

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the May/June 2014 series

4037 ADDITIONAL MATHEMATICS

4037/21

Paper 2, maximum raw mark 80

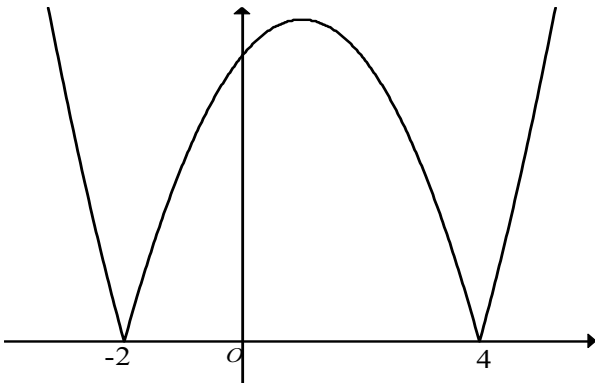
This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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1	$x^2 + x \geq 0$ critical values 0 and -1 soi $-1 < x < 0$	M1 A1 A1	expands and rearranges condone space, comma, “and” but not “or” Mark final answer.
2	$\frac{6}{(1+\sqrt{3})^2}$ or $6 = (a+b\sqrt{3})(1+\sqrt{3})^2$ $\frac{6}{4+2\sqrt{3}}$ or $6 = (a+b\sqrt{3})(4+2\sqrt{3})$ $\frac{6}{4+2\sqrt{3}} \times \frac{4-2\sqrt{3}}{4-2\sqrt{3}}$ AND attempting to multiply out $6 - 3\sqrt{3}$ isw	M1 M1 M1 A1	for dealing with the negative index (condone treating 6 as have negative index at this stage) for squaring for rationalising or for obtaining a pair of simultaneous equations $4a + 6b = 6$ and $2a + 4b = 0$
3 (i)		B1 B1	correct shape x intercepts marked or implied by tick marks, for example or seen nearby; condone y intercept omitted
(ii)	$x = 1$ (only) soi $y = \pm 9$ (only) $0 < k < 9$	B1 B1 B1	can be implied by second B1 or $k = \pm 9, +9$ or -9 or both; must be strict inequality in k ; condone space, comma, “and”, “or”
4	Attempt to find $f(4)$ or $f(1)$ or division to a remainder $128 + 16a + 4b + 12 = 0$ or better $(16a + 4b = -140)$ $2 + a + b + 12 = -12$ or better ($a + b = -26$) Solves linear equations in a and b $a = -3, b = -23$	M1 A1 A1 M1 A1	condone one error both

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5	(i)	$2\left(x - \frac{1}{4}\right)^2 + \frac{47}{8}(5.875)$ isw	B3,2,1,0	one mark for each of p, q, r correct; allow correct equivalent values. If B0 , then SC2 for $2\left(x - \frac{1}{4}\right) + \frac{47}{8}$, or SC1 for correct values but incorrect format
	(ii)	$\frac{47}{8}$ is min value when $x = \frac{1}{4}$	B1ft + B1ft	strict ft <i>their</i> $\frac{47}{8}$ and <i>their</i> $\frac{1}{4}$; each value must be correctly attributed; condone $y = \frac{47}{8}$ for B1 , or $\left(\frac{1}{4}, \frac{47}{8}\right)$ for B1B1
6	(a)	${}^8C_3 \times 3^3 \times (\pm 2)^5$ or $3^8 \left[{}^8C_3 \left(\pm \frac{2}{3} \right)^5 \right]$ –48384	M1 A1	condone ${}^8C_5, -2x^5$ can be in expansion
	(b) (i)	$1 + 12x + 60x^2$	B2,1,0	ignore additional terms. If B0 , allow M1 for 3 correct unsimplified terms
	(ii)	Coefficient of x correct or correct ft $(12+a)$ soi Coefficient of x^2 correct or correct ft $(60+12a)$ soi $1.5 \times \text{their}(12 + a) = \text{their}(60 + 12a)$ – 4	B1ft B1ft M1 A1	ft <i>their</i> $1 + 12x + 60x^2$ ft <i>their</i> $1 + 12x + 60x^2$ no x or x^2
7	(i)	$-\frac{1}{x^2} + \frac{1}{x^{1/2}}$	B1 + B1	or equivalent with negative indices
	(ii)	$\frac{2}{x^3} - \frac{1}{2x^{3/2}}$	B1ft + B1ft	or equivalent with negative indices. Strict ft
	(iii)	Attempting to solve <i>their</i> $\frac{dy}{dx} = 0$ $x = 1 \quad y = 3$ Substitute <i>their</i> $x = 1$ into <i>their</i> $\frac{d^2y}{dx^2}$; or examines $\frac{dy}{dx}$ or y on both sides of <i>their</i> $x = 1$ Complete and correct determination of nature. If correct, minimum.	M1 A1 M1 A1	must achieve $x = \dots$ (allow slips) SC2 for $(1, 3)$ stated, nfw for using <i>their</i> value from $\frac{dy}{dx} = 0$ must be from correct work

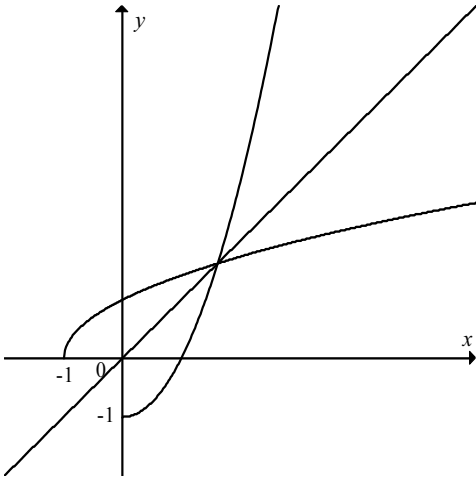
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8	(i)	$2r + r\theta = 30$ giving $\theta = \frac{30 - 2r}{r}$ Substitute <i>their</i> expression for θ into $A = \frac{1}{2}r^2\theta$ Correct simplification to $A = 15r - r^2$ AG	M1 M1 A1	correct arc formula + (2)r rearranged
	(ii)	$15 - 2r = 0$ $r = 7.5$ 56.25	M1 A1 A1	<i>their</i> $\frac{dA}{dr} = 0$ 56.3 is A0 unless 56.25 seen; if M0 , then SC2 for $A = 56.25$ with no working; or SC1 for $r = 7.5$ with no working
9	(i)	(3, 5)	B1B1	column vector B0B1
	(ii)	$m_{BD} \left(= \frac{6-4}{1-5} \right) = -\frac{1}{2}$ $m_{AC} \left(= -1 \div -\frac{1}{2} \right)$ seen or used $y - 5 = 2(x - 3)$ or $y = 2x + c$, $c = -1$ or better	M1 M1 A1	can be implied by second M1
	(iii)	$p = 1$ $q = 7$ [$A(1, 1)$ $C(4, 7)$] Method for finding area numerically 15	M1 M1 A1	could be in (ii) e.g. $24 - \left(\frac{1}{2} \times 1 \times 3 + \frac{1}{2} \times 1 \times 3 + \frac{1}{2} \times 4 \right)$ or shoelace method SC2 for 15 with no working
10	(i)	$-2 \sin 2x$ and $\frac{1}{3} \cos \left(\frac{x}{3} \right)$ Attempt at product rule $\frac{1}{3} \cos 2x \cos \left(\frac{x}{3} \right) - 2 \sin 2x \sin \left(\frac{x}{3} \right)$ isw	B1+B1 M1 A1ft	each trig function correctly differentiated ft $k_1 \sin 2x$ and $k_2 \cos \left(\frac{x}{3} \right)$ provided k_1 , k_2 are non-zero
	(ii)	$\sec^2 x$ and $\frac{1}{x}$ Attempt at quotient rule (with given quotient) $\frac{(\sec^2 x)(1 + \ln x) - \frac{1}{x}(\tan x)}{(1 + \ln x)^2}$ isw	B1 + B1 M1 A1	or rearrangement to correct product and attempt at product rule penalise poor bracketing if not recovered

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11	(a)	$2^{x^2-5x} = 2^{-6}$ $x^2 - 5x + 6 = 0$ Correct method of solution of their 3 term quadratic $x = 2$ or $x = 3$	M1 M1 M1 A1	Or $(x^2 - 5x)\ln 2 = \ln\left(\frac{1}{64}\right) = -6\ln 2$ their “6”
	(b)	Correct change of base to $\frac{\log_a 4}{\log_a 2a}$ $\frac{\log_a 4}{\log_a 2 + \log_a a}$ $\log_a a = 1$ used soi simplification to $\log_a 4$	B1 M1 M1 A1	base a only at this stage but can recover at end for $\log 2a = \log 2 + \log a$

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12	(i)	$f(3)$ $\frac{6}{4}$ oe	M1 A1	$\text{or } fg(x) = \frac{2\sqrt{(x+1)}}{\sqrt{(x+1)}+1}$
	(ii)	$\frac{2\left(\frac{2x}{x+1}\right)}{\frac{2x}{x+1}+1}$ A correct and valid step in simplification	M1 dM1	allow omission of $2(\dots)$ in numerator or $(\dots)+1$ in denominator, but not both. e.g. multiplying numerator and denominator by $x+1$, or simplifying $\frac{2x}{x+1}+1$ to $\frac{2x+x+1}{x+1}$
	(iii)	Putting $y = g(x)$, changing subject to x and swapping x and y or vice versa $g^{-1}(x) = x^2 - 1$ (Domain) $x > 0$ (Range) $g^{-1}(x) > -1$	M1 A1 B1 B1	condone $x = y^2 - 1$; reasonable attempt at correct method condone $y = \dots$, $f^{-1} = \dots$ condone $y > -1$ $f^{-1} > -1$
	(iv)		B1 + B1 B1	correct graphs; -1 need not be labelled but could be implied by 'one square' idea of reflection or symmetry in line $y = x$ must be stated.