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Cambridge Ordinary Level

ADDITIONAL MATHEMATICS

4037/12

Paper 1

October/November 2016

MARK SCHEME

Maximum Mark: 80

Published

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Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

Question	Answer	Marks	Part Marks
1 (a) (i)	10	B1	
(ii)	22	B1	
(iii)	4	B1	
(b) (i)	$Q \subset R$	B1	
(ii)	$P \cap Q = \emptyset$, or $\{ \}$	B1	
2	$a = 1, b = -3, c = -1$	B3	B1 for each
3	$3y^2 + 5y - 2 = 0$ $y = \frac{1}{3}, y = -2$ $x = 3^{\frac{1}{3}}, x = 3^{-2}$ $x = 1.44, x = \frac{1}{9}$	B1, B1 M1 M1 A1, A1	B1 for $5y$ or $5\log_3 x$, B1 for -2 for correct attempt at the solution of <i>their</i> quadratic equation for dealing with one base 3 logarithm correctly A1 for each
4 (i)	$32x^{10} - \frac{80}{3}x^7 + \frac{80}{9}x^4$	B3	B1 for each term, powers of x must be simplified
(ii)	Coefficients needed: $\left(3 \times \text{their} - \frac{80}{3} \right) + (1 \times \text{their } 32)$ $= -48$	M1 A1	for dealing with 2 terms Allow A1 for $-48x^7$

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Question	Answer	Marks	Part Marks
5 (i)	$\frac{dy}{dx} = \frac{3}{2(3x+2)}$	B1	for correct derivative of log function
	When $x = -\frac{1}{3}$, $y = 0$, $\frac{dy}{dx} = \frac{3}{2}$	B1	for $y = 0$
	Equation of normal: $y = -\frac{2}{3}\left(x + \frac{1}{3}\right)$	M1 A1	M1 for attempt at a gradient of a perpendicular from differentiation and the equation of the normal
(ii)	$Q\left(0, -\frac{2}{9}\right)$ or $(0, 0.22)$ or better	B1 ft	Follow through on <i>their c</i> from part (i)
	$R\left(0, \frac{1}{2}\ln 2\right)$ or $(0, 0.35)$ or better	B1	
	Area of $PQR = \frac{1}{2}\left(\frac{1}{2}\ln 2 + \frac{2}{9}\right) \times \frac{1}{3}$		
	$= 0.0948$	B1	Allow 0.095
6 (a)	YX, XZ	B2	B2 for both with no extras B1 for 1 correct with or without extras B1 for both correct with extras B0 for anything else
(b) (i)	$\frac{1}{18}\begin{pmatrix} 7 & 1 \\ -4 & 2 \end{pmatrix}$	B1, B1	B1 for $\frac{1}{18}$, B1 for $\begin{pmatrix} 7 & 1 \\ -4 & 2 \end{pmatrix}$
(ii)	$\mathbf{C} = \mathbf{A}^{-1}\mathbf{B}$		
	$= \frac{1}{18}\begin{pmatrix} 7 & 1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} -4 & 2 \\ 10 & 4 \end{pmatrix}$	M1	for pre-multiplication
	$= \begin{pmatrix} -1 & 1 \\ 2 & 0 \end{pmatrix}$	A1, A1	A1 for any correct pair of elements, but must be from correct matrices

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Question	Answer	Marks	Part Marks
7 (i)	$(0, \sqrt{3})$ or $(0, 1.73)$ or better	B1	
(ii)	$\left(\frac{\pi}{6}, 2\right)$ or $(0.524, 2)$ or better	B1, B1	B1 for each
(iii)	$\cos\left(x - \frac{\pi}{6}\right) = 0$ $x = \frac{2\pi}{3}$ oe or 2.09 or better	M1 A1	for correct attempt to solve trigonometric equation
(iv)	$2\sin\left(x - \frac{\pi}{6}\right)$ (+c)	B1	
(v)	Area = $\left[2\sin\left(x - \frac{\pi}{6}\right)\right]_0^{\frac{2\pi}{3}}$ = 2 + 1 = 3	M1 A1	for correct use of their limits, in radians, into $k\sin\left(x - \frac{\pi}{6}\right)$.
8 (i)	$47 - 24 = 12\theta$ $\theta = \frac{23}{12}$, so $\theta = 1.917$ or better $\theta = 1.92$ to 2dp	M1 A1	for complete correct method to get $\theta =$ must have evidence of working to more than 2 dp, allow if 1.916 seen (truncated)
(ii)	$\sin\frac{\theta}{2} = \frac{CD/2}{12}$ $CD = \text{awrt } 19.6 \text{ or } 19.7$	M1 A1	for a complete method, may use cosine rule to get CD
(iii)	Area of sector = awrt 138 Area of triangle AOB = awrt 67 or 68 Area of segment = awrt 70 or 71 $AD \times AB + \text{segment area} = 425$ leading to $AD = \text{awrt } 18.1 \text{ or } 18.0$ Alternative method: Area of sector = awrt 138 Difference in length between BC (or AD) and OM where M is the midpoint of $CD = 6.88$, allow awrt 6.9 Remaining area consists of two trapezia each of width 9.85 and each of area 143.4 $\frac{1}{2}(2BC - 6.88) \times 9.85 = 143.4$ oe leading to $AD = \text{awrt } 18.1 \text{ or } 18.0$	B1 M1 M1 M1 A1 B1 M1 M1 M1 A1	for sector area, allow unsimplified for a correct attempt at area for segment area (<i>their</i> sector area – <i>their</i> triangle area) for complete method to find AD Allow A1 for 18 for sector area for attempt to find difference between parallel sides for area of one trapezium $\frac{1}{2}(2BC - \text{their } 6.88) \times \text{their } 9.85$ oe for attempt to find either BC or AD

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9 (i)	$p\left(\frac{3}{2}\right): \frac{27a}{8} - \left(4 \times \frac{9}{4}\right) + \frac{3b}{2} + 18 (=0)$	M1	for attempt at $p\left(\frac{3}{2}\right)$
	$p'\left(\frac{3}{2}\right) = \left(3a \times \frac{9}{4}\right) - \left(8 \times \frac{3}{2}\right) + b (=0)$	M1	for differentiation and attempt at $p'\left(\frac{3}{2}\right)$
	leading to $9a + 4b + 24 = 0$ oe	M1	for solution of simultaneous equations, to get either a or b
	and $27a + 4b - 48 = 0$ oe	A1	for both
	leading to $a = 4, b = -15$		
(ii)	$(x+2)(2x-3)^2$ oe	M1, A1	M1 for attempt at long division or factorisation
(iii)	$(x+2)(2x-3)^2 = x+2$		
	$x+2=0, x=-2$	B1	Must be using $(x+2)$ correctly using part (ii) to get $x=-2$
	$(2x-3)^2 = 1$	M1	for solution of the quadratic equation
	leading to $x=1, x=2$	A1	
10 (a) (i)	$20U + \frac{1}{2}\left(U + \frac{U}{2}\right)10 = 165$	M1	for realising that area under the graph is needed and attempt to find an area
	leading to $U = 6$	DM1	for equating their area to 165 and attempt to solve
		A1	
	(ii) Gradient of line: -0.3	M1, A1	M1 for use of the gradient, must be negative
	(b) (i) 27	B1	
	(ii) $t^2 = 8 \ln 4$	M1	for a correct attempt to solve $e^{\frac{t^2}{8}} = 4$
	$t = 3.33$ or better	A1	
	(iii) acceleration $= 3 \frac{2t}{8} e^{\frac{t^2}{8}} \left(e^{\frac{t^2}{8}} - 4 \right)^2$	M1, A1	M1 for a correct attempt to differentiate using the chain rule
	When $t = 1, a = 6.98$	M1, A1	M1 for use of $t = 1$ in their acceleration

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Question	Answer	Marks	Part Marks
11 (i)	$\ln y = \ln A + x \ln b$	B1	may be implied, if equation not seen specifically, by correct values for A and b for use of gradient to obtain $\ln b$
	Gradient: $\ln b = -\frac{0.12}{8}, = -0.015$	M1	
	$b = 0.985$	A1	Allow A1 for $e^{-0.015}$
	Intercept: $\ln A = 0.26$	DM1	for use of one of the given points correctly
	$A = 1.30$	A1	Allow A1 for $e^{0.26}$ or 1.3
	Alternative 1		
	$\ln y = \ln A + x \ln b$	B1	
	$0.2 = 4 \ln b + \ln A$	M1	for one correct equation
	$0.08 = 12 \ln b + \ln A$	DM1	for attempt to obtain either $\ln A$ or $\ln b$ from simultaneous equations
	$A = 1.30$ and $b = 0.985$	A1, A1	Allow A1 for $b = e^{-0.015}$ and $a = e^{0.26}$ or 1.3
(ii)	Alternative 2		
	$1.22 = Ab^4$	B1	
	$1.08 = Ab^{12}$	B1	
		M1	for correct attempt to obtain b or A , must already have B2
	$A = 1.30$ and $b = 0.985$	A1, A1	Allow A1 for $b = e^{-0.015}$ and $a = e^{0.26}$ or 1.3
	When $x = 6$, $\ln y = 0.17$	M1	for $\ln y = \text{their } \ln A + 6 \text{ their } \ln b$ or $y = \text{their } A \times (\text{their } b)^6$
	$y = 1.19$	A1	allow awrt 1.18 to 1.20
	When $y = 1.1$, $\ln y = 0.095$	M1	for $\ln 1.1 = \text{their } \ln A + x \text{ their } \ln b$ or $1.1 = \text{their } A \times (\text{their } b)^x$
	$x = 11$	A1	allow 10.5 to 11.5