Cambridge O Level

Cambridge Assessment International Education
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS	**4037/22**
Paper 2	**October/November 2019**
MARK SCHEME	
Maximum Mark: 80	

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Cambridge Assessment International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

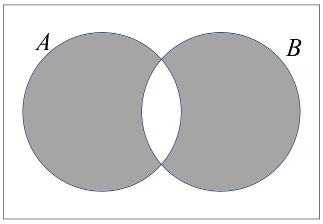
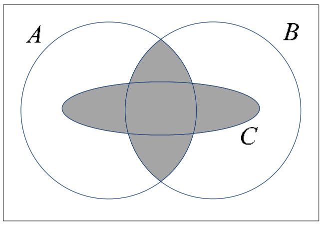
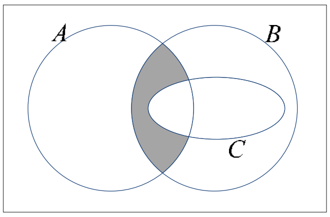
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1		**B1**	
		**B1**	
		**B 1**	
2	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6\cos 3x$	**B1**	
	$-3\sin 3x$	**B1**	
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -18\sin 3x - 9\cos 3x$	**B1**	**FT** Correct derivative of *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$
	Insert and collect like terms	**M1**	Must insert for y, *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ correctly resulting in 6 terms.
	$k = -15$	**A1**	Allow $-15\sin 3x$ seen nfww
3(i)	${}^{14}P_5$ or $14 \times 13 \times 12 \times 11 \times 10$	**M1**	
	240 240	**A1**	cao
3(ii)	${}^{3}P_1 \times {}^{5}P_2 \times {}^{6}P_2$ or $3 \times (5 \times 4) \times (6 \times 5)$	**M1**	Two of the three elements multiplied by …
	= 1800	**A1**	
3(iii)	${}^{6}P_2 \times {}^{8}P_3$ or $(6 \times 5) \times (8 \times 7 \times 6)$	**M1**	One element multiplied by … Clear intention to multiply
	= 10 080	**A1**	

Question	Answer	Marks	Guidance
4	$kx + 3 = x^2 + 5x + 12$ $\rightarrow x^2 + (5 - k)x + 9(= 0)$	M1	Equate and attempt to simplify to all terms on one side.
	Use discriminant of *their* quadratic.	M1	**dep**
	$(5 - k)^2 - 36$ oe	A1	Unsimplified
	$k = -1$ and 11	A1	Both boundary values
	$-1 < k < 11$	A1	Must be in terms of k.
	OR $2x + 5 \sim k$	M1	Connect gradients of line and curve
	$y = (2x+5)x + 3 \rightarrow$ $2x^2 + 5x + 3 = x^2 + 5x + 12$	M1	Eliminate k and y.
	$x^2 = 9 \rightarrow x = \pm 3$	A1	
	$k = 11$ or $k = -1$	A1	
	$-1 < k < 11$	A1	
5(i)	$\frac{dy}{dx} = \frac{-2k}{(x+1)^3}$	B1	oe Unsimplified
	Gradient of normal = $\frac{(x+1)^3}{2k}$ or Gradient of tangent = -3	M1	Gradient of normal = $\frac{-1}{\text{gradient of tangent}}$
	$\frac{8}{2k} = \frac{1}{3}$ or $\frac{2k}{8} = -3$	M1	Equate gradient of normal to $\frac{1}{3}$ at $x = 1$ or equate gradient of tangent to -3 at $x = 1$
	$k = 12$	A1	
5(ii)	$x = 2 \rightarrow \frac{dy}{dx} = -\frac{8}{9}$ or *their* $\frac{-2k}{27}$	B1	**FT**
	$y = \frac{4}{3}$ or *their* $\frac{k}{9}$	B1	**FT**
	$\frac{y - \frac{4}{3}}{x - 2} = -\frac{8}{9}$ or $y = -\frac{8}{9}x + \frac{28}{9}$	B1	isw

Question	Answer	Marks	Guidance
6(i)	$\dfrac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}+1}+\dfrac{\frac{1}{\cos x}+1}{\frac{\sin x}{\cos x}}$	M1	Use $\tan x = \dfrac{\sin x}{\cos x}$ and $\sec x = \dfrac{1}{\cos x}$ throughout
	$\dfrac{\sin x}{1+\cos x}+\dfrac{1+\cos x}{\sin x}$	M1	**dep** Multiply by cos$x$
	$\dfrac{\sin^2 x+1+2\cos x+\cos^2 x}{(1+\cos x)\sin x}$	M1	**dep** Add *their* fractions correctly and expand $(1 + \cos x)^2$ correctly
	$\dfrac{2(1+\cos x)}{(1+\cos x)\sin x}$	M1	**dep** Use $\sin^2 x + \cos^2 x = 1$ and take out a factor of 2.
	All correct AG	A1	Do not award if brackets missing at any point **or** x missing more than twice **or** x misplaced. Do not credit mixed variables.
	OR $\dfrac{\tan^2 x+(\sec x+1)^2}{\tan x(\sec x+1)}$	M1	Add fractions
	$=\dfrac{2\sec^2 x+2\sec x}{\tan x(\sec x+1)}$	M1	**dep** Expand brackets correctly and use $1 + \tan^2 x = \sec^2 x$
	$\dfrac{2\sec x}{\tan x}$	M1	**dep** Cancel sec$x$ + 1
	$\dfrac{2}{\cos x}\times\dfrac{\cos x}{\sin x}$	M1	**dep** Use $\tan x = \dfrac{\sin x}{\cos x}$ and $\sec x = \dfrac{1}{\cos x}$ oe
	All correct AG	A1	Do not award if brackets missing at any point **or** x missing more than twice **or** x misplaced. Do not credit mixed variables.
6(ii)	$3\sin^2 x + \sin x - 2 = 0$ oe	B1	Obtain three term quadratic.
	$(3\sin x - 2)(\sin x + 1) = 0$	M1	Solve three term quadratic
	41.8° awrt	A1	
	138.2° awrt	A1	Mark final answers This mark is not awarded if there are more solutions in the range.

Question	Answer	Marks	Guidance
7(a)	$2 \times 4 \times p = 40 \to p = 5$	**B1**	May be obtained later.
	$(x-2)(x-4)(x-p) = 0$	**M1**	Factorise cubic
	$a = -11$	**A1**	Expand and identify
	$b = 38$	**A1**	
	OR $2 \times 4 \times p = 40 \to p = 5$	**B1**	May be obtained later.
	Obtain equations $4a + 2b = 32$ $16a + 4b = -24$ and attempt to solve	**M1**	
	$a = -11$	**A1**	
	$b = 38$	**A1**	
7(b)	Find $x = -1$	**M1**	Trial value/s and finds a root or shows that $(x+1)$ or $(x+4)$ or $(x-10)$ divides into $x^3 - 5x^2 - 46x - 40$.
	$(x+1)(x^2 - 6x - 40)$ $(= 0)$ or $(x+4)(x^2 - 9x - 10)(= 0)$ or $(x-10)(x^2 + 5x + 4)(= 0)$	**A1**	Factorise to give linear and quadratic factor
	$(x+1)(x+4)(x-10)$ $(= 0)$	**M1**	Solve the quadratic to give 2 roots
	$x = -1, -4, 10$	**A1**	
	OR Uses factor theorem to find a root $(-1)^3 - 5(-1^2) - 46(-1) - 40$ or $-1 - 5 + 46 - 40 = 0$ $\to x = -1$	**M1**	This may be awarded for $x = -4$ or $x = 10$.
	Uses factor theorem to attempt to find further roots	**M1**	At least two more trials.
	$(-4)^3 - 5(-4)^2 - 46(-4) - 40$ or $-64 - 80 + 184 - 40 = 0$ $\to x = -4$	**A1**	
	$(10)^3 - 5(10)^2 - 46(10) - 40$ or $1000 - 500 - 460 - 40 = 0$ $\to x = 10$	**A1**	

Question	Answer	Marks	Guidance
8(i)	$\sqrt{5^2+12^2}=13$	M1	
	$\boldsymbol{v}_A=-\frac{5}{2}\mathbf{i}-6\mathbf{j}$ or $\frac{1}{2}(-5\mathbf{i}-12\mathbf{j})$	A1	
8(ii)	$\lvert v_B\rvert=\sqrt{12^{12}+(-9)^2}$	M1	Use Pythagoras
	15	A1	Do not allow ± 15. Mark final answer.
8(iii)	$\boldsymbol{r}_A=\begin{pmatrix}20\\-7\end{pmatrix}+t\begin{pmatrix}-2.5\\-6\end{pmatrix}$ or $\boldsymbol{r}_A=(20-2.5t)\mathbf{i}+(-7-6t)\mathbf{j}$	B1	**FT** on *their* $\boldsymbol{v}_A$ only if of the form $k(-5\mathbf{i}-12\mathbf{j})$ where $k\neq 1$ or 0.
	$\boldsymbol{r}_B=\begin{pmatrix}-67\\11\end{pmatrix}+t\begin{pmatrix}12\\-9\end{pmatrix}$ or $\boldsymbol{r}_B=(-67+12t)\mathbf{i}+(11-9t)\mathbf{j}$	B1	
8(iv)	$20-2.5t=-67+12t$ or $-7-6t=11-9t$	M1	Equate x or y coordinates. Must have two terms in both coordinates.
	$t=6$	A1	nfww Ignore other value of t.
	$\boldsymbol{r}=\begin{pmatrix}5\\-43\end{pmatrix}$ only or $\boldsymbol{r}=5\mathbf{i}-43\mathbf{j}$	A1	**A0** if further value of $\boldsymbol{r}$ found.
9(i)	Midpoint (1, 2)	B1	May be seen on diagram
	Gradient of $AB=-\frac{3}{4}$	B1	
	Gradient of PM $=\frac{-1}{\textit{their}\text{ gradient of }AB}=\frac{4}{3}$	M1	Use $m_1\times m_2=-1$
	Equation PM $\frac{y-2}{x-1}=\frac{4}{3}$	M1	**dep** Attempt to find equation of line with *their* midpoint and *their* gradient of PM. If $y=mx+c$ used c must be found.
	$y=\frac{4}{3}x+\frac{2}{3}$	A1	
9(ii)	$s=\frac{4}{3}r+\frac{2}{3}$	B1	**FT**

Question	Answer	Marks	Guidance
			Insert (r, s) into *their* linear equation to obtain $s = \ldots$
9(iii)	$(r-1)^2 + (s-2)^2 = 100$ oe	B1	**FT** Use Pythagoras with *their* (1, 2)
	Eliminate r or s	M1	From one linear and one quadratic expression. Unsimplified
	$25r^2 - 50r - 875 = 0$ oe or $25s^2 - 100s - 1500 = 0$ oe	A1	
	$(5r + 25)(5r - 35) = 0$ oe or $(5s - 50)(5s + 30) = 0$ oe	M1	Solve three term quadratic Can be implied by correct solution.
	$r = 7$, $s = 10$	A1	Do not award if negative values of r and s are also given nfww
	OR Equivalent method such as: $\overrightarrow{MP} = \begin{pmatrix} a \\ b \end{pmatrix} \rightarrow a^2 + b^2 = 100$ and $\frac{b}{a} = \frac{4}{3}$	B1	Using distance $=10$ and gradient $= \frac{4}{3}$.
	Eliminate a or b	M1	
	$a^2 + \left(\frac{4a}{3}\right)^2 = 100$ or $\left(\frac{3b}{4}\right)^2 + b^2 = 100$	A1	
	$\rightarrow a = (\pm)6$ and $b = (\pm)8$	M1	Solve
	$r = 7$, $s = 10$	A1	
10(i)	Quotient rule or product rule	M1	
	$\frac{x - 2x\ln x}{x^4}$ or $\frac{x - \ln x.2x}{x^4}$ oe isw	A2/1/0	Minus one each error. Allow unsimplified.
10(ii)	$x - 2x\ln x = 0$	M1	Set $\frac{dy}{dx} = 0$ and attempt to solve. Must have two terms and obtain $\ln x = k$ only.
	$x = 1.65$ awrt or $\sqrt{e}$	A1	
	$y = 0.184$ awrt or $\frac{1}{2e}$	A1	

Question	Answer	Marks	Guidance
10(iii)	$\frac{\ln x}{x^2} = \int \frac{1}{x^3} - \frac{2\ln x}{x^3} \mathrm{d}x$	M1	Integrate *their* derivative from (i) which must have two terms. Condone omission of dx.
	$\frac{-1}{2x^2}$	A1	Find $\int \frac{1}{x^3} \mathrm{d}x$
	$\int \frac{\ln x}{x^3} \mathrm{d}x = -\frac{1}{4x^2} - \frac{\ln x}{2x^2} + (C)$	A1	oe Rearrange and complete
10(iv)	Insert limits and subtract correctly	M1	**dep** Must be inserting into **two** terms in x from (iii). Values explicitly seen if expression is incorrect.
	$\frac{3}{16} - \frac{\ln 2}{8}$ or 0.101 awrt	A1	
11	$\left(\sqrt{5}-3\right)\left(\sqrt{5}+3\right) = -4$	B1	Seen anywhere
	Attempt formula	M1	
	$x = \frac{-3 \pm 5}{2\left(\sqrt{5}-3\right)}$	A1	
	Multiply by *their* $\left(\sqrt{5}+3\right)$	M1	Attempt must be seen with a further line of working. oe
	$x = \sqrt{5}+3$	A1	oe Mark final answer
	$x = \frac{-1\left(\sqrt{5}+3\right)}{4}$	A1	oe Mark final answer