

**CAMBRIDGE INTERNATIONAL EXAMINATIONS**

**GCE Ordinary Level**

**MARK SCHEME for the October/November 2013 series**

**4037 ADDITIONAL MATHEMATICS**

**4037/12**

Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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## Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
  - A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
  - B Accuracy mark for a correct result or statement independent of method marks.
- When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep\*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
  - The symbol  $\nabla$  implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously ‘correct’ answers or results obtained from incorrect working.
  - Note: B2 or A2 means that the candidate can earn 2 or 0.  
B2, 1, 0 means that the candidate can earn anything from 0 to 2.

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1	$a = 3, b = 2, c = 1$	B1, B1, B1 [3]	B1 for each
2	<p>Using <math>b^2 - 4ac, 9 = 4(k + 1)^2</math>  <math>4k^2 + 8k - 5 = 0</math></p> $k = -\frac{5}{2}, \left(\frac{1}{2}\right)$ <p>To be below the <math>x</math>-axis <math>k &lt; -\frac{5}{2}</math></p> <p><b>Or:</b> <math>\frac{dy}{dx} = 2(k + 1)x - 3</math>  when <math>\frac{dy}{dx} = 0, x = \frac{3}{2(k + 1)}</math>  <math>\therefore y = (k + 1)\frac{9}{4(k + 1)^2} - \frac{9}{2(k + 1)} + (k + 1)</math>  To lie under the <math>x</math>-axis, <math>y &lt; 0</math>  <math>\therefore (k + 1)\frac{9}{4(k + 1)^2} - \frac{9}{2(k + 1)} + (k + 1) &lt; 0</math>  leading to <math>9 = 4(k + 1)^2</math> or equivalent  then as for previous method</p>	<p>M1 DM1</p> <p>A1</p> <p>A1 [4]</p> <p>M1</p>	<p>M1 for any use of <math>b^2 - 4ac</math>  DM1 for solution of their quadratic in <math>k</math></p> <p>A1 for critical value(s), <math>\frac{1}{2}</math> not necessary</p> <p>A1 for <math>k &lt; -\frac{5}{2}</math> only</p> <p>M1 for a complete method to this point.</p>

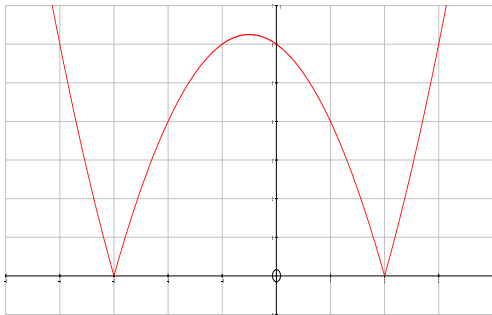
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<p>3</p> $\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} + \frac{(1 + \sin \theta)^2 + \cos^2 \theta}{\cos \theta(1 + \sin \theta)}$ $= \frac{1 + 2 \sin \theta + \sin^2 \theta + \cos^2 \theta}{\cos \theta(1 + \sin \theta)}$ $= \frac{2 + 2 \sin \theta}{\cos \theta(1 + \sin \theta)}$ $= \frac{2(1 + \sin \theta)}{\cos \theta(1 + \sin \theta)}$ $= 2 \sec \theta$ <p><b>Alternative solution:</b></p> $\sec \theta + \tan \theta + \frac{1}{\sec \theta + \tan \theta}$ $= \frac{(\sec \theta + \tan \theta)^2 + 1}{\sec \theta + \tan \theta}$ $= \frac{\sec^2 \theta + 2 \sec \theta \tan \theta + \tan^2 \theta + 1}{\sec \theta + \tan \theta}$ $= \frac{2 \sec^2 \theta + 2 \sec \theta \tan \theta}{\sec \theta + \tan \theta}$ $= \frac{2 \sec \theta (\sec \theta + \tan \theta)}{\sec \theta + \tan \theta}$ $= 2 \sec \theta$	<p>M1</p> <p>DM1</p> <p>DM1</p> <p>A1</p> <p>[4]</p> <p>M1</p> <p>DM1</p> <p>DM1</p> <p>A1</p>	<p>M1 for dealing with the fractions, denominator must be correct, be generous with numerator</p> <p>M1 for expansion and use of <math>\cos^2 \theta + \sin^2 \theta = 1</math></p> <p>M1 for attempt to factorise</p> <p>A1 for obtaining final answer correctly</p> <p>M1 for dealing with the fractions</p> <p>M1 for expansion and use of <math>\tan^2 \theta + 1 = \sec^2 \theta</math></p> <p>DM1 for attempt to factorise</p> <p>A1 for obtaining final answer correctly</p>
<p>4 (i) <math>n(A) = 3</math></p> <p>(ii) <math>n(B) = 4</math></p> <p>(iii) <math>A \cup B = \{60^\circ, 240^\circ, 300^\circ, 420^\circ, 600^\circ\}</math></p> <p>(iv) <math>A \cap B = \{60^\circ, 420^\circ\}</math></p>	<p>B1</p> <p>[1]</p> <p>B1</p> <p>[1]</p> <p>√B1</p> <p>[1]</p> <p>√B1</p> <p>[1]</p>	<p>If elements listed for (i), then they must be correct elements to get B1 leading to <math>n(A) = 3</math>. If they are not listed and correct answer given then B1.</p> <p>If elements listed for (ii), then they must be correct elements leading to <math>n(B) = 4</math> to get B1. If they are not listed and correct answer given then B1.</p> <p>Follow through on any sets listed in (i) and (ii). Do not allow any repetitions.</p> <p>Follow through on any sets listed in (i) and (ii).</p>

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<p><b>5 (i)</b> <math>9x - \frac{1}{3}\cos 3x (+c)</math></p> <p><b>(ii)</b> <math>\left[9x - \frac{1}{3}\cos 3x\right]_{\frac{\pi}{9}}^{\pi}</math>  <math>= \left(9\pi - \frac{1}{3}\cos 3\pi\right) - \left(\pi - \frac{1}{3}\cos \frac{\pi}{3}\right)</math>  <math>= 8\pi + \frac{1}{2}</math></p>	<p>B1, B1, B1 [3]</p> <p>M1</p> <p>A1, A1 [3]</p>	<p>B1 for <math>9x</math>, B1 for <math>\frac{1}{3}</math> or <math>\cos 3x</math></p> <p>B1 for <math>-\frac{1}{3}\cos 3x</math></p> <p>Condone omission of <math>+c</math></p> <p>M1 for correct use of limits in their answer to <b>(i)</b></p> <p>A1 for each term</p>
<p><b>6</b> <math>f\left(\frac{1}{2}\right) = \frac{a}{8} + 1 + \frac{b}{2} - 2</math></p> <p>leading to <math>a + 4b - 8 = 0</math></p> <p><math>f(2) = 2f(-1)</math></p> <p><math>8a + 16 + 2b - 2 = 2(-a + 4 - b - 2)</math></p> <p>leading to <math>10a + 4b + 10 = 0</math> or equivalent</p> <p><math>\therefore a = -2, b = \frac{5}{2}</math></p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>DM1 A1 [6]</p>	<p>M1 for substitution of <math>x = \frac{1}{2}</math> into <math>f(x)</math></p> <p>A1 for correct equation in any form</p> <p>M1 for attempt to substitute <math>x = 2</math> <b>or</b> <math>x = -1</math> into <math>f(x)</math> <b>and</b> use <math>f(2) = \pm 2f(-1)</math> or <math>2f(2) = \pm f(-1)</math></p> <p>A1 for a correct equation in any form</p> <p>DM1 (on both previous M marks) for attempt to solve simultaneous equations to obtain either <math>a</math> or <math>b</math></p> <p>A1 for both correct</p>

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<p>7 (a) (i) 360</p> <p>(ii) 120</p> <p>(b) (i) 924</p> <p>(ii) 28</p> <p>(iii) <math>924 - ({}^8C_3 \times {}^4C_3) - ({}^8C_2 \times {}^4C_4)</math>  (i.e. <math>924 - 3M\ 3W - 2M\ 4W</math>)</p> <p><math>924 - 224 - 28</math>  <math>= 672</math></p> <p>Or: <math>4M\ 2W\ {}^8C_4 \times {}^4C_2 = 420</math></p> <p><math>5M\ 1W\ {}^8C_3 \times {}^4C_1 = 224</math></p> <p><math>6M\ {}^8C_6 = 28</math></p> <p>Total <math>= 672</math></p>	<p>B1 [1]</p> <p>B1 [1]</p> <p>B1 [1]</p> <p>B1 [1]</p> <p>M1</p> <p>A1 [3]</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>M1 for 3 terms, at least 2 of which must be correct in terms of <math>C</math> notation or evaluated.</p> <p>A1 for any pair (must be evaluated)  A1 for final answer</p> <p>M1 for 3 terms, at least 2 of which must be correct in terms of <math>C</math> notation or evaluated.  A1 for any pair (must be evaluated)</p> <p>A1 for final answer</p>
<p>8 (i)</p>  <p>(ii) <math>\left(-\frac{1}{2}, \frac{25}{4}\right)</math></p> <p>(iii) <math>k &gt; \frac{25}{4}</math> or <math>\frac{25}{4} &lt; k \leq 14</math></p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1 [4]</p> <p>B1, B1 [2]</p> <p>B1 [1]</p>	<p>B1 for correct shape</p> <p>B1 for <math>(-3, 0)</math> or <math>-3</math> seen on graph</p> <p>B1 for <math>(2, 0)</math> or <math>2</math> seen on graph</p> <p>B1 for <math>(0, 6)</math> or <math>6</math> seen on graph or in a table</p> <p>B1 for each</p>

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<p>9 (a) <math>12x^2 \ln(2x+1) + 4x^3 \left( \frac{2}{2x+1} \right)</math></p> <p>(b) (i) <math>\frac{dy}{dx} = \frac{(x+2)^{\frac{1}{2}} 2 - 2x(x+2)^{-\frac{1}{2}} \frac{1}{2}}{x+2}</math></p> $= \frac{(x+2)^{\frac{1}{2}}}{(x+2)} (2(x+2) - x)$ $= \frac{x+4}{(x+2)^{\frac{3}{2}}}$ <p>Or:</p> $\frac{dy}{dx} = 2x \left( -\frac{1}{2} \right) (x+2)^{-\frac{3}{2}} + (x+2)^{-\frac{1}{2}} (2)$ $= (x+2)^{-\frac{3}{2}} (2(x+2) - x)$ $= \frac{x+4}{(x+2)^{\frac{3}{2}}}$ <p>(ii) <math>\frac{10x}{\sqrt{x+2}} (+c)</math></p> <p>(iii) <math>\left[ \frac{10x}{\sqrt{x+2}} \right]_2^7 = \frac{70}{3} - \frac{20}{2}</math></p> $= \frac{40}{3}$	<p>M1 A2, 1, 0 [3]</p> <p>M1, A1</p> <p>DM1</p> <p>A1 [4]</p> <p>M1, A1</p> <p>DM1 A1</p> <p>M1, A1 [2]</p> <p>M1 A1 [2]</p>	<p>M1 for differentiation of a correct product –1 for each error</p> <p>M1 for differentiation of a quotient involving <math>(x+2)^{\frac{1}{2}}</math></p> <p>A1 all correct unsimplified DM1 for attempt to simplify</p> <p>A1 for correct simplification to obtain the given answer</p> <p>M1 for differentiation of a product involving <math>(x+2)^{-\frac{1}{2}}</math></p> <p>A1 all correct unsimplified DM1 for attempt to simplify A1 for correct simplification to obtain the given answer</p> <p>M1 for <math>\frac{1}{5} \times \frac{2x}{\sqrt{x+2}}</math> or <math>5 \times \frac{2x}{\sqrt{x+2}}</math> A1 correct only, allow unsimplified. Condone omission of <math>+c</math></p> <p>M1 for correct application of limits in their answer to (b)(ii)</p>
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<p><b>10 (i)</b> <math>\sqrt{20}</math> or 4.47</p> <p><b>(ii)</b> <math>\text{Grad } AB = \frac{1}{2}, \perp \text{ grad} = -2</math>  <math>\perp \text{ line } y - 4 = -2(x - 1)</math>  <math>(y = -2x + 6)</math></p> <p><b>(iii)</b> Coords of <math>C(x, y)</math> and <math>BC^2 = 20</math>  <math>(x - 1)^2 + (y - 4)^2 = 20</math> <b>or</b>          Coords of <math>C(x, y)</math> and <math>AC^2 = 40</math>  <math>(x + 3)^2 + (y - 2)^2 = 40</math></p> <p>Need intersection with <math>y = -2x + 6</math>,</p> <p>leads to <math>5x^2 - 10x - 15 = 0</math> <b>or</b>  <math>5y^2 - 40y - 0 = 0</math></p> <p>giving <math>x = 3, -1</math>          and <math>y = 0, 8</math></p> <p><b>Or, using vector approach:</b>  <math>\overrightarrow{AB} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}</math>  <math>\overrightarrow{OC} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 8 \end{pmatrix}</math>  <math>\overrightarrow{OC} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}</math></p>	<p>B1 [1]</p> <p>M1</p> <p>M1, A1</p> <p>[3]</p> <p>M1</p> <p>A1</p> <p>DM1</p> <p>DM1 A1, A1 [6]</p> <p>B1</p> <p>M1 A1, A1</p> <p>A1, A1</p>	<p>M1 for attempt at a perp gradient</p> <p>M1 for attempt at straight line equation, must be perpendicular and passing through <math>B</math>. A1 allow unsimplified</p> <p>M1 for attempt to obtain relationship using an appropriate length and the point <math>(1, 4)</math> or <math>(-3, 2)</math> A1 for a correct equation</p> <p>DM1 for attempt to solve with <math>y = -2x + 6</math> and obtain a quadratic equation in terms of one variable only</p> <p>M1 for attempt to solve quadratic A1 for each 'pair'</p> <p>May be implied</p> <p>M1 for correct approach A1 for each element correct</p> <p>A1 for each element correct</p>
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<p><b>11 (a) (i)</b> <math>\begin{pmatrix} 4 &amp; 3 \\ 4 &amp; 3 \end{pmatrix}</math></p> <p><b>(ii)</b> <math>\mathbf{A}^2 = \begin{pmatrix} 16 &amp; 9 \\ 12 &amp; 13 \end{pmatrix}</math></p> <p><b>(iii)</b> <math>\mathbf{B}</math> is the inverse matrix of <math>\mathbf{A}^2</math>  <math>= \frac{1}{100} \begin{pmatrix} 13 &amp; -9 \\ -12 &amp; 16 \end{pmatrix}</math></p> <p><b>(b)</b> <math>\det \mathbf{C} = x(x-1) - (-1)(x^2 - x + 1)</math>  <math>= 2x^2 - 2x + 1</math></p> <p><math>b^2 - 4ac &lt; 0, 4 - 8 &lt; 0</math></p> <p>No real solutions (so <math>\det \mathbf{C} \neq 0</math>)</p>	<p>B1 [1]</p> <p>B1, B1 [2]</p> <p>✓B1, ✓B1 [2]</p> <p>M1 A1</p> <p>DM1</p> <p>A1 [4]</p>	<p>B1 for any 2 correct elements B1 for all correct</p> <p>Follow through on their <math>\mathbf{A}^2</math></p> <p>M1 for attempt to obtain <math>\det \mathbf{C}</math> A1 for this correct quadratic expression from a correct <math>\det \mathbf{C}</math></p> <p>DM1 for use of discriminant or attempt to solve using the formula, or attempt to complete the square in order to show there are no real roots.</p> <p>A1 for correct reasoning or statement that there are no real roots.</p>
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12	(a) (i)	$f(-10) = 299, f(8) = 191$ Min point at $(0, -1)$ or when $y = -1$	M1 B1	M1 for substitution of either $x = -10$ or $x = 8$ , may be seen on diagram B1 May be implied from final answer, may be seen on diagram Must have $\leq$ for A1, do not allow $x$
		$\therefore$ range $-1 \leq y \leq 299$	A1	
			[3]	
	(ii)	$x \geq 0$ or equivalent	B1	Allow any domain which will make $f$ a one-one function Assume upper and lower bound when necessary.
			[1]	
	(b) (i)	$g^{-1}(x) = \ln\left(\frac{x+2}{4}\right)$	M1	M1 for complete method to find the form inverse function, must involve $\ln$ or $\lg$ if appropriate. May still be in terms of $y$ .
		or $\frac{\lg\left(\frac{x+2}{4}\right)}{\lg e}$	A1	
			[2]	
	(ii)	$gh(x) = g(\ln 5x)$ $= 4e^{\ln 5x} - 2$	M1 A1	M1 for correct order A1 for correct expression $4e^{\ln 5x} - 2$
		$20x - 2 = 18, x = 1$	A1	
			[3]	
	Or	$h(x) = g^{-1}(18)$ $\ln 5x = \ln 5$	M1 A1	M1 for correct order A1 for correct equation
		leading to $x = 1$	A1	A1 for correct solution <b>from correct working</b>