



Cambridge International Examinations
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS**4037/11**

Paper 1

May/June 2017

MARK SCHEME

Maximum Mark: 80

Published

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This document consists of **8** printed pages.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

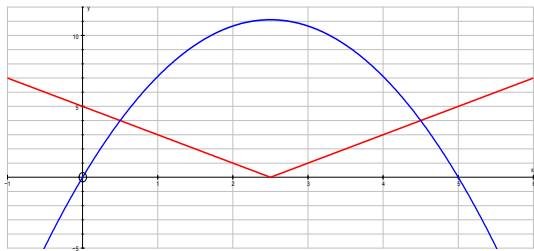
Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(i)	$kx - 5 = x^2 + 4x$ $x^2 + (4 - k)x + 5 = 0$	M1	equating line and curve equation and collecting terms to form an equation of the form $ax^2 + bx + c = 0$ x terms must be gathered together, maybe implied by later work
	For a tangent $(4 - k)^2 = 20$	DM1	correct use of discriminant
	$k = 4 + 2\sqrt{5}$	A1	Accept $k = 4 + \sqrt{20}$
	Alternative Gradient of line = k Gradient of curve = $\frac{dy}{dx} = 2x + 4$ Equating: $k = 2x + 4$	M1	
	substitution of $k = 2x + 4$ or $x = \frac{k - 4}{2}$ in $kx - 5 = x^2 + 4$ and simplify to a quadratic equation in k or x	DM1	
	$k = 4 + 2\sqrt{5}$	A1	Accept $k = 4 + \sqrt{20}$
1(ii)	Normal gradient = $-\frac{1}{4 + 2\sqrt{5}} \times \frac{4 - 2\sqrt{5}}{4 - 2\sqrt{5}}$	M1	use of negative reciprocal and attempt to rationalise using a form of $a - b\sqrt{5}$ or $a - \sqrt{20}$ or <i>their</i> equivalent from (i)
	$= -\frac{4 - 2\sqrt{5}}{-4}$ oe $= 1 - \frac{\sqrt{5}}{2}$	A1	$-\frac{4 - 2\sqrt{5}}{-4}$ oe leading to $1 - \frac{\sqrt{5}}{2}$
2	$p(3) = 27 + 9a + 3b - 48$	M1	attempt to find $p(3)$
	$3a + b = 9$ oe	A1	
	$p'(x) = 3x^2 + 2ax + b$ $p'(1) = 3 + 2a + b$	M1	attempt to differentiate and find $p'(1)$ must have 2 terms correct
	$2a + b = -3$ oe	A1	
	$a = 12, b = -27$	A1	for both
3(a)	$x^3 y^7$	B2	B1 for each term

Question	Answer	Marks	Guidance
3(b)(i)	for $(t-2)^{\frac{3}{2}} = (t-2)^{\frac{1}{2}}(t-2)$ soi	M1	
	$(t-2)^{\frac{1}{2}}(4+5(t-2))$	A1	
	$(t-2)^{\frac{1}{2}}(5t-6)$	A1	
3(b)(ii)	2 and $\frac{6}{5}$	B1	FT on <i>their</i> $(t-2)^{\frac{1}{2}}(5t-6)$, must have 2
4(a)(i)	$f > 5$, $f(x) > 5$	B1	
4(a)(ii)	$\frac{y-5}{3} = e^{-4x}$ or $\frac{x-5}{3} = e^{-4y}$	B1	
	$-4x = \ln\left(\frac{y-5}{3}\right)$ or $-4y = \ln\left(\frac{x-5}{3}\right)$	B1	
	leading to $f^{-1}(x) = -\frac{1}{4}\ln\left(\frac{x-5}{3}\right)$ or $f^{-1}(x) = \frac{1}{4}\ln\left(\frac{3}{x-5}\right)$ or $f^{-1}(x) = \frac{1}{4}(\ln 3 - \ln(x-5))$ or $f^{-1}(x) = -\frac{1}{4}(\ln(x-5) - \ln 3)$	B1	
	Domain $x > 5$	B1	
4(b)	$\ln(x^2 + 5) = 2$	B1	
	$x^2 + 5 = e^2$	B1	
	$x = 1.55$ or better or $\sqrt{e^2 - 5}$	B1	
5(a)(i)	$\overrightarrow{OM} = \overrightarrow{OC} + \frac{1}{2}(\overrightarrow{OA} - \overrightarrow{OC})$ oe	M1	may be implied by correct answer.
	$\frac{1}{2}(\mathbf{a} + \mathbf{c})$	A1	

Question	Answer	Marks	Guidance
5(a)(ii)	$\mathbf{b} = \frac{5}{2}\overline{OM}$ oe, $\frac{5}{2}$ (their (i)) or $\overline{OM} = \frac{2}{3}(\mathbf{b} - \overline{OM})$	M1	dealing with ratio correctly to relate $\mathbf{b}$ or $\overline{OB}$ to $\overline{OM}$
	$= \frac{5}{4}(\mathbf{a} + \mathbf{c})$	A1	
5(b)(i)	$ -10\mathbf{i} + 24\mathbf{j} = 26$ $\mathbf{p} = \frac{39}{26}(-10\mathbf{i} + 24\mathbf{j})$	M1	magnitude of $-10\mathbf{i} + 24\mathbf{j}$ and use with 39
	$\mathbf{p} = -15\mathbf{i} + 36\mathbf{j}$	A1	
5(b)(ii)	If parallel to the y -axis, $\mathbf{i}$ component is zero	M1	realising $\mathbf{i}$ component is zero
	so $2\mathbf{p} + \mathbf{q} = 12\mathbf{j}$	DM1	use of 12
	$\mathbf{q} = 30\mathbf{i} - 60\mathbf{j}$	A1	
5(b)(iii)	$ \mathbf{q} = 30\sqrt{1^2 + (-2)^2}$ or $\sqrt{900} \times \sqrt{5}$	M1	attempt at magnitude of <i>their</i> $\mathbf{q}$
	$ \mathbf{q} = 30\sqrt{5}$	A1	Answer Given: must have full and correct working
6(i)	$\frac{1}{2} \times 12^2 \times \theta = 150$	M1	use of sector area
	$\theta = 2.083$, so $\theta = 2.08$ to 2dp	A1	
6(ii)	Area of triangle $AOB = \frac{1}{2} \times 12^2 \sin 2.08$	M1	correct method for area of triangle
	Area of segment $= 150 - \frac{1}{2} \times 12^2 \times \sin 2.08$	A1	allow unsimplified, using $\theta = 2.08, 2.083$ or $\frac{150}{72}$
	$\frac{AB}{\sin 1.04} = \frac{2}{12}$	M1	correct trigonometric statement using $\theta = 2.08, 2.083$ or $\frac{150}{72}$ with attempt to obtain AB
	$AB = \text{awrt } 20.7$	A1	
	Shaded area $= \text{their } AB \times 8 - \text{their segment area}$	M1	execution of a correct 'plan' (rectangle – segment)
	$\text{awrt } 78.4$ or 78.5	A1	

Question	Answer	Marks	Guidance
6(iii)	Arc $AB = 25$ or 24.96	B1	
	Perimeter = $25 + \text{their } AB + 16$	M1	correct 'plan' (arc + <i>their</i> $AB + 2 \times 8$)
	awrt 61.7	A1	
7	differentiation to obtain answer in the form $p(3x^2 + 8)^{\frac{2}{3}}$ or $qx(3x^2 + 8)^{\frac{2}{3}}$	M1	
	$6x(3x^2 + 8)^{\frac{2}{3}}$	B1	
	$\frac{dy}{dx} = \frac{5}{3} \times 6x(3x^2 + 8)^{\frac{2}{3}}$	A1	all correct
	When $\frac{dy}{dx} = 0$ only solution is $x = 0$	DM1	$qx(3x^2 + 8)^{\frac{2}{3}} = 0$ and attempt to solve
	$x = 0$ and $3x^2 + 8 = 0$ has no solutions	A1	
	Stationary point at $(0, 32)$	A1	
	correct gradient method with substitution of x values either side of zero or equivalent valid method	M1	
	correct conclusion from correct work using a correct $\frac{dy}{dx}$	A1	
8(i)		B5	B1 for shape of modulus function B1 for y intercept = 5 (for modulus graph only) B1 for x intercept = 2.5 at the V of a modulus graph B1 for shape of quadratic function for $-1 \leq x \leq 6$ B1 for intercepts at $x = 0$ and $x = 5$ for a quadratic graph
8(ii)	$2x - 5 = \pm 4$	B1	one correct answer
	$x = \frac{9}{2}$	M1	solution of two different correct linear equations or solution of an equation obtained from squaring both sides or use of symmetry from first solution.
	$x = \frac{1}{2}$	A1	second correct solution

Question	Answer	Marks	Guidance
8(iii)	$16\left(\frac{1}{2}\right)^2 - 80\left(\frac{1}{2}\right) + 36 = 4$ and $16\left(\frac{9}{2}\right)^2 - 80\left(\frac{9}{2}\right) + 36 = 4$	B1	verification using both x values or for forming and solving $16x^2 - 80x + 36 = 0$
8(iv)	using <i>their</i> values from (ii) in an equality of the form $a \leq x \leq b$ or $a < x < b$	M1	
	$\frac{1}{2} \leq x \leq \frac{9}{2}$ cao	A1	
9(i)	$5 + 4\left(\sec^2\left(\frac{x}{3}\right) - 1\right)$ leading to given answer	B1	use of correct identity
9(ii)	$3 \tan\left(\frac{x}{3}\right) (+c)$	B1	
9(iii)	attempt to integrate using (i) and/or (ii)	M1	
	$\text{Area} = \int_{\frac{\pi}{2}}^{\pi} 4 \sec^2\left(\frac{x}{3}\right) + 1 \, dx$	A1	all correct
	$\left[12 \tan\left(\frac{x}{3}\right) + x \right]_{\frac{\pi}{2}}^{\pi}$	DM1	correct method for evaluation using limits in correct order
	$= \left(12 \tan \frac{\pi}{3} + \pi \right) - \left(12 \tan \frac{\pi}{6} + \frac{\pi}{2} \right)$	A1	
	$= 8\sqrt{3} + \frac{\pi}{2}$	A1	
10(a)	differentiation of a quotient or equivalent product	M1	
	correct differentiation of e^{3x}	B1	
	$\frac{dy}{dx} = \frac{3e^{3x}(4x^2 + 1) - 8xe^{3x}}{(4x^2 + 1)^2}$ or $\frac{dy}{dx} = \frac{3e^{3x}}{4x^2 + 1} - \frac{8xe^{3x}}{(4x^2 + 1)^2}$	A1	everything else correct including brackets where needed, allow unsimplified

Question	Answer	Marks	Guidance
10(b)(i)	one term differentiated correctly	M1	
	$\frac{dy}{dx} = -4\sin\left(x + \frac{\pi}{3}\right) + 2\sqrt{3}\cos\left(x + \frac{\pi}{3}\right)$	A1	all correct
	When $x = \frac{\pi}{2}$, $\frac{dy}{dx} = -5$	A1	
10(b)(ii)	$\frac{dy}{dx} \times \frac{dx}{dt} = \frac{dy}{dt}$ $-5 \times \frac{dx}{dt} = 10$ oe	M1	correct use of rates of change
	$\frac{dy}{dt} = -2$	A1	FT answer to (i)