CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the May/June 2014 series

4037 ADDITIONAL MATHEMATICS

4037/11 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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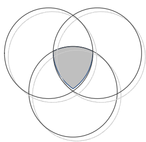
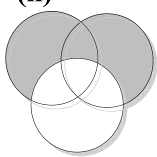
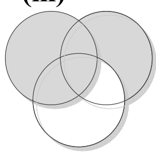
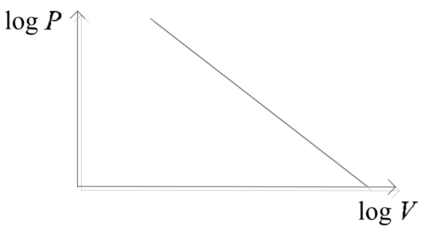
1	$\text{LHS} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}$	B1	B1 for use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$
	$= \frac{\sin\theta(1+\sin\theta)+\cos^2\theta}{\cos\theta(1+\sin\theta)}$	M1	M1 for attempt to obtain a single fraction
	$= \frac{1+\sin\theta}{\cos\theta(1+\sin\theta)}$	DM1	DM1 for use of $\sin^2\theta+\cos^2\theta=1$
	$= \frac{1}{\cos\theta}$ leading to $\sec\theta$	A1	A1 for 'finishing off'
	Alternative solution:		
	$\text{LHS} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta\,(1\text{-}\sin\theta)}{(1+\sin\theta)(1\text{-}\sin\theta)}$	B1	B1 for use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$
	$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$	M1	M1 for multiplication by $(1-\sin\theta)$
	$= \frac{\sin\theta}{\cos\theta} + \frac{(1-\sin\theta)}{\cos\theta}$	DM1	DM1 for use of $\sin^2\theta+\cos^2\theta=1$
	$= \frac{1}{\cos\theta}$ leading to $\sec\theta$	A1	A1 for 'finishing off'
	Alternative solution:		
	$\text{LHS} = \frac{\tan\theta(1+\sin\theta)+\cos\theta}{1+\sin\theta}$	M1	M1 for attempt to obtain a single fraction
	$= \frac{\frac{\sin\theta}{\cos\theta}+\frac{\sin^2}{\cos\theta}+\cos\theta}{1+\sin\theta}$	B1	B1 for use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$
	$= \frac{\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$		
	$= \frac{1+\sin\theta}{\cos\theta(1+\sin\theta)}$	DM1	DM1 for use of $\sin^2\theta+\cos^2\theta=1$
	$= \frac{1}{\cos\theta}$ leading to $\sec\theta$	A1	A1 for 'finishing off'

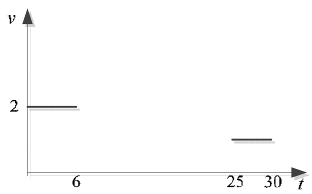
2 **(i)**	$\lvert\mathbf{a}\rvert = \sqrt{4^2 + 3^2} = 5$ $\lvert\mathbf{b}+\mathbf{c}\rvert = \sqrt{(-3)^2 + 4^2} = 5$	**M1** **A1**	**M1** for finding the modulus of either **a** or **b** + **c** **A1** for completion
(ii)	$\lambda\begin{pmatrix}4\\3\end{pmatrix} + \mu\begin{pmatrix}2\\2\end{pmatrix} = 7\begin{pmatrix}-5\\2\end{pmatrix}$ $4\lambda + 2\mu = -35$ and $3\lambda + 2\mu = 14$ leading to $\lambda = -49$, $\mu = 80.5$	**M1** **DM1** **A1**	**M1** for equating like vectors and obtaining 2 linear equations **DM1** for solution of simultaneous equations **A1** for both
3 **(a)**	(i)  (ii)  (iii) 	**B1** **B1** **B1**	**B1** for each
(b) **(i)**	2	**B1**	
(ii)	0	**B1**	
4	$k(4x-3) = 4x^2 + 8x - 8$ $4x^2 + x(8-4k) + 3k - 8 = 0$ $b^2 - 4ac = (8-4k)^2 - 16(3k-8)$ $= 16k^2 - 112k + 192$ $b^2 - 4ac < 0$, $k^2 - 7k + 12 < 0$ critical values $k = 3, 4$ $\therefore 3 < k < 4$	**M1** **DM1** **DM1** **A1** **A1**	**M1** for equating the line and the curve and attempt to obtain a quadratic equation in k **DM1** for use of $b^2 - 4ac$ with k **DM1** for solution of a 3 term quadratic equation, dependent on both previous M marks **A1** for both critical values **A1** for the range
5 **(i)**	$\frac{dy}{dx} = 2xe^{x^2}$	**B1B1**	**B1** for e^{x^2}, **B1** for $2xe^{x^2}$
(ii)	$\frac{1}{2}e^{x^2}$	**M1A1**	**M1** for ke^{x^2} **A1** for $\frac{1}{2}e^{x^2}$
(iii)	$\left(\frac{1}{2}e^4\right) - \left(\frac{1}{2}\right) = 26.8$	**DM1** **A1**	**DM1** for correct use of limits **A1** for 26.8, allow exact value

6	(i)	$\mathbf{AB} = \begin{pmatrix} 10 & 19 \\ 32 & 37 \\ 14 & 14 \end{pmatrix}$	M1 A1	**M1** for at least 3 correct elements of a 3×2 matrix **A1** for all correct
	(ii)	$\mathbf{B}^{-1} = \frac{1}{7}\begin{pmatrix} 5 & -1 \\ -3 & 2 \end{pmatrix}$	B1 B1	**B1** for $\frac{1}{7}$, **B1** for $\begin{pmatrix} 5 & -1 \\ -3 & 2 \end{pmatrix}$
	(iii)	$2\begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -22 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 5 & -1 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} -1.5 \\ -11 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 3.5 \\ -17.5 \end{pmatrix}$ $x = 0.5,\ y = -2.5$	M1 M1 A1	**M1** for obtaining in matrix form **M1** for pre-multiplying by $\mathbf{B}^{-1}$ **A1** for both
7	(i)	$y = 2x^2 - \frac{1}{x+1}(+c)$ when $x = \frac{1}{2}, y = \frac{5}{6}$ so $\frac{5}{6} = \frac{1}{2} - \frac{2}{3} + c$ leading to $c = 1$ $\left(y = 2x^2 - \frac{1}{x+1} + 1\right)$	B1 B1 M1 A1	**B1** for each correct term **M1** for attempt to find $+c$, must have at least 1 of the previous B marks Allow **A1** for $c = 1$
	(ii)	When $x = 1, y = \frac{5}{2}$ $\frac{dy}{dx} = \frac{17}{4}$ so gradient of normal $= -\frac{4}{17}$ Equation of normal $y - \frac{5}{2} = -\frac{4}{17}(x-1)$ $(8x + 34y - 93 = 0)$	M1 B1 DM1 A1	**M1** for using $x = 1$ in their **(i)** to find y **B1** for gradient of normal **DM1** for attempt at **normal** equation **A1** – allow unsimplified (fractions must not contain decimals)

<table>
<tr><td>8 (i)</td><td>$\log p = n\log V + \log k$
<table><tr><td>$\ln V$</td><td>2.30</td><td>3.91</td><td>4.61</td><td>5.30</td></tr><tr><td>$\ln p$</td><td>4.55</td><td>2.14</td><td>1.10</td><td>0.10</td></tr></table><table><tr><td>$\lg V$</td><td>1</td><td>1.70</td><td>2</td><td>2.30</td></tr><tr><td>$\lg p$</td><td>1.98</td><td>0.93</td><td>0.48</td><td>0.04</td></tr></table></td><td>B1

M1
A2,1,0</td><td>B1 for statement, but may be implied by later work.

M1 for plotting a suitable graph
–1 for each error in points plotted</td></tr>
<tr><td>(ii)</td><td>Use of gradient = n
$n = -1.5$ (allow –1.4 to –1.6)</td><td>DM1
A1</td><td>DM1 for equating numerical gradient to n</td></tr>
<tr><td>(iii)</td><td>Allow 13 to 16</td><td>DM1
A1</td><td>DM1 for use of their graph or substitution into their equation.</td></tr>
<tr><td>9 (a)</td><td>Distance travelled = area under graph
$= \frac{1}{2}(60+20)\times 12 = 480$</td><td>M1

A1</td><td>M1 for realising that area represents distance travelled and attempt to find area</td></tr>
<tr><td>(b)</td><td></td><td>B1
B1

B1</td><td>B1 for velocity of 2 ms^{-1} for $0 \leqslant t \leqslant 6$
B1 for velocity of zero for their ‘6’ to their ‘25’
B1 for velocity of 1 ms^{-1} for $25 \leqslant t \leqslant 30$</td></tr>
<tr><td>(c) (i)</td><td>$v = 4 - \frac{16}{t+1}$

When $v = 0$, $t = 3$</td><td>M1

DM1
A1</td><td>M1 for attempt at differentiation

DM1 for equating velocity to zero and attempt to solve</td></tr>
<tr><td>(ii)</td><td>$a = \frac{16}{(t+1)^2}$

$0.25(t+1)^2 = 16$

$t = 7$</td><td>M1

A1</td><td>M1 for attempt at differentiation and equating to 0.25 with attempt to solve</td></tr>
</table>

10 (a)	1 digit even numbers 2	**B1**	
	2 digit even numbers $4 \times 2 = 8$	**B1**	
	3 digit even numbers $3 \times 3 \times 2 = 18$	**B1**	
	Total = 28	**B1**	
(b) (i)	3M 5W = 35 4M 4W = 175 5M 3W = 210	**B1** **B1** **B1**	
	Total = 420	**B1**	**B1** for addition to obtain final answer, must be evaluated.
	or ${}^{12}C_8 -$ 6M 2W – 7M 1W 495 – 70 – 5 = 420		**or**: as above, final **B1** for subtraction to get final answer
(ii)	Oldest man in, oldest woman out and vice–versa		
	${}^{10}C_7 \times 2 = 240$	**B1, B1**	**B1** for ${}^{10}C_7$, B1 for realising there are 2 identical cases
	Alternative: 1 man out 1 woman in 6 men 4 women		
	6M 1W : ${}^6C_6 \times {}^4C_1 = 4$ 5M 2W : ${}^6C_5 \times {}^4C_2 = 36$ 4M 3W : ${}^6C_4 \times {}^4C_3 = 60$ 3M 4W : ${}^6C_3 \times {}^4C_4 = 20$ Total = 120	**B1**	All separate cases correct for **B1**
	There are 2 identical cases to consider, so 240 ways in all.	**B1**	**B1** for realising there are 2 identical cases, which have integer values

11 (a)	$5\sin 2x + 3\cos 2x = 0$ $\tan 2x = -0.6$ $2x = 149°, 329°$ $x = 74.5°, 164.5°$	M1 DM1 A1,A1	**In each case the last A mark is for a second correct solution and no extra solutions within the range** **M1** for use of tan **DM1** for dealing with $2x$ correctly **A1** for each
	Alternatives: $\sin(2x + 31°) = 0$ or $\cos(2x - 59°) = 0$	M1	**M1** for either, then mark as above
(b)	$2\cot^2 y + 3\text{cosec}\, y = 0$ $2(\text{cosec}^2 y - 1) + 3\text{cosec}\, y = 0$ $2\text{cosec}^2 y + 3\text{cosec}\, y - 2 = 0$	M1	**M1** for use of correct identity
	$(2\text{cosec}\, y - 1)(\text{cosec}\, y + 2) = 0$	M1	**M1** for attempt to factorise a 3 term quadratic equation
	One valid solution $\text{cosec}\, y = -2,\ \sin y = -\frac{1}{2}$ $y = 210°, 330°$	A1,A1	**A1** for each
	Alternative: $2\frac{\cos^2 y}{\sin^2 y} + \frac{3}{\sin y} = 0$	M1	**M1** for use of $\cot y = \frac{\cos y}{\sin y}$ and $\text{cosec}\, y = \frac{1}{\sin y}$
	leads to $2\sin^2 y - 3\sin y - 2 = 0$ and $\sin y = -\frac{1}{2}$ only	M1	**M1** for attempt to factorise a 3 term quadratic equation
	$y = 210°, 330°$	A1A1	
(c)	$3\cos(z + 1.2) = 2$ $\cos(z + 1.2) = \frac{2}{3}$		
	$(z + 1.2) = 0.8411, 5.442, 7.124$	M1	**M1** for correct order of operations to end up with 0.8411 radians or better
	$z = 4.24, 5.92$	A1 A1A1	**A1** for one of 5.441 or 7.124 (or better) **A1** for each valid solution