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ADDITIONAL MATHEMATICS

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MARK SCHEME

Maximum Mark: 80

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

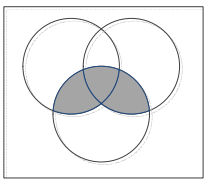
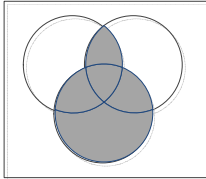
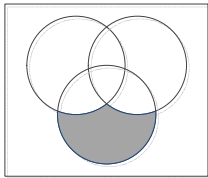
Types of mark

- M Method marks, awarded for a valid method applied to the problem.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	 $(A \cup B) \cap C$  $(A \cap B) \cup C$  $(A' \cap B') \cap C$	B3	B1 for each
2	attempt at differentiating a quotient, must have minus sign and $(x+1)^2$ in the denominator	M1	
	for $(5x^2 + 4)^{-\frac{1}{2}}$	B1	
	for $\frac{1}{2}(10x)(5x^2 + 4)^{-\frac{1}{2}}$	DB1	
	$\frac{dy}{dx} = \frac{(x+1)\frac{1}{2}(10x)(5x^2 + 4)^{-\frac{1}{2}} - (5x^2 + 4)^{\frac{1}{2}}}{(x+1)^2}$	A1	all else correct
	When $x = 3$, $\frac{dy}{dx} = \frac{11}{112}$	A1	must be exact
	Alternative		
	$y = (5x^2 + 4)^{\frac{1}{2}}(x+1)^{-1}$	M1	attempt to differentiate a product
	for $(5x^2 + 4)^{-\frac{1}{2}}$	B1	
	for $\frac{1}{2}(10x)(5x^2 + 4)^{-\frac{1}{2}}$	DB1	
	$\frac{dy}{dx} = \frac{1}{2}10x(5x^2 + 4)^{-\frac{1}{2}}(x+1)^{-1} + (5x^2 + 4)^{\frac{1}{2}}(-(x+1)^{-2})$	A1	all else correct
	When $x = 3$, $\frac{dy}{dx} = \frac{11}{112}$	A1	A1 must be exact

Question	Answer	Marks	Partial Marks
3(a)	$\mathbf{v} = 3\sqrt{5} \times \frac{1}{\sqrt{5}}(\mathbf{i} - 2\mathbf{j})$	M1	attempt to find the magnitude of $(\mathbf{i} - 2\mathbf{j})$ and use
	$= 3\mathbf{i} - 6\mathbf{j}$	A1	for $3\mathbf{i} - 6\mathbf{j}$ only
3(b)	$\mathbf{w} = 2 \cos 30^\circ \mathbf{i} + 2 \sin 30^\circ \mathbf{j}$	M1	attempt to use trigonometry correctly to obtain components
	$= \sqrt{3}\mathbf{i} + \mathbf{j}$	A1	
4	$3^n - n3^{n-1}\left(\frac{x}{6}\right) + n(n-1)3^{n-2}\left(\frac{x}{6}\right)^2$ $3^n = 81$, so $n = 4$	B1	
	$4 \times 3^3 \times -\frac{1}{6} = a$	M1	for $-n3^{n-1}\left(\frac{x}{6}\right)$, ${}^nC_1 3^{n-1}\left(-\frac{x}{6}\right)$ or $\binom{n}{1} 3^{n-1}\left(-\frac{x}{6}\right)$, with/without <i>their n</i>
	$a = -18$	A1	using <i>their n</i> and equating to a to obtain $a = -18$
	$\frac{4 \times 3}{2} \times 3^2 \times \frac{1}{36} = b$	M1	for $n(n-1)3^{n-2}\left(\frac{x}{6}\right)^2$, ${}^nC_2 3^{n-2}\left(\frac{x}{6}\right)^2$ or $\binom{n}{2} 3^{n-2}\left(\frac{x}{6}\right)^2$, with/without <i>their n</i>
	$b = \frac{3}{2}$	A1	using <i>their n</i> and equating to b to obtain $b = \frac{3}{2}$
5(i)	$v = -12 \sin 3t$	B1	
5(ii)	12	B1	FT on <i>their</i> (i) of the form $k \sin 3t$, must be $ k $
5(iii)	$a = -36 \cos 3t$	B1	allow unsimplified
	$3t = \frac{\pi}{2}$, 1.57 or better	B1	
	$t = \frac{\pi}{6}$ or 0.524	B1	
5(iv)	4 cao	B1	may be obtained from knowledge of cosine curve

Question	Answer	Marks	Partial Marks
6(i)	$\frac{1}{\sin \theta} \times \frac{1}{\frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta}}$	M1	for $\cot \theta = \frac{\cos \theta}{\sin \theta}$, $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$
	dealing with the fractions correctly	M1	
	$\frac{1}{\sin \theta} \times \frac{\sin \theta \cos \theta}{\cos^2 \theta + \sin^2 \theta}$	M1	use of identity
	$= \cos \theta$	A1	correct simplification, with all correct
	Alternative 1 $\frac{\operatorname{cosec} \theta}{\frac{1}{\tan \theta} (1 + \tan^2 \theta)}$	M1	dealing with fractions
	$= \frac{\tan \theta \operatorname{cosec} \theta}{\sec^2 \theta}$	M1	use of appropriate identity
	$= \frac{\sin \theta}{\cos \theta} \times \frac{1}{\sin \theta} \times \cos^2 \theta$	M1	for $\cot \theta = \frac{1}{\tan \theta}$, $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\sec \theta = \frac{1}{\cos \theta}$, $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$
	$= \cos \theta$	A1	correct simplification, with all correct
	Alternative 2 $\frac{\operatorname{cosec} \theta}{\frac{1}{\cot \theta} (\cot^2 \theta + 1)}$	M1	dealing with fractions
	$= \frac{\cot \theta \operatorname{cosec} \theta}{\operatorname{cosec}^2 \theta}$	M1	use of appropriate identity
	$= \frac{\cot \theta}{\operatorname{cosec} \theta}$ $= \frac{\cos \theta}{\sin \theta} \times \sin \theta$	M1	for $\cot \theta = \frac{\cos \theta}{\sin \theta}$, $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$
	$= \cos \theta$	A1	correct simplification, with all correct

Question	Answer	Marks	Partial Marks
6(ii)	$\int_0^a \cos 2\theta \, d\theta = \left[\frac{1}{2} \sin 2\theta \right]_0^a$	B1	
	$\frac{1}{2} \sin 2a = \frac{\sqrt{3}}{4}$	M1	use of $\left[k \sin 2\theta \right]_0^a = \frac{\sqrt{3}}{4}$ to obtain $k \sin 2a = \frac{\sqrt{3}}{4}$
	$2a = \frac{\pi}{3}$	DM1	attempt to solve equation of the form $k \sin 2a = \frac{\sqrt{3}}{4}$, with $-1 \leq \frac{\sqrt{3}}{4k} \leq 1$, must have a correct order of operations dealing with the double angle
	$a = \frac{\pi}{6}$, 0.167π or better	A1	
7(i)	$\lg y = \lg A + bx$	B1	straight line form, may be implied by correct values of both A and b later
	Gradient = b ,	M1	equating gradient to b
	$b = 3$	A1	
	Use of substitution into one of the following $2.2 = \lg A + 0.5b$ $3.7 = \lg A + b$ $158.489 = A \times 10^{0.5b}$ $5011.872 = A \times 10^b$ or equivalent valid method leads to $\lg A = 0.7$	M1	
	$A = 5$, 5.01 or $10^{0.7}$	A1	
	Alternative 1 $\lg y = \lg A + bx$	B1	straight line form, may be implied by correct work later
	$2.2 = \lg A + 0.5b$	M1	one correct equation
	$3.7 = \lg A + b$	A1	both equations correct
	attempt to solve 2 correct equations	M1	
	leading to $b = 3$ and $A = 5$, 5.01 or $10^{0.7}$	A1	

Question	Answer	Marks	Partial Marks
7(i)	Alternative 2 $y = A(10^{bx})$ $158.489 = A \times 10^{0.5b}$	M1	one correct equation
	$5011.872 = A \times 10^b$	A1	both correct
	$\frac{5011.872}{158.489} = 10^{0.5b}$	M1	attempt to solve 2 correct equations
	leading to $b = 3$	A1	correct b
	Use of substitution leads to $A = 5, 5.01$ or $10^{0.7}$	A1	correct A
7(ii)	Substitute A and b correctly into either $y = A(10^{0.6b})$, $\lg y = \lg A + 0.6b$ or $\lg y = \lg A + 0.6 \lg 10^b$ or using $\lg y = 1.8 + 0.7$	M1	correct statement using <i>their</i> A and b correctly in either equation or using $\lg y = 3x + 0.7$
	$y = 316, 315$ or $10^{2.5}$	A1	
7(iii)	Substitute A and b correctly into either $600 = A(10^{bx})$, $\lg 600 = \lg A + bx$ or $\lg 600 = \lg A + x \lg 10^b$ or using $\lg 600 = 3x + 0.7$	M1	correct statement using <i>their</i> A and b correctly in either equation or using $\lg y = 3x + 0.7$
	$x = 0.693$	A1	
8(a)(i)	2520	B1	
8(a)(ii)	360	B1	
8(a)(iii)	1080	B1	
8(a)(iv)	6 or 8 to start with No of ways = $2 \times 5 \times 4 \times 3 \times 2$ = 240	B1	
	9 to start with No of ways = $1 \times 5 \times 4 \times 3 \times 3$ = 180	B1	
	Total number of ways = 420	DB1	Dependent on both previous B marks

Question	Answer	Marks	Partial Marks
8(a)(iv)	Alternative 1 All numbers > 6000 – all odd numbers > 6000	B1	plan and attempt to use, must be using 1080
	1080 – 180 – 480	B1	for 180 and 480
	Total number of ways = 420	DB1	Dependent on both previous B marks
	Alternative 2 Even numbers > 60000 : Odd numbers > 60000 7 : 11	B1	correct ratio
	Total number of ways = $\frac{7}{18} \times 1080$	B1	
	= 420	DB1	Dependent on both previous B marks
8(b)(i)	480700	B1	
8(b)(ii)	26460	B1	
8(b)(iii)	With brother and sister ${}^{23}C_5 = 33649$	B1	for ${}^{23}C_5$ or ${}^{23}C_5 \times {}^kC_k$
	Without brother and sister ${}^{23}C_7 = 245157$	B1	for ${}^{23}C_7$ or ${}^{23}C_7 \times {}^kC_k$
	Total number of ways = 278806	B1	for ${}^{23}C_5 + {}^{23}C_7$ and evaluation
9(a)(i)	3×2	B1	
9(a)(ii)	correct attempt to multiply the 2 matrices	M1	
	$\mathbf{C} = \begin{pmatrix} 6 & -6 \\ 5 & 2 \\ 19 & -8 \end{pmatrix}$	A2	–1 for each incorrect element
9(b)(i)	$\mathbf{X}^{-1} = \frac{1}{13} \begin{pmatrix} -7 & 12 \\ -4 & 5 \end{pmatrix}$	B2	B1 for correct use of determinant B1 for correct matrix
9(b)(ii)	$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{13} \begin{pmatrix} -7 & 12 \\ -4 & 5 \end{pmatrix} \begin{pmatrix} 26 \\ 52 \end{pmatrix}$	B1	
	attempt to evaluate using inverse from (i) together with pre-multiplication to obtain a 2×1 matrix	M1	
	$x = 34, y = 12$	A2	A1 for each
10(i)	0.5	B1	for 0.5 from correct work only

Question	Answer	Marks	Partial Marks
10(ii)	$15^2 = 8^2 + 8^2 - (2 \times 8 \times 8 \times \cos AOB)$ $AOB = 2.43075$ rads	M1	use of cosine rule (or equivalent) to obtain angle AOB .
	$DOC = AOB - 2(\text{their } AOD)$	M1	use of angle AOD and symmetry
	$DOC = 1.43$ to 2 dp	A1	Answer Given: need to have seen either 2.431 or better, or 1.431 or better in previous calculations
	Alternative 1 $15 = 2 \times 8 \times \sin\left(\frac{1 + DOC}{2}\right)$	M1	use of basic trigonometry
	use of $\frac{1 + 0.5DOC}{2}$	M1	may be implied
	$DOC = 1.43$ to 2 dp	A1	Answer Given: need to have seen either 2.431 or better, or 1.431 or better or 1.215 or better in previous calculations
	Alternative 2		
	$15^2 = 8^2 + 8^2 - (2 \times 8 \times 8 \times \cos AOB)$ $AOB = 2.43075$ rads $\angle AOB \times 8 = \text{arc } AB$	M1	use of cosine rule (or equivalent) to obtain angle AOB .
	$\frac{\text{arc } AB - 8}{8} = \angle DOC$	M1	attempt at DOC , must be a complete method with AOB found
	$DOC = 1.43$ to 2 dp	A1	Answer Given: need to have seen either 2.431 or better, or 1.431 or better or 1.215 or better in previous calculations
	Alternative 3 Equating 2 different forms for the area of triangle AOB $\frac{15\sqrt{31}}{4} = \frac{1}{2} \times 8^2 \sin AOB$, $AOB = 2.43075$ rads	M1	using both different forms of the area of triangle AOB
	$DOC = AOB - 2(\text{their } AOD)$	M1	use of angle AOD and symmetry
	$DOC = 1.43$ to 2 dp	A1	Answer Given: need to have seen either 2.431 or better, or 1.431 or better in previous calculations

Question	Answer	Marks	Partial Marks
10(iii)	$\sin\left(\frac{1.43}{2}\right) = \frac{DC}{8} \text{ or}$ $DC^2 = 8^2 + 8^2 - (2 \times 8 \times 8 \times \cos 1.43)$	M1	use of cosine rule or basic trigonometry to obtain DC
	$DC = 10.49$	A1	awrt 10.5, may be implied
	Perimeter = $10.49 + 4 + 4 + 15$ = 33.5	A1	awrt 33.5
10(iv)	$\frac{1}{2} \times 8^2 (2.43 - \sin 2.43) - \frac{1}{2} \times 8^2 (1.431 - \sin 1.431)$	B1	area of one appropriate sector; allow unsimplified; may be implied by a correct segment
	area of one appropriate triangle, allow unsimplified	B1	
	an appropriate segment, allow unsimplified	B1	
	= 42.8 (allow awrt 42.8)	B1	final answer
	Alternative 1 Area of a trapezium + 2 small segments	B1	one appropriate small sector, allow unsimplified (could be doubled)
	Each small segment = $\frac{1}{2} \times 8^2 (0.5 - \sin 0.5)$	B1	an appropriate triangle, allow unsimplified (could be doubled)
	Area of trapezium = $\frac{1}{2} (15 + 10.5) \times (6.041 - 2.784)$	B1	attempt at trapezium, must have a correct attempt at finding the distance between the parallel sides – allow unsimplified
	Total area = 42.8 (allow awrt 42.8)	B1	final answer
	Alternative 2 Area of 2 small sectors + area of triangle ODC – the area of triangle OAB Area of a small sector = $\frac{1}{2} \times 8^2 \times \frac{1}{2}$	B1	area of small sector, allow unsimplified, (could be doubled)
	Area of triangle ODC = $\frac{1}{2} \times 8^2 \times \sin 1.43$	B1	area of triangle ODC , allow unsimplified
	Area of triangle OAB = $\frac{1}{2} \times 8^2 \times \sin 2.43$	B1	area of triangle OAB , allow unsimplified
	Total area = 42.8 (allow awrt 42.8)	B1	final answer

Question	Answer	Marks	Partial Marks
10(iv)	Alternative 3 Area of rectangle + 2 small triangles + 2 small segments Each small segment = $\frac{1}{2} \times 8^2 (0.5 - \sin 0.5)$	B1	area of a small segment, allow unsimplified, could be doubled
	$\frac{1}{2} \times \frac{(15 - 10.49)}{2} (6.041 - 2.784)$	B1	area of a small triangle, allow unsimplified, could be doubled
	Area of rectangle = $10.49 \times (6.041 - 2.784)$	B1	allow unsimplified, could be doubled
	Total area = 42.8 (allow awrt 42.8)	B1	final answer
	Alternative 4 Sector AOB – sector AOD – sector COB – triangle DOC	B1	area of one appropriate sector; allow unsimplified; may be implied by a correct segment
	$\left(\frac{1}{2} \times 8^2 \times 2.43 \right) - 2 \left(\frac{1}{2} \times 8^2 \times 0.5 \right) - \left(\frac{1}{2} \times 8^2 \sin 1.43 \right)$ Area = sector AOB – segment DC – triangle AOB	B1	area of one appropriate triangle, allow unsimplified
	$\left(\frac{1}{2} \times 8^2 \times 2.43 \right) - (\text{their segment}) - \left(\frac{1}{2} \times 8^2 \sin 2.43 \right)$	B1	an appropriate segment, allow unsimplified
	Total area = 42.8 (allow awrt 42.8)	B1	final answer
11(i)	me^{2x-1} where m is numeric constant	M1	
	$f(x) = \frac{1}{2}e^{2x-1} (+c)$	A1	condone omission of +c
	$\frac{7}{2} = \frac{1}{2} + c$	DM1	correct attempt to find arbitrary constant
	$f(x) = \frac{1}{2}e^{2x-1} + 3$	A1	must be an equation
11(ii)	ke^{2x-1} where k is a numeric constant	M1	
	$f''(x) = 2e^{2x-1}$	A1	
	$2x - 1 = \ln\left(\frac{4}{k}\right)$	DM1	attempt to equate to 4 and use logarithms
	$x = \frac{1}{2} + \ln\sqrt{2}$	A1	