

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the October/November 2014 series

4037 ADDITIONAL MATHEMATICS

4037/12

Paper 1, maximum raw mark 80

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1	$\frac{dy}{dx} = 2x - \frac{16}{x^2}$ When $\frac{dy}{dx} = 0$, $x = 2, y = 12$	M1 A1 DM1 A1	for attempt to differentiate all correct for equating $\frac{dy}{dx}$ to zero and an attempt to solve for x . A1 for both, but no extra solutions
2 (a)		B1 B1 B1 B1 B1	for correct shape for max value of 2, starting at (0, 2) and finishing at (180°, 2) for min value of -4 must be positive
(b) (i)	4	B1	must be positive
(ii)	60° or $\frac{\pi}{3}$ or 1.05 rad	B1	
3 (i)	$y = 4(x+3)^{\frac{1}{2}} + c$ $10 = 4\left(9^{\frac{1}{2}}\right) + c$ $c = -2$ $y = 4(x+3)^{\frac{1}{2}} - 2$	M1, A1 M1 A1	M1 for $(x+3)^{\frac{1}{2}}$, A1 for $4(x+3)^{\frac{1}{2}}$ for a correct attempt to find c , but must be from an attempt to integrate Allow A1 for $c = -2$
(ii)	$6 = 4(x+3)^{\frac{1}{2}} - 2$ $x = 1$	A1 ft	ft for substitution into <i>their</i> equation to obtain x ; must have the first M1

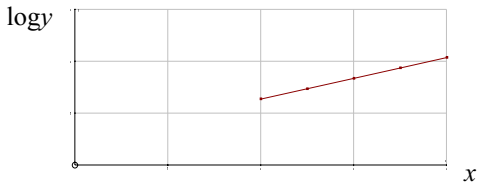
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4	(i)	$5y^2 - 7y + 2 = 0$	B1, B1	B1 for 5, B1 for -7
	(ii)	$(5y - 2)(y - 1) = 0$ $y = \frac{2}{5}, x = \frac{\ln 0.4}{\ln 5}$ $x = -0.569$ $y = 1, x = 0$	M1 M1 A1 B1	for solution of quadratic equation from (i) for use of logarithms to solve equation of the type $5^x = k$ must be evaluated to 3sf or better
5	(i)	$\frac{dy}{dx} = 3x^2 - \frac{1}{x}$ When $x = 1, y = 1$ and $\frac{dy}{dx} = 2$ Tangent: $y - 1 = 2(x - 1)$ $(y = 2x - 1)$	M1 B1 DM1 A1	for attempt to differentiate for $y = 1$ for attempt to find equation of tangent allow equation unsimplified
	(ii)	Mid-point (5, 9) $9 = 2(5) - 1$ Alternative Method: Tangent equation $y = 2x - 1$ Equation of line joining (-2, 16) and (12, 2) $y = -x + 14$ Solve simultaneously $x = 5, y = 9$ Mid-point (5, 9)	B1 B1 B1 B1	for midpoint from given coordinates for checking the mid-point lies on tangent for a complete method to find the coordinates of the point of intersection for midpoint from given coordinates
6	(i)	$(2 + px)^6 = 64 + 192px + 240p^2x^2 \dots$ $240p^2 = 60$ $p = \frac{1}{2}$	B1 M1 A1	for $240p^2$ or $240p^2x^2$ or ${}^6C_2 \times 2^4 \times (px)^2$ or ${}^6C_2 \times 2^4 \times p^2$ or ${}^6C_2 \times 2^4 \times p^2x^2$ for equating <i>their</i> term in x^2 to 60 and attempt to solve
	(ii)	$(3 - x)(64 + 192px + 240p^2x^2 \dots)$ Coefficient of x^2 is $180 - 192p$ $= 84$	B1 ft M1 A1	ft for $192p, 96$ or $192 \times \text{their } p$ for $180 - 192p$

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7	(i)	$\mathbf{A}^{-1} = \frac{1}{5ab} \begin{pmatrix} b & -2b \\ a & 3a \end{pmatrix}$	B1, B1	B1 for $\frac{1}{5ab}$, B1 for $\begin{pmatrix} b & -2b \\ a & 3a \end{pmatrix}$
	(ii)	$\mathbf{X} = \mathbf{BA}^{-1}$ $= \begin{pmatrix} -a & b \\ 2a & 2b \end{pmatrix} \begin{pmatrix} \frac{1}{5a} & -\frac{2}{5a} \\ \frac{1}{5b} & \frac{3}{5b} \end{pmatrix}$ $= \begin{pmatrix} 0 & 1 \\ \frac{4}{5} & \frac{2}{5} \end{pmatrix}$	M1 DM1 A1 A1	for post-multiplication by inverse matrix for correct attempt at matrix multiplication, needs at least one term correct for their $\mathbf{BA}^{-1}$ (allow unsimplified) for each correct pair of elements, must be simplified
8	(i)	$\overline{AB} = \begin{pmatrix} 12 \\ 16 \end{pmatrix}, \text{ at } P, x = -2 + \frac{1}{4}(12)$ so at $P, x = 1$ $y = 3 + \frac{1}{4}(16), y = 7$	B1 B1	for convincing argument for $x = 1$ for $y = 7$
	(ii)	Gradient of $AB = \frac{16}{12}$, so perp gradient = $-\frac{3}{4}$ Perp line: $y - 7 = -\frac{3}{4}(x - 1)$ $(3x + 4y = 31)$	M1 M1 A1	for finding gradient of perpendicular for equation of perpendicular through their P Allow unsimplified
	(iii)	$Q\left(0, \frac{31}{4}\right)$ Area $AQB = 12.5$	B1 ft M1 A1	ft on their perpendicular line, may be implied for any valid method of finding the area of the correct triangle, allow use of <i>their</i> Q ; must be in the form $(0, q)$.

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9	(i)	$\log y = \log a + x \log b$ <table><tr><td>x</td><td>2</td><td>2.5</td><td>3</td><td>3.5</td><td>4</td></tr><tr><td>$\lg y$</td><td>1.27</td><td>1.47</td><td>1.67</td><td>1.87</td><td>2.07</td></tr></table> <table><tr><td></td><td>2</td><td>2.5</td><td>3</td><td>3.5</td><td>4</td></tr><tr><td>$\ln y$</td><td>2.93</td><td>3.39</td><td>3.84</td><td>4.31</td><td>4.76</td></tr></table> 	x	2	2.5	3	3.5	4	$\lg y$	1.27	1.47	1.67	1.87	2.07		2	2.5	3	3.5	4	$\ln y$	2.93	3.39	3.84	4.31	4.76	B1	for the statement, may be seen or implied in later work,
	x	2	2.5	3	3.5	4																						
$\lg y$	1.27	1.47	1.67	1.87	2.07																							
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$\ln y$	2.93	3.39	3.84	4.31	4.76																							
(ii)	Gradient = $\log b$ $\lg b = 0.4$ or $\ln b = 0.92$ $b = 2.5$ (allow 2.4 to 2.6) Intercept = $\log a$ $\lg a = 0.47$ or $\ln a = 1.10$ $a = 3$ (allow 2.8 to 3.2) Alternative method: Simultaneous equations may be used provided points that are on the plotted straight line are used. $a = 3$ (allow 2.8 to 3.2) $b = 2.5$ (allow 2.4 to 2.6)	M1 A2,1,0 DM1 A1 DM1 A1 DM1 DM1 A1 A1	for attempt to draw graph of x against $\log y$ –1 each error in points plotted for attempt to find gradient and equate it to $\log b$, dependent on M1 in (i) for attempt to equate y-intercept to $\log a$ or use <i>their</i> equation with <i>their</i> gradient and a point on the line, dependent on M1 in (i) for a pair of equations using points on the line, dependent on M1 in (i) for solution of these equations, dependent on M1 in (i) A1 for each																									

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10	(a) (i)	360	B1	
	(ii)	60	B1	
	(iii)	36	B1	
	(b) (i)	${}^8C_5 \times {}^{12}C_5$ $56 \times 792 = 44352$	B1, B1 B1	B1 for each, allow unevaluated with no extra terms Final answer must be evaluated and from multiplication
	(ii)	4 places are accounted for Gender no longer 'important' Need ${}^{16}C_6 = 8008$ Alternative Method $({}^6C_6 \times {}^{10}C_0) + ({}^6C_5 \times {}^{10}C_1) \dots ({}^6C_0 \times {}^{10}C_6)$ $1 + 60 + 675 + 2400 + 3150 + 1512 + 210 = 8008$	M1 A1 M1 A1	for realising that 4 places are accounted or that gender is no longer important for 8008 for at least 5 of the 7 cases, allow unsimplified
11	(a)	$2 \cos 3x - \frac{\cos 3x}{\sin 3x} = 0$ $\cos 3x \left(2 - \frac{1}{\sin 3x} \right) = 0$ Leading to $\cos 3x = 0$, $3x = 90^\circ, 270^\circ$ $x = 30^\circ, 90^\circ$	M1	for use of $\cot 3x = \frac{\cos 3x}{\sin 3x}$, may be implied
		and $\sin 3x = \frac{1}{2}$, $3x = 30^\circ, 150^\circ$ $x = 10^\circ, 50^\circ$	DM1 A1	for attempt to solve $\cos 3x = 0$ correctly from correct factorisation to obtain x A1 for both, no excess solutions in the range
			DM1 A1	for attempt to solve $\sin 3x = \frac{1}{2}$ correctly to obtain x A1 for both, condone excess solutions
			M1	for dealing with $\sec \left(y + \frac{\pi}{2} \right)$ correctly
	(b)	$\cos \left(y + \frac{\pi}{2} \right) = -\frac{1}{2}$ $y + \frac{\pi}{2} = \frac{2\pi}{3}, \frac{4\pi}{3}$ so $y = \frac{\pi}{6}, \frac{5\pi}{6}$ (0.524, 2.62)	DM1 A1, A1	for correct order of operations, must not mix degrees and radians

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12 (i)	$\overrightarrow{AQ} = \lambda \mathbf{b} - \mathbf{a}$	B1	
(ii)	$\overrightarrow{BP} = \mu \mathbf{a} - \mathbf{b}$	B1	
(iii)	$\overrightarrow{OR} = \mathbf{a} + \frac{1}{3}(\lambda \mathbf{b} - \mathbf{a})$ or $\lambda \mathbf{b} - \frac{2}{3}(\lambda \mathbf{b} - \mathbf{a})$ $= \frac{2}{3}\mathbf{a} + \frac{1}{3}\lambda \mathbf{b}$	M1 A1	for $\mathbf{a} + \frac{1}{3}$ their (i) Allow unsimplified
(iv)	$\overrightarrow{OR} = \mathbf{b} + \frac{7}{8}(\mu \mathbf{a} - \mathbf{b})$ or $\mu \mathbf{a} - \frac{1}{8}(\mu \mathbf{a} - \mathbf{b})$ $= \frac{1}{8}\mathbf{b} + \frac{7}{8}\mu \mathbf{a}$	M1 A1	for $\mathbf{b} + \frac{7}{8}$ their (ii) Allow unsimplified
(v)	$\frac{2}{3}\mathbf{a} + \frac{1}{3}\lambda \mathbf{b} = \frac{1}{8}\mathbf{b} + \frac{7}{8}\mu \mathbf{a}$ $\frac{2}{3} = \frac{7}{8}\mu, \mu = \frac{16}{21}$ Allow 0.762 $\frac{1}{3}\lambda = \frac{1}{8}, \lambda = \frac{3}{8}$ Allow 0.375	M1 A1 A1	for equating (iii) and (iv) and then equating like vectors