Cambridge Assessment International Education

Cambridge O Level

ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **May/June 2022**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **14** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

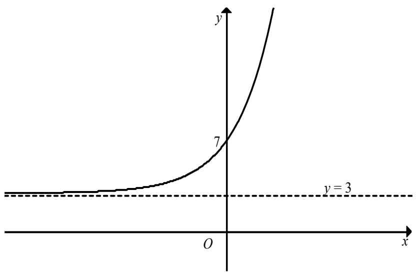
Question	Answer	Marks	Partial Marks
1	$[y=]\dfrac{6+\sqrt{6}}{3+\sqrt{6}}\times\dfrac{3-\sqrt{6}}{3-\sqrt{6}}$ oe, soi	**M1**	
	Correctly multiplies out correct expression: $[y=]\dfrac{18-6\sqrt{6}+3\sqrt{6}-6}{9-6}$ oe	**M1**	
	$[y=]\ 4-\sqrt{6}$	**A1**	not from wrong working
2	$\mathrm{f}(x)=-2x+5$ or $\mathrm{g}(x)=x-1$ soi	**B1**	
	Uses correct f(x) and g(x) to find the critical value 2 soi	**B1**	
	Valid method to find other CV e.g. $2x-5 * x-1$ oe seen, where * is = or any inequality sign	**M1**	**FT** *their* equations of form $y = mx + c$, for non-zero m and c; **dep** on first **B1**
	Correct critical value 4 soi	**A1**	
	$2 \leqslant x \leqslant 4$ mark final answer	**A1**	
	Alternative method 1 $\mathrm{f}(x)=\ 2x-5$ or $\mathrm{g}(x)=x-1$ soi	**(B1)**	
	Uses correct f(x) and g(x) to find the critical value 4 soi	**(B1)**	
	Valid method to find other CV e.g. $-2x+5 * x-1$ oe seen, where * is = or any inequality sign	**(M1)**	**FT** *their* equations of form $y = mx + c$, for non-zero m and c; **dep** on first **B1**
	Correct critical value 2 soi	**(A1)**	
	$2 \leqslant x \leqslant 4$ mark final answer	**(A1)**	
	Alternative method 2 f(x) = $-2x + 5$ or $2x - 5$ OR g(x) = $x - 1$ soi	**(B1)**	
	Squares, equates, simplifies correct f(x) and g(x): $3x^2-18x+24[*\,0]$	**(B1)**	where * is = or any inequality sign
	Attempts to solve or factorise	**(M1)**	FT *their* 3-term quadratic from $(ax + b)^2 = (cx + d)^2$ for non-zero a, b, c and d; **dep** on first **B1**
	Correct critical values 2, 4	**(A1)**	
	$2 \leqslant x \leqslant 4$ mark final answer	**(A1)**	

Question	Answer	Marks	Partial Marks
3	Uses $b^2 - 4ac$ oe: $(k+5)^2 - 4k(-4)$ [* 0, where * could be = or any inequality sign]	**M1**	
	Forms a correct 3-term expression: $k^2 + 26k + 25$	**A1**	
	Factorises $k^2 + 26k + 25$ or solves $k^2 + 26k + 25 = 0$ oe	**M1**	**dep** on first **M1**, **FT** *their* 3-term quadratic in k
	Correct critical values -1, -25 soi	**A1**	
	$k \leqslant -25$, $k \geqslant -1$	**A1**	mark final answer
4	Substitutes $y = 4$ and rearranges to correct 3-term quadratic $3x^2 - 2x - 1 = 0$ oe	**B1**	
	Solves *their* 3-term quadratic in x as far as $x = \ldots$	**M1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -2x^{-2} - 2x^{-3}$ oe, isw	**B1**	
	$\frac{0.01}{\delta x} = their\left(\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\vert_{x = their\ 1}\right)$ or better	**M1**	**FT** *their* derivative and *their x,* providing *their* $x > 0$ and *their* $x \neq 4$ unless 4 is a genuine solution of *their* 3-term quadratic; must see a power decrease for attempted differentiation in two out of the three terms
	$-\frac{1}{400}$ oe as the only solution	**A1**	**dep** on all previous marks being awarded

Question	Answer	Marks	Partial Marks
4	**Alternative method** $x = \dfrac{1}{\sqrt{y}-1}$ or $x = \dfrac{1+\sqrt{y}}{y-1}$ or $x = \dfrac{-1-\sqrt{y}}{1-y}$ oe	**(B2)**	**B1** for $y = \dfrac{(x+1)^2}{x^2}$ or better or for $x = \dfrac{2+\sqrt{4-4(y-1)(-1)}}{2(y-1)}$ or for $x = \dfrac{-2-\sqrt{4-4(1-y)}}{2(1-y)}$
	$\dfrac{\mathrm{d}x}{\mathrm{d}y} = -\left(\sqrt{y}-1\right)^{-2}\left(\dfrac{1}{2}y^{-\frac{1}{2}}\right)$ oe isw or $\dfrac{\mathrm{d}x}{\mathrm{d}y} = \dfrac{(y-1)\left(\dfrac{1}{2}y^{-\frac{1}{2}}\right)-(1+\sqrt{y})}{(y-1)^2}$ oe	**(M1)**	or $\dfrac{\mathrm{d}x}{\mathrm{d}y} = \dfrac{(1-y)\left(-\dfrac{1}{2}y^{-\frac{1}{2}}\right)-(-1-\sqrt{y})(-1)}{(1-y)^2}$ oe
	$\dfrac{\delta x}{0.01} = their\left(\left.\dfrac{\mathrm{d}x}{\mathrm{d}y}\right\rvert_{y=4}\right)$ or better	**(M1)**	**FT** *their* derivative ; must have attempted derivative
	$-\dfrac{1}{400}$ oe as the only solution	**(A1)**	**dep** on all previous marks being awarded

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Partial Marks</th></tr>
<tr><td rowspan="3">5(a)</td><td>$\dfrac{(5^4)^{\frac{x^3-1}{2}}}{(5^3)^{x^3}} = 5$ oe or $\dfrac{(25^2)^{\frac{x^3-1}{2}}}{\left(25^{\frac{3}{2}}\right)^{x^3}} = 25^{\frac{1}{2}}$
or $\dfrac{\left(\sqrt{625}\right)^{x^3} \times 625^{-\frac{1}{2}}}{125^{x^3}} = 5$
or $\log 625^{\frac{x^3-1}{2}} - \log 125^{x^3} = \log 5$ oe</td><td>B1</td><td>converts the terms given to powers of 5 or 25

or separates the power in the numerator correctly

or applies a correct log law</td></tr>
<tr><td>$5^{2x^3-2-3x^3} = 5^1$ oe
$\Rightarrow -x^3 - 2 = 1$ oe
or
$25^{x^3-1-1.5x^3} = 25^{\frac{1}{2}}$ oe
$\Rightarrow -0.5x^3 - 1 = 0.5$ oe
or
$\left(\dfrac{1}{5}\right)^{x^3} \times \dfrac{1}{25} = 5$ oe
$\Rightarrow x^3 \log\dfrac{1}{5} = \log 125$ oe
or
$\dfrac{x^3-1}{2}\log 625 - x^3 \log 125 = \log 5$ oe</td><td>M1</td><td>FT their exponential equation in the same base
or their logarithmic equation with any consistent base,
providing their exponential or logarithmic equation has at most one sign or arithmetic error</td></tr>
<tr><td>$[x =]\sqrt[3]{-3}$ oe
or –1.442249... rot to 3 or more figs.</td><td>A1</td><td>mark final answer; not from wrong working</td></tr>
<tr><td>5(b)</td><td></td><td>B2</td><td>B1 for correct shape; tending to $y = 3$

B1 for shape with correct curvature and correct intercept of 7 marked or (0, 7) indicated</td></tr>
</table>

Question	Answer	Marks	Partial Marks
6(a)	$4\cos75\mathbf{i} + 4\sin75\mathbf{j}$ or $4\sin15\mathbf{i} + 4\cos15\mathbf{j}$ or $4\sin15\mathbf{i} + 4\sin75\mathbf{j}$ or $4\cos75\mathbf{i} + 4\cos15\mathbf{j}$ oe, isw	**B2**	**B1** for $x = 4\cos75$ or $x = 4\sin15$ oe, soi or $y = 4\sin75$ or $y = 4\cos15$ oe, soi or **B1** for a correct pair of implicit statements for x and y e.g. **both** $\frac{x}{4} = \sin15$ **and** $\frac{y}{4} = \cos15$ oe or **both** $\frac{x}{4} = \cos75$ **and** $\frac{y}{4} = \sin75$ oe If 0 scored, **SC1** for a correct expression with missing brackets such as $\sqrt{6} - \sqrt{2}\mathbf{i} + \sqrt{6} + \sqrt{2}\mathbf{j}$ or for $\begin{pmatrix} 1.04 \\ 3.86 \end{pmatrix}$ oe

Question	Answer	Marks	Partial Marks
6(b)	(–6cos30–2cos40)**i** +(6sin30 – 2sin40)**j** or (–6sin60 –2sin50)**i** +(6sin30 –2sin40)**j** or (–6cos30 –2cos40)**i** +(6cos60 –2cos50)**j** or $(-3\sqrt{3}-1.532...)\mathbf{i}+(3-1.285...)\mathbf{j}$ oe, soi	**B1**	
	$\left[r^2=\right](-6.7282...)^2+(1.7144...)^2$	**M1**	**FT** *their* (–6.728… **i** + 1.714…**j**)
	$[r=]$ 6.94 or 6.9432329... rot to 4 or more sf	**A1**	**dep** on **B1**
	$\alpha=\tan^{-1}\left(\dfrac{6.7282...}{1.7144...}\right)$ or awrt 75.7 or $\beta=\tan^{-1}\left(\dfrac{1.7144...}{6.7282...}\right)$ or awrt 14.3	**M1**	**FT** *their* (–6.728… **i** + 1.714…**j**)
	284 or 284.2[95…] rot to 4 or more sf	**A1**	**dep** on **B1**
	Alternative method $[r^2=]\,2^2+6^2-2\times2\times6\times\cos(\textit{their}\,110)$	**(M1)**	
	$[r=]$ 6.94 or 6.9432329... rot to 4 or more sf	**(A1)**	
	$\dfrac{\sin\theta}{2}=\dfrac{\sin(\textit{their}\,110)}{\textit{their}\,6.943...}$ oe or $\dfrac{\sin\phi}{6}=\dfrac{\sin(\textit{their}\,110)}{\textit{their}\,6.943...}$ oe	**(M1)**	$\cos\theta=\dfrac{2^2-\textit{their}\,6.943...^2-6^2}{-2(\textit{their}\,6.943...)(6)}$ or $\cos\phi=\dfrac{6^2-\textit{their}\,6.943...^2-2^2}{-2(\textit{their}\,6.943...)(2)}$
	$[\theta=]$ awrt 15.7 oe or $[\phi=]$ awrt 54.3	**(A1)**	
	284 or 284.2[95…] rot to 4 or more sf	**(A1)**	

Question	Answer	Marks	Partial Marks
7	$\frac{d}{dx}(e^{4x}) = 4e^{4x}$ soi	B1	
	$4e^{4x}\tan x + e^{4x}\sec^2 x$	M1	**FT** *their* $4e^{4x}$
	$\frac{(\ln x)(4e^{4x}\tan x + e^{4x}\sec^2 x) - \frac{1}{x}e^{4x}\tan x}{(\ln x)^2}$	M1	**FT** *their* $4e^{4x}\tan x + e^{4x}\sec^2 x$
	Fully correct derivative, isw	A1	
	Alternative method for final two marks Applies correct product rule to $y = (e^{4x}\tan x)(\ln x)^{-1}$ $\left(4e^{4x}\tan x + e^{4x}\sec^2 x\right)(\ln x)^{-1} + \left(-(\ln x)^{-2}\left(\frac{1}{x}\right)\right)\left(e^{4x}\tan x\right)$	(M1)	**FT** *their* $4e^{4x}\tan x + e^{4x}\sec^2 x$
	Fully correct derivative, isw	(A1)	
8(a)	$3(2\sin x\cos x) - (-2\sin x)\quad [=0]$ or better	B2	**B1** for the correct derivative for either term; may be unsimplified
	Factors out $\sin x$ and equates to 0: $[2](\sin x)(3\cos x + 1) = 0$ oe	B1	**FT** an expression of the form $a\sin x\cos x + b\sin x$ for non-zero constants a and b
	$\sin x = 0 \quad [\mathit{their}\, a \cos x = -\mathit{their}\, b]$	M1	**FT** an expression of the form $a\sin x\cos x + b\sin x$ for non-zero constants a and b; **dep** on previous **B1**
	$x = \pi$ as only solution	A1	**dep** on all previous marks awarded If **B2 B0 M0** then **SC1** for dividing by $\sin x$ and using $\cos^{-1}\left(-\frac{1}{3}\right)$ to find $x = 1.91$ and 4.37 and clearly reject them.

Question	Answer	Marks	Partial Marks
8(b)	For use of $\sin^2 x = 1 - \cos^2 x$ to write in terms of $\cos x$ only e.g. $3(1-\cos^2 x) - 2\cos x = 1 - 3\cos x$	**M1**	
	Collects terms e.g. $3\cos^2 x - \cos x - 2 = 0$	**A1**	
	Factorises the left-hand side or solves: $(3\cos x + 2)(\cos x - 1) = 0$	**M1**	
	$x = 2.3[0]$, $x = 3.98$ and no extras	**A2**	not from wrong working **A1** for either; not from wrong working
9(a)	[area sector =] $2 \times \frac{1}{2}a^2\phi$ or $\frac{1}{2}a^2(2\phi)$ oe	**B1**	or [area kite =] $2a^2\phi$ or [area OPT =] $a^2\phi$ nfww
	[shaded area =] $\left[2 \times \frac{1}{2} \times\right]\frac{1}{2}a(a\tan\phi)$ oe or $a(a\tan\phi) - \frac{1}{2}a^2(2\phi)$ oe soi	**B1**	or $[a^2\phi = \frac{1}{2}a \times PT \therefore]$ $PT = 2a\phi$ **<u>and</u>** $PT = a\tan\phi$ oe, nfww
	Correct equation using correct areas e.g. $a^2\phi = \frac{1}{2}a(a\tan\phi)$ or $a(a\tan\phi) - a^2\phi = a^2\phi$ soi	**M1**	or equates expressions for PT
	Correct completion to given equation $\tan\phi = 2\phi$	**A1**	
	Alternative method [$\frac{1}{2}$ area sector =] $\frac{1}{2}a^2\phi$	**(B1)**	
	[$\frac{1}{2}$ shaded area =] $\frac{1}{2} \times \frac{1}{2}a(a\tan\phi)$ oe or $\frac{1}{2}a(a\tan\phi) - \frac{1}{2}a^2\phi$ oe soi	**(B1)**	
	Correct equation using correct areas e.g. $\frac{1}{2}a^2\phi = \frac{1}{4}a(a\tan\phi)$ or $\frac{1}{2}a^2\tan\phi - \frac{1}{2}a^2\phi = \frac{1}{2}a^2\phi$ soi	**(M1)**	
	Correct completion to given equation $\tan\phi = 2\phi$	**(A1)**	

Question	Answer	Marks	Partial Marks
9(b)	$2a + a(2\phi) = \frac{1}{2}(2a\tan\phi + a(2\phi))$ oe or $a\tan\phi = 2a + a\phi$	M2	**M1** for arc length = $2a\phi$ soi or for $PT = a\tan\phi$ **and** $PT = 2a + a\phi$
	$\tan\phi = 2 + \phi$	A1	
10(a)(i)	$ar = 8$ and $ar^2 + ar^3 = 160$ soi	B2	**B1** for each
	Correct unsimplified quadratic equation in r e.g. $\left[a = \frac{8}{r} \text{ and so}\right] \frac{8}{r} \times r^2 + \frac{8}{r} \times r^3 = 160$ oe	M1	
	Correct simplification to $r^2 + r - 20 = 0$	A1	
10(a)(ii)	Factorises or solves: $(r + 5)(r - 4) = 0$	M1	
	$r = 4$	A1	
	$a = 2$	A1	
	Alternative method $5a^2 - 2a - 16 = 0$ oe	(B1)	
	Factorises or solves *their* 3-term quadratic in a	(M1)	
	$a = 2$	(A1)	
10(b)	$p + 2(q - 1) = 14$ oe	B1	
	$\frac{q}{2}\{2p + 4(q - 1)\} = 168$ oe	B1	
	Eliminates p or q e.g. $\frac{q}{2}\{2(14 - 2(q - 1)) + 4(q - 1)\} = 168$ OR simplifies S_q : $q\{p + 2(q - 1)\} = 168$ and writes $q(14) = 168$	M1	condone one error in rearrangement of either equation before substituting
	Correctly solves *their* equation for *their* p or *their* q	M1	**dep** on previous **M1**
	$q = 12$ and $p = -8$	A2	**A1** for $q = 12$ or $p = -8$

Question	Answer	Marks	Partial Marks
11	Full, complete and actioned method to find the first area: A or $\frac{1}{2}A$ or B or $\frac{1}{2}B$	5	**B1 for** $[\mathrm{F}(x)=]\int(1+\cos x)\mathrm{d}x = x+\sin x$ oe **B1** for the area of an appropriate rectangle or rectangles soi **M1** for correct use of correct limits to find an appropriate area under the curve **A1** for the accurate area under the curve **B1** for area A or $\frac{1}{2}A$ or B or $\frac{1}{2}B$ OR **M1** for attempting to integrate sin$x$ or ±cos$x$ **M1** dep for using correct limits **A1** for a correctly integrated expression with correct limits **M1** for correct use of correct limits **A1** for exact value $A = 2$ OR equivalent correct plan
	Full, complete and actioned method to find the second, corresponding area	3	**B1** for Area($A + B$) or $\frac{1}{2}$Area($A + B$) oe **M1** for using Area($A + B$) to find A or B or for using $\frac{1}{2}$Area($A + B$) to find $\frac{1}{2}A$ or $\frac{1}{2}B$ oe **A1** for exact value OR **B1** for $4\pi - \left(2\int_0^{\frac{\pi}{2}}(1+\cos x)\mathrm{d}x + \left(\frac{3\pi}{2}-\frac{\pi}{2}\right)\right)$ oe **M1** for correct use of correct limits **A1** for exact value $B = 2\pi - 2$ OR equivalent correct plan
	$k = \pi - 1$ cao	B1	**dep** on all previous marks; allow $k = \frac{2\pi - 2}{2}$

Question	Answer	Marks	Partial Marks
12	$x^{\frac{1}{2}}+2+x^{-\frac{1}{2}}$	**B2**	**B1** for $(x^{\frac{1}{4}}+x^{-\frac{1}{4}})^2$ or $\dfrac{x+2\sqrt{x}+1}{\sqrt{x}}$ oe seen or for two terms correct in $x^{\frac{1}{2}}+2+x^{-\frac{1}{2}}$
	At least two terms correct in *their* $\left[\dfrac{dy}{dx}=\right]\dfrac{2}{3}x^{\frac{3}{2}}+2x+2x^{\frac{1}{2}}(+c)$	**M1**	**FT** *their* $x^{\frac{1}{2}}+2+x^{-\frac{1}{2}}$ providing at least two terms correct and no extra spurious terms
	$\dfrac{4}{3}=\dfrac{2}{3}\left[1^{\frac{3}{2}}\right]+2[1]+2\left[1^{\frac{1}{2}}\right]+c$	**M1**	**dep** on previous **M1** and having an arbitrary constant; condone one sign or arithmetic slip
	$\displaystyle\int\left(\frac{2}{3}x^{\frac{3}{2}}+2x+2x^{\frac{1}{2}}-\frac{10}{3}\right)dx$ $=\dfrac{4}{15}x^{\frac{5}{2}}+x^2+\dfrac{4}{3}x^{\frac{3}{2}}-\dfrac{10}{3}x+A$	**A1**	**FT** *their* $-\dfrac{10}{3}$
	$-1=\dfrac{4}{15}\left[1^{\frac{5}{2}}\right]+1^{[2]}+\dfrac{4}{3}\left[1^{\frac{3}{2}}\right]-\dfrac{10}{3}[1]+A$	**M1**	**dep** on previous **A1**; condone one sign or arithmetic slip
	$y=\dfrac{4}{15}x^{\frac{5}{2}}+x^2+\dfrac{4}{3}x^{\frac{3}{2}}-\dfrac{10}{3}x-\dfrac{4}{15}$ oe	**A1**	