) $y-5=-\frac{1}{2}(x-4)$ or $2y=-x+14$ solves equation BC with equation AX $X(0, 7)$ areaΔ + area rectangle $= \frac{1}{2} \sqrt{20} \times \sqrt{20} + \sqrt{20} \times \sqrt{5}$ $= 20$	B1 B1√ M1 A1 M1 A1 M1 M1 A1 [8] B1 B1 M1 A1 M1 A1 M1 A1	correct gradient correct equation for AD, ft their m_{AD} uses $m_1 m_2 = -1$ and $x=1$ and $y=9$ in equation of line correct equation for CD solving equations for a value of x or y $x=5, y=7$ a correct method to calculate the area of the trapezium correct answer correct gradient correct equation for BC uses $m_1 m_2 = -1$ and $x=4$ and $y=5$ in equation of line correct equation for AX solving equations for a value of x or y $x=0, y=7$ a correct method to calculate the area correct answer

Page 8	Mark Scheme	Syllabus	Paper
	GCE O LEVEL – October/November 2012	4037	23

9 (i) x^3 (ii) <table><tr><td>x^3</td><td>1</td><td>8</td><td>27</td><td>64</td></tr><tr><td>x^2y</td><td>9.41</td><td>5.16</td><td>-6.21</td><td>-28.32</td></tr></table> (iii) $a = 9.5$ to 10.5 gradient = $\frac{y_2 - y_1}{x_2 - x_1}$ $b = -0.6 \pm 0.01$ (iv) $y = \frac{a}{13.69} + 3.7b$ or $13.69y = a + 50.653b$ $= -1.48 \pm 0.04$	x^3	1	8	27	64	x^2y	9.41	5.16	-6.21	-28.32	B1 [1] M1 A2,1,0 [3] B1 M1 A1 [3] M1 A1 [2]	correct answer graph of x^2y against x^3, linear axes 4, 1–3, 0 points plotted correctly correct answer finding numerical value for the gradient correct answer appropriate substitutions or read graph at 50.653 and divide value by 13.69 correct answer
x^3	1	8	27	64								
x^2y	9.41	5.16	-6.21	-28.32								
10 (i) $x^2 + 80^2$ seen time = $\frac{\text{distance}}{\text{speed}}$, oe (ii) $\left(\frac{dT}{dx} = \right) \frac{-1}{10} + \frac{x}{6\sqrt{x^2 + 6400}}$$\frac{x}{6\sqrt{x^2 + 6400}} = \frac{1}{10} \text{ oe}$$x = 60$$T = 30\frac{2}{3}(30.7)$	B1 B1 [2] M1* A1A1 M1dep* A1 A1 [6]	attempt to differentiate given expression A1 each correct unsimplified term attempt to solve $\frac{dT}{dx} = 0$, to include squaring both sides correct answer for x correct answer for T										

Page 9	Mark Scheme	Syllabus	Paper
	GCE O LEVEL – October/November 2012	4037	23

11 (a) $2^{x-2} = 100^2, \frac{x-2}{2} = \log_2 100$ or $2^{\left(\frac{x-1}{2}\right)} = 100$ $x = 2 + \frac{4}{0.301...}$ $= 15.3$ (b) $\log_y 512 = 3 \text{ or } 3 = \log_y y^3$ or $\log_y k = \frac{\log k}{\log y}$ (twice) $y^3 = 512 \text{ or } 2 = \frac{y^3}{256}$ $y = 8$ (c) $\frac{6^{5z-2}}{6^{2z}} = \frac{6^{3(z-1)}}{6^{2(3-z)}}$ or $\log 6^{(5z-2)} - \log 6^{2z} = \log 6^{3(z-1)} - \log 6^{2(3-z)}$ $5z - 2 - 2z = 3z - 3 - (6 - 2z) \text{ oe}$ $z = 3.5$	B1 M1 A1 [3] B1 M1 A1 [3] M1 A1 M1 A1 [4]	correct expression valid attempt to obtain value for x correct answer correct relevant use of rule for logarithms attempt to solve correct answer attempt to express at least two elements in terms of 6^z or $\log 6$ correct expression uses rule of indices or logarithms correctly, accept index/log format correct answer
12E (i) $(2x + 8)^2 - 9$ or $a = 2, b = 8, c = -9$ (ii) $f^{-1}(x) = \frac{\sqrt{(x+9)} - 8}{2}$ oe (iii) $\left(\frac{2}{x} + 8\right)^2 - 9 = 135 \text{ or } \frac{4}{x^2} + \frac{32}{x} + 55 = 135$ $\frac{2}{x} + 8 = 12 \text{ (or } -12) \text{ or } 80x^2 - 32x - 4 = 0$ $x = 0.5$ oe, only	B1B1B1 [3] M1 A2,1,0√ [3] M1 A1 M1 A1 [4]	B1 for each correct value inverse of form $\frac{\sqrt{(x \pm c)} \pm b}{a}$ 3, 1 - 2, 0 correct values, ft their a, b and c apply fg (not gf) or replace x by $\frac{1}{x}$ correct equation valid method for solving their equation correct answer

Page 10	Mark Scheme	Syllabus	Paper
	GCE O LEVEL – October/November 2012	4037	23

120 (i) 3.5 (ii) $y^2 + 7 = 2x$ $h^{-1}(x) = \frac{x^2 + 7}{2}$ (iii) $\frac{3x - 4}{x - 2} = x, x^2 - 5x + 4 = 0$ $(x - 4)(x - 1)$ $x = 4$ only (iv) $\frac{3\left(\frac{3x - 4}{x - 2}\right) - 4}{\left(\frac{3x - 4}{x - 2}\right) - 2}$ $\frac{3(3x - 4) - 4(x - 2)}{3x - 4 - 2(x - 2)}$ $5 - \frac{4}{x}$	B1 [1] M1 A1 [2] M1 M1 A1 [3] M1 A1 M1 A1 [4]	correct answer attempt at inverse, involving squaring correct inverse equate $k(x)$ with x and obtain quadratic equation solve three term quadratic correct answer substitute to obtain expression for k^2 correct unsimplified expression multiply numerator and denominator by $(x - 2)$, oe correct answer
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