CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the May/June 2014 series

4037 ADDITIONAL MATHEMATICS

4037/22 Paper 2, maximum raw mark 80

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CAMBRIDGE International Examinations

1	rationalise the denominator to get $\dfrac{(2+\sqrt{5})^2(\sqrt{5}+1)}{5-1}$ or better	**M1**	or squaring to get $\dfrac{(4+4\sqrt{5}+5)}{\sqrt{5}-1}$ or better
	squaring to get $\dfrac{(4+4\sqrt{5}+5)(\sqrt{5}+1)}{their 4}$ or better	**M1**	or rationalising the denominator to get $\dfrac{their(9+4\sqrt{5})(\sqrt{5}+1)}{5-1}$ or better
	$\dfrac{29}{4}+\dfrac{13}{4}\sqrt{5}$ oe isw	**A1 + A1**	correct simplification Allow $\dfrac{29+13\sqrt{5}}{4}$ etc.
2	Correctly eliminate y	**M1**	$-kx+2=2x^2-9x+4$ oe
	$2x^2+(k-9)x+2[=0]$ oe	**A1**	allow even if x terms not collected; condone … $=y$ provided later work implies it should be 0
	Use b^2-4ac oe	**M1**	must be applied to a 3 term quadratic expression containing k as a coefficient; condone <0 etc.
	Reach $their(k-9=\pm 4)$ or solves $their(k^2-18k+65)=0$	**M1**	condone $9-k=\pm 4$; condone an inequality at this stage
	$k=5$ and 13 cao	**A1**	mark final answer, do not isw; **A0** if inequalities for final answers

3 **(i)**	$3(-1)^3-14(-1)^2-7(-1)+d=0$ with completion to $d=10$	**B1**	at least $-3-14+7+d=0$, $d=10$; N.B. $=0$ must be seen or implied by $\ldots=d$ or $\ldots=-d$, may be seen in following step. or convincingly showing $3(-1)^3-14(-1)^2-7(-1)+10=0$; at least $-3-14+7+10=0$ or correct synthetic division at least as far as $\begin{array}{r\|rrrr} -1 & 3 & -14 & -7 & 10 \\ & & -3 & 17 & -10 \\ \hline & 3 & -17 & 10 & \end{array}$
(ii)	$3x^2-17x+10$ isw or $a=3$, $b=-17$, $c=10$ isw	**B2, 1, 0**	-1 each error; must be seen or referenced in **(ii)** even if found in **(i)** or **(iii)**
(iii)	$(x+1)(x-5)(3x-2)$	**M1**	for factorising quadratic **ft** correct; condone omission of $(x+1)$ or for **ft** correct use of formula or **ft** correct completing the square
	$-1,\ 5,\ \frac{2}{3}$	**A1**	If **M0** then **SC1** for all three roots stated without working or verified/found by trials

4	**(i)**	$12\left(x-\frac{1}{4}\right)^2+\frac{17}{4}$ isw	**B3, 2, 1,0**	one mark for each of p, q, r correct in a correctly formatted expression; allow correct equivalent values; If **B0** then **SC2** for $12\left(x-\frac{1}{4}\right)+\frac{17}{4}$ or **SC1** for correct 3 values seen in incorrect format e.g. $12\left(x-\frac{1}{4}x\right)+\frac{17}{4}$ or $12\left(x^2-\frac{1}{4}\right)+\frac{17}{4}$ or for a correct completed square form of the original expression in a different but correct format. e.g. $3\left(2x-\frac{1}{2}\right)^2+\frac{17}{4}$
	(ii)	*their* $\frac{4}{17}$ or *their* 0.235	**B1ft**	strict **ft** ; *their* $\frac{4}{17}$ must be a proper fraction or decimal rounded to 3sig figs or more or truncated to 4 figs or more
		their $x=\frac{1}{4}$ oe	**B1ft**	strict **ft** ; x must be correctly attributed
5	**(i)**	$1-20x+160x^2$	**B2, 1, 0**	-1 each error if **B0** then **M1** for 3 correct terms seen; may be unsimplified e.g. $1,\ 5(-4x),\ \frac{5\times 4}{2}(-4x)^2$
	(ii)	$a+(their\ -20)=-23$ soi	**M1**	condone sign errors only; must be *their* -20 from **(i)**
		$a=-3$	**A1**	validly obtained
		$b+(their\ -20)a+(their160)=222$ soi	**M1**	condone sign errors only ; must be *their* -20 and *their*160 from **(i)** and *their* a if used
		$b=2$	**A1**	validly obtained

6 (a) (i)	1	**B1**	
(ii)	$x = -1$ or -2	**B1 + B1**	as final answers
(b)	$\dfrac{\log_3 5}{\log_3 a}$ seen or implied	**B1***	may be implied by $2\log_3 15 - \log_3 5$
	$2\log_3 15 = \log_3 15^2$ seen or implied	**B1**	
	$\log_3 15^2 - \log_3 5 = \log_3\left(\dfrac{15^2}{5}\right)$	**B1dep***	not from wrong working
	$\log_3 45$ cao	**B1**	must be 45 not e.g. $\dfrac{225}{5}$; with no wrong working seen
7 (i)	$x^4\left(3e^{3x}\right) + 4x^3e^{3x}$ isw	**B1 + B1**	each term of the **sum** correct; must be a sum of two terms
(ii)	$\dfrac{1}{2+\cos x} \times (-\sin x)$ isw	**B2**	or **B1** for $\dfrac{1}{2+\cos x} \times (k \pm \sin x)$ and k a constant
(iii)	$\dfrac{d}{dx}(\sin x) = \cos x$ soi	**B1**	
	$\dfrac{d}{dx}\left(1+\sqrt{x}\right) = \dfrac{1}{2}x^{-\frac{1}{2}}$ soi	**B1**	
	$\dfrac{\left(1+\sqrt{x}\right)\mathit{their}\cos x - \left(\mathit{their}\,\dfrac{1}{2}x^{-\frac{1}{2}}\right)\sin x}{\left(1+\sqrt{x}\right)^2}$ isw	**B1ft**	for correct form of quotient rule **ft** their $\cos x$ and their $\dfrac{1}{2}x^{-\frac{1}{2}}$; allow correct use of product and chain rules to obtain $\sin x\left(-\left(1+\sqrt{x}\right)^{-2} \times \dfrac{1}{2}x^{\frac{1}{2}}\right) +$ $\cos x\left(1+\sqrt{x}\right)^{-1}$ oe

8	Substitution of either $x-5$ or $y+5$ into equation of curve and brackets expanded	**M1**	condone one sign error in either equation of curve or expansion of brackets; condone omission of = 0, BUT $x-5$ or $y+5$ must be correct
	$2x^2-8x-10[=0]$ or $2y^2+12y[=0]$ obtained	**A1**	
	Solving their quadratic	**M1**	dep on a valid substitution attempt
	$(-1, -6)$ oe and $(5, 0)$ oe isw	**A1*+A1***	or **A1** for correct pair of x coordinates or correct pair of y coordinates
	$\sqrt{72}$ or $6\sqrt{2}$ cao isw	**B1dep***	
9 (i)	$[y=]\dfrac{(2x+1)^{\frac{3}{2}}}{2\times\frac{3}{2}}(+c)$ oe	**B2**	or **B1** for $(2x+1)^{\frac{1}{2}+1}$
	$10=\dfrac{2}{6}(2(4)+1)^{\frac{3}{2}}+c$ oe	**M1**	for valid attempt to find c; condone slips e.g. omission of power or sign error
	$y=\dfrac{(2x+1)^{\frac{3}{2}}}{2\times\frac{3}{2}}+c$ seen and $c=1$ or $y=\dfrac{(2x+1)^{\frac{3}{2}}}{2\times\frac{3}{2}}+1$ isw	**A1**	must have $y = \ldots$; condone $f(x) = \ldots$
(ii)	$\int\left(\frac{1}{3}(2x+1)^{\frac{3}{2}}+1\right)dx=\frac{1}{15}(2x+1)^{\frac{5}{2}}+x(+const)$	**B1 + B1**	**B1** for $(2x+1)^{\frac{3}{2}+1}$, **B1** for $\frac{1}{15}(2x+1)^{\frac{5}{2}}$
		B1ft	**B1 ft** *their* c from **(i)** provided $c \neq 0$
	$\left[\frac{1}{15}(2x+1)^{\frac{5}{2}}+x\right]_0^{1.5}=$ $\left[\frac{1}{15}(2(1.5)+1)^{\frac{5}{2}}+(1.5)\right]-\left[\frac{1}{15}(2(0)+1)^{\frac{5}{2}}+0\right]$	**M1**	for a genuine attempt to find F(1.5) – F(0) in an attempt to integrate *their* y; if their F(0) is 0 must see at least their F(1.5)– 0; condone + c as long as their c is **not** numerical.
	$\dfrac{107}{30}$ oe isw	**A1**	if decimal 3.57 or more accurate e.g. 3.566

10 (i)	Taking logs of both sides	**M1**	any base; must be an explicitly correct statement
	$\log y = \log A + x \log b$	**A1**	correct form; any base; no recovery from incorrect method steps
(ii)	b: awrt 3 to one sf isw or awrt 4 to one sf isw	**B2**	or **M1** for $b = e^{\textit{their}\text{ gradient}}$ soi; their gradient must be correctly evaluated as rise/run
	A: awrt 0.5 to one sf	**B2**	or **B1** for $A = e^{-0.6}$ or **SC1** for $A = e^{-0.3} = 0.7$ (giving an awrt 0.7)
(iii)	Evidence of graph used at $\ln y = 5.4$ soi	**M1**	or $\dfrac{220}{\textit{their}0.5} = (\textit{their}4)^x$ or 5.39…= *their*(1.4)x + *their* −0.6 or $\ln(220) = x\ln(\textit{their}4) + \ln(\textit{their}0.5)$
	awrt 4.4 to two sf	**A1**	

11 (i)	$f(x) > 3$ or $[f(x) \in](3, \infty)$	**B1**	condone $y > 3$
(ii)	$x+1=2^y$ $f^{-1}(x)=\log_2(x+1)$	**M1** **A1**	or $y+1=2^x$ mark final answer or $\log_2(y+1)=x$ and $f^{-1}(x)=\log_2(x+1)$ or for $f^{-1}(x)=\dfrac{\log(x+1)}{\log 2}$ (any base for this form)
	Domain $x > 3$	**B1ft**	**ft** their **range** of f provided mathematically valid inequality or interval
	Range $f^{-1}(x) > 2$	**B1**	condone $f(x) > 2$ or $y > 2$
(iii)	$2^x(2^x-1)$ oe isw	**B1**	e.g. $(2^x-1)^2+(2x-1)$ or $2^{2x}-2\times2^x+1+2^x-1$
	$2^x(2^x-1)=0$ **leading to** $2^x=0$, impossible oe	**B1**	or $2^x=0$ which is outside domain of gf
	$2^x=1 \Rightarrow x=0$	**M1**	or $2^x(2^x-1)=2^{2x}-2^x=0$ $[2^{2x}=2^x] \Rightarrow x=0$
	0 is not in the domain (and so $gf(x)=0$ has no solutions)	**A1**	

12 **(i)**	$\frac{dy}{dx} = 3x^2 - 18x + 24$ Solving their $3x^2 - 18x + 24 \geqslant 0$ by factorising or quadratic formula or completing the square	**B1** **M1**	attempt at differentiation resulting in quadratic expression with two terms correct; allow = or $\leqslant$ or < or > or $\geqslant$ 0 omitted here.
	Critical values 2 and 4 $x \leqslant 2, x \geqslant 4$	**A1** **A1**	**A0** if spurious attempt to combine; mark final answer
(ii)	Evaluating their $\frac{dy}{dx}$ at $x = 3$	**M1**	
	Use of $m_1 m_2 = -1$ to get $m_{normal} = -\frac{1}{their(-3)}$	**M1**	must be explicit statement of gradient of normal ; may be seen in equation
	$y = 18$ soi	**B1**	
	$y - their18 = \left(their\frac{1}{3}\right)(x-3)$ or $y = their\frac{1}{3}x + c$ and $c = their17$ isw	**A1ft**	**ft** *their m* provided a genuine attempt at m_{normal} ; no **ft** if $m = their\ m_{\text{tangent}}$
	$P(0, 17)$ cao	**B1**	