

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the May/June 2015 series

4037 ADDITIONAL MATHEMATICS

4037/22

Paper 2, maximum raw mark 80

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Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

1	(i)		B3,2,1,0	2 correctly placed in Venn diagram; 1, 3, 4, 6 correctly placed; 12, 8, 0, 7, 9, 10 correctly placed; 11, 5 correctly placed
	(ii)	3	B1ft	correct or correct ft <i>their</i> (i), provided non-zero
	(iii)	{4, 6}	B1ft	correct or correct ft <i>their</i> (i), provided not the empty set
2	(i)	$[P] = \begin{pmatrix} 60 & 70 & 58 \\ 50 & 52 & 34 \end{pmatrix}$ and $[Q] = (120 \quad 300)$	B2	or $[P] = \begin{pmatrix} 50 & 52 & 34 \\ 60 & 70 & 58 \end{pmatrix}$ and $[Q] = (300 \quad 120)$ or B1 if one error may be written as an unevaluated product; B0 if choice of P and Q offered
	(ii)	(22 200 24 000 17 160)	B2	must have brackets and must not have commas; must be a 1 by 3 matrix; must be from correct product; working may be seen in (i) or B1 for any two elements correct
	(iii)	The total (amount of revenue) from all (three) flights. oe	B1	do not accept, e.g. The total amount from each flight; must be a comment not just a figure; must not contain a contradiction

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3	(i)	$\frac{(36 + 15\sqrt{5})}{(6 + 3\sqrt{5})} \times \frac{(6 - 3\sqrt{5})}{(6 - 3\sqrt{5})} \text{ oe}$ $\frac{216 + 90\sqrt{5} - 108\sqrt{5} - 225}{-9}$ $1 + 2\sqrt{5} \text{ cao}$ Alternative method: $36 + 15\sqrt{5} = (6a + 15b) + (3a + 6b)\sqrt{5}$ $6a + 15b = 36$ $3a + 6b = 15$ $a = 1 \text{ and } b = 2$	M1 DM1 A1 M1 DM1 A1	$\text{or } \frac{(12 + 5\sqrt{5})}{(2 + \sqrt{5})} \times \frac{(2 - \sqrt{5})}{2 - \sqrt{5}} \text{ oe}$ $\text{or } \frac{24 + 10\sqrt{5} - 12\sqrt{5} - 25}{-1}$ $\text{or } -(24 + 10\sqrt{5}) - 12\sqrt{5} - 25$ allow $a = 1$ and $b = 2$
	(ii)	$\left[AC^2 = (6 + 3\sqrt{5})^2 + \text{their } (1 + 2\sqrt{5})^2 \right]$ $= 36 + 36\sqrt{5} + 45 + \text{their } (1 + 4\sqrt{5} + 20)$ $102 + 40\sqrt{5} \text{ cao}$	M1 A1	correct or correct ft expansions, using Pythagoras with $(6 + 3\sqrt{5})$ and <i>their</i> BC ignore attempts to square root after correct answer seen
4	(i)	$\cos(x) = \frac{2}{3} \text{ oe soi}$ $48.189...^\circ \text{ or } 131.810...^\circ \text{ or } 0.8410... \text{ rad or } 2.3(00...) \text{ rad oe isw}$ with reference axis indicated by comment, e.g. “to the bank” or “upstream”, etc. or clearly marked on a diagram	M1 A1	Alternatively $\sin(y) = \frac{2}{3} \text{ oe soi}$ $41.810...^\circ \text{ or } 0.7297... \text{ or } 0.73(0) \text{ rad oe isw}$ with reference axis indicated by comment, e.g. “to the perpendicular with the bank”, etc. or clearly marked on a diagram If M0 then SC1 for an unsupported answer of $138.189...^\circ$ or $2.4118... \text{ rad}$ or $318.189...^\circ$ or $5.5534... \text{ rad}$ with reference axis indicated by comment, e.g. “on a bearing of” or “from North” or clearly marked on a diagram

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(ii)	Speed = $\sqrt{9-4} (= \sqrt{5})$ or $3 \sin 48.2$ or $2 \tan 48.2$ or $3 \cos 41.8$ or $\frac{2}{\tan 41.8}$ or $\sqrt{2^2 + 3^2 - 2 \times 2 \times 3 \cos 48.2}$ oe or 2.236(0...) rot to 4 or more figs or 2.24 [m/s] soi time = $\frac{80}{\text{their } \sqrt{5}}$ oe 35.66 to 35.8 (seconds) oe	B1 M1 A1	Or Distance = $\frac{80}{\sin 48.2} = 107.(33\dots)$ oe soi time = $\frac{\text{their } 107.33\dots}{3}$ ignore subsequent rounding or attempted conversion to, e.g. minutes but A0 if answer spoiled by continuation of method if no working, so B0 M0, then allow B3 for an answer 35.66 to 35.8 oe
5	Substitution of either $4 - x$ or $4 - y$ into equation of curve and brackets expanded $12x^2 - 52x + 48 [= 0]$ or $12y^2 - 44y + 32 [= 0]$ oe Solve their 3-term quadratic $x = \frac{4}{3}$ and 3 isw $y = \frac{8}{3}$ and 1 isw	M1 A1 M1 A1 A1	condone one sign error or slip in either equation of curve or expansion of brackets; condone omission of $= 0$, BUT $4 - x$ or $4 - y$ must be correct dep on a valid substitution attempt or $x = \frac{4}{3}$ $y = \frac{8}{3}$ not from wrong working or $x = 3$ $y = 1$ not from wrong working if no working, allow full marks for fully correct answer only.
6 (a)	$(x-2) \log 6 = \log \left(\frac{1}{4} \right)$ oe or $\log_6 \left(\frac{1}{4} \right) = x-2$ oe 1.23 or 1.226(29...) rot to 4 or more figures isw	M1 A1	or $x \log 6 = \log \left(\frac{36}{4} \right)$ oe or $x \log 6 - \log 36 = \log 1 - \log 4$ oe correct answer or 1.22 implies M1

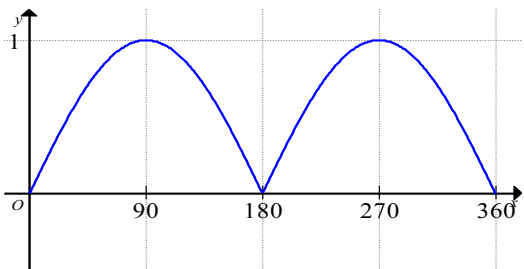
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(b)	Method 1 $\log\left(\frac{8 \times 2y^2 \times 16y}{64y}\right) = \log 4^2$ oe $y = 2$	B3 B1	or B2 if at most one error or omitted step or B1 if at most two errors or omitted steps not from wrong working
	Method 2 $\log 2 + 2 \log y + 3 \log 2 + 4 \log 2 + \log y - 6 \log 2 - \log y = 4 \log 2$ $y = 2$	B3,2,1,0 B1	<u>LHS terms</u> $\log 2y^2 = \log 2 + 2 \log y$; $\log 8 = 3 \log 2$; $\log 16y = 4 \log 2 + \log y$; $-\log 64y = -6 \log 2 - \log y$; <u>RHS term</u> $2 \log 4 = 4 \log 2$ not from wrong working
7	$\frac{n(n-1)(n-2)(n-3)(2^4)}{4 \times 3 \times 2 \times 1} = 10 \frac{n(n-1)(2^2)}{2 \times 1}$ or better $n^2 - 5n - 24 [= 0]$ oe $(n+3)(n-8) [= 0]$ $n = 8$ only	M3 A1 M1 A1	condone omitting the factor of n and/or $n-1$; must have dealt with factorials M2 if one slip/omission or M1 if two slips/omissions or B1 for $\frac{n(n-1)}{2}(2)^2[x^2]$ seen and B1 for $\frac{n(n-1)(n-2)(n-3)}{24}(2)^4[x^4]$ seen equivalent must be 3-terms, e.g. $n^2 - 5n = 24$ or any valid method of solution for their 3-term quadratic A0 if -3 also given as a final solution, i.e. not discarded If zero scored, allow SC1 for $n = 8$ unsupported or without correct method

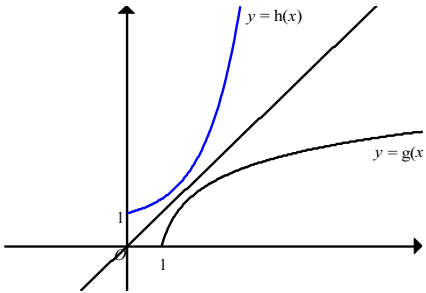
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8	Method 1 (Separate areas subtracted) $[x_B = x_C =] 7$ soi $\left[\int (x^2 - 6x + 10) dx = \right] \frac{x^3}{3} - \frac{6x^2}{2} + 10x$ Correct or correct ft substitution of limits 0 and <i>their</i> 7 into <i>their</i> $\left[\frac{x^3}{3} - \frac{6x^2}{2} + 10x \right]$ $\frac{1}{2}(10+17) \times 7$ oe or $\int_0^7 (x+10) dx = \left[\frac{x^2}{2} + 10x \right]_0^7 = \frac{(7)^2}{2} + 10(7)$ oe <i>their</i> $\left(\frac{189}{2} - \frac{112}{3} \right)$ $\frac{343}{6}$ or $57\frac{1}{6}$ or 57.2 to 3 sf or 57.16(6...) rot to 4 figs isw Method 2 (Subtracting and using integration once) $[x_B = x_C =] 7$ soi $\int (-x^2 + 7x) dx$ $\left[-\frac{x^3}{3} + \frac{7x^2}{2} \right]$ oe or $\left[\frac{x^3}{3} - \frac{7x^2}{2} \right]$ oe Correct or correct ft substitution of limits 0 and <i>their</i> 7 into <i>their</i> $\left[-\frac{x^3}{3} + \frac{7x^2}{2} \right]$ $\frac{343}{6}$ or $57\frac{1}{6}$ or 57.2 to 3 sf or 57.16(6...) rot to 4 figs isw	B1 M2 DM1 B2 M1 A1 B1 B1 M3 M2 A1	or M1 for at least one term correct dep on at least M1 being earned; evidence of substitution must be seen in <i>their</i> integral which must be at least two terms; condone omission of lower limit; or M1 for $\frac{1}{2}(\text{their } 10 + \text{their } 17) \times \text{their } 7$ oe or B1 for $\int (x+10) dx = \frac{x^2}{2} + 10x$ dep on a genuine attempt to integrate the equation of the curve; must be <i>their</i> area trapezium/under the line – <i>their</i> attempt at area under curve from full and correct working with no omitted steps condone omission of dx or M2 for $\int (px^2 + qx) dx = \frac{px^3}{3} + \frac{qx^2}{2}$ oe either with $p = \pm 1$ or $q = \pm 7$ or M1 for $\int (px^2 + qx) dx = \frac{px^3}{3} + \frac{qx^2}{2}$ with non-zero constants p and q, with $p \neq \pm 1$ and $q \neq \pm 7$ dep on a valid integration attempt; evidence of substitution must be seen; condone omission of lower limit; from full and correct working with no omitted steps
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9	(i)	$10 = 2m + 4$ soi $m = 3$	M1	or $[m =]\frac{10-4}{2-0}$ oe soi
	(ii)	1	A1	
	(iii)	$\frac{10 - y_R}{2 - -1} = 1$ oe soi $(-1, 7)$ or $x = -1$ and $y = 7$	B1	
			M1	or $y = x + 8$ oe
			A1	if $y = 7$ only stated, provided that $x = -1$ is soi in working allow both marks
				if M0 then B1 for $y = 7$ only with no working
	(iv)	Use of $m_1 m_2 = -1$ with <i>their</i> m from (i) $y - 10 = \left(\text{their} - \frac{1}{3} \right) (x - 2)$ $3y + x = 32$ isw	M1	may be implied by perpendicular gradient seen in equation
			A1	or $\left(\text{their} - \frac{1}{3} \right) x + c$ and
				$10 = \left(\text{their} - \frac{1}{3} \right) 2 + c$
			A1	allow for correct equation with integer coefficients in any simplified form
	(v)	$\left(\frac{1}{2}, \text{their} \frac{11}{2} \right)$ oe isw	B1, B1ft	ft <i>their</i> y_Q
				or M1 for $\left(\frac{2-1}{2}, \frac{10+1}{2} \right)$ seen
	(vi)	4.5 oe cao	B2	not from wrong working
				or M1 for any correct method with correct coordinates
10	(a)		B2, 1, 0	correct sinusoidal/reflected sinusoidal shape, all above x -axis with intent to have all maximum points of equal height; 2 maximum points of intended equal height only over 0 to 360; all max points clearly at $y = 1$; cusp at 180

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	(b)(i)	$[hg(x)] = \frac{e^{\ln(4x-3)} + 3}{4}$ fully correct and completion to $[hg(x)] = x$	M1	Alternative method $y = \ln(4x - 3)$ and change of subject to x oe,
	(ii)		A1	fully correct and comment that $h(x) = g^{-1}(x)$ oe
	(iii)	$x \geq 0$ or $[0, \infty)$	B1	correct shape; 1 marked on the y-axis or (0, 1) stated close by; curve with positive gradient in first quadrant only
	(iv)	$y \geq 1$ or $[1, \infty)$	B1	not domain ≥ 0 or $h(x) \geq 1$, $h \geq 1$ etc.
11	(i)	$\frac{8-h}{8}$ or $8 : 8 - h$ soi $\frac{8-h}{8} \times 4$ oe $h\left(\frac{8-h}{8} \times 4\right)^2$ oe expand and simplify to $\frac{h^3}{4} - 4h^2 + 16h$ AG	M1	or $\frac{8}{8-h}$ or $8 - h : 8$ soi
			A1	or $4 \div \frac{8}{8-h}$ oe
			M1	h must be in the numerator of the expression for this mark;
			A1	
	(ii)	$\frac{3}{4}h^2 - 8h + 16$ oe $their\left(\frac{3}{4}h^2 - 8h + 16\right) = 0$ and attempt to solve $\frac{8}{3}$ oe only	B1	
			M1	must be a 3-term quadratic; must be an attempt at a derivative
			A2	or A1 for $h = \frac{8}{3}$ and 8 allow 2.67 or 2.66(6...) rot to 4 or more figs for $\frac{8}{3}$

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12	(i)	$-120 + 104 + 22 - 6 = 0$ or correct unsimplified form, e.g. $15(-2)^3 + 26(-2)^2 - 11(-2) - 6 = 0$ or $15(-8) + 26(4) - 11(-2) - 6 = 0$	B1	or correct synthetic division $\begin{array}{r rrrr} -2 & 15 & 26 & -11 & -6 \\ & & -30 & 8 & 6 \\ \hline & 15 & -4 & -3 & 0 \end{array}$
	(ii)	Substituting $x = 3$ into $15x^3 + 26x^2 - 11x - 6$ 600	M1	or correct synthetic division $\begin{array}{r rrrr} 3 & 15 & 26 & -11 & -6 \\ & & 45 & 213 & 606 \\ \hline & 15 & 71 & 202 & 600 \end{array}$
	(iii)	$(x - 1)(15x^3 + 26x^2 - 11x - 6)$ soi	B1	by inspection or division; may be implied by e.g. $(ax + b)(15x^3 + 26x^2 - 11x - 6)$ and $a = 1, b = -1$ seen in later work comparing coefficients
		Multiply out $(x \pm 1)(15x^3 + 26x^2 - 11x - 6)$ and compare coefficients of x^3 or x to quartic $p = 11$ $q = 5$	M1 A1 A1	or multiply out, e.g. $(ax + b)(15x^3 + 26x^2 - 11x - 6)$ and compare coefficients of x^3 or x to quartic correct p or q implies M1; correct p and q www implies B1 M1