



Cambridge O Level

ADDITIONAL MATHEMATICS**4037/22**

Paper 2

October/November 2022**MARK SCHEME**Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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This document consists of **14** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$3y - (-5y - 4)y = 6$ oe or $3\left(\frac{-4-x}{5}\right) - x\left(\frac{-4-x}{5}\right) = 6$ or $x + 5\left(\frac{6}{3-x}\right) = -4$ oe	M1	
	$5y^2 + 7y - 6 = 0$ or $x^2 + x - 42 = 0$	A1	
	Factorises <i>their</i> 3-term quadratic expression or solves <i>their</i> 3-term quadratic equation, e.g. $(5y - 3)(y + 2) [= 0]$ or $(x - 6)(x + 7) [= 0]$	M1	
	$x = 6, y = -2$ $x = -7, y = 0.6$	A2	A1 for either $x = 6, x = -7$ or $y = -2, y = 0.6$ or for an x, y pair from a correct factorisation or correct solving of a correct equation. The method of solution must be seen in this case.
2	$e^{2x-3-(5-x)} = \frac{7}{4}$ oe or $e^{5-x-(2x-3)} = \frac{4}{7}$ oe	M1	
	$e^{3x-8} = \frac{7}{4}$ oe, soi or $e^{8-3x} = \frac{4}{7}$ oe, soi	A1	
	$3x - 8 = \ln \frac{7}{4}$ oe or $8 - 3x = \ln \frac{4}{7}$ oe	M1	FT expression of similar structure which has at most one sign or arithmetic slip
	$x = \frac{\ln 1.75 + 8}{3}$ oe or $x = \frac{8 - \ln \frac{4}{7}}{3}$ oe or 2.85[32...] rot to 3 or more sf	A1	

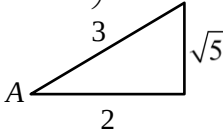
Question	Answer	Marks	Guidance
2	Alternative method		
	$\ln 4 + \ln e^{2x-3} = \ln 7 + \ln e^{5-x}$ oe, soi	(M1)	
	$\ln 4 + 2x - 3 = \ln 7 + 5 - x$ oe	(A1)	
	$2x + x = \ln 7 + 5 - \ln 4 + 3$ or $3x - 8 = \ln \frac{7}{4}$ oe or $8 - 3x = \ln \frac{4}{7}$ oe	(M1)	FT expression of similar structure which has at most one sign or arithmetic slip
	$x = \frac{\ln 7 - \ln 4 + 8}{3}$ oe or 2.85[32...] rot to 3 or more sf	(A1)	
3	$[m_{\text{tangent}} =] -ax^{-2} + 3$ oe	B1	
	[When $x = 1$, $m_{\text{normal}} =$] $\frac{-1}{3-a}$ oe or gradient of tangent = 4 soi	B1	FT $\frac{-1}{\text{their } \frac{dy}{dx} _{x=1}}$ if appropriate
	$\frac{-1}{\text{their}(3-a)} = -\frac{1}{4}$ oe or $\text{their}(3-a) = 4$ oe	M1	FT $\frac{-1}{\text{their } \frac{dy}{dx} _{x=1}}$ or $\text{their } \frac{dy}{dx} _{x=1}$ and their evaluation of $\frac{-1}{-\frac{1}{4}}$
	$a = -1$ nfw	A1	
	[When $x = 1$] $\text{their } 0 = -\frac{1}{4}[1] + b$ oe	M1	FT $y = (\text{their } a) + 1$ providing at least 2 of the first 3 marks awarded
	$b = \frac{1}{4}$ nfw	A1	

Question	Answer	Marks	Guidance
4	$\log_3\left(\frac{11x-8}{x^2}\right)=1$ or $\log_3(11x-8)=\log_3(3x^2)$ soi OR $\log_x\left(\frac{11x-8}{3}\right)=2$ or $\log_x(11x-8)=\log_x(3x^2)$ soi	M2	M1 for correct use change of base in a correct equation so that all logs have consistent base: $\log_x 3 = \frac{\log_3 3}{\log_3 x}$ or $\log_x 3 = \frac{1}{\log_3 x}$ oe, soi OR $\log_3(11x-8) = \frac{\log_x(11x-8)}{\log_x 3}$ oe, soi
	$3x^2 - 11x + 8 [= 0]$ oe, nfww	A1	
	$(3x-8)(x-1) [= 0]$	M1	FT <i>their</i> 3-term quadratic dep on at least M1 previously awarded
	$x = \frac{8}{3}$ or 2.67 or 2.666[6...] rot to 3 or more dp as only solution	A1	
5	$6x^3 - 5x^2 - 13x + 12 [= 0]$	B1	
	Uses the correct factor $x-1$ to find a quadratic factor with at least 2 terms correct	M1	
	$6x^2 + x - 12$	A1	
	Factorises or solves <i>their</i> 3-term quadratic: $(2x+3)(3x-4) [= 0]$ or $[x =] \frac{-1 \pm \sqrt{1-4(6)(-12)}}{2(6)}$ oe	M1	dep on previous M1
	$x = 1, -1.5, \frac{4}{3}$ nfww	A1	dep on all previous marks awarded

Question	Answer	Marks	Guidance
6(a)	510	3	M2 for a fully correct method e.g. [starts with 5, 7, 9 and ends in 3 or two of 5, 7, 9] $3 \times 6 \times 5 \times 3$ or 270 and [starts with 6, 8 and ends in 3, 5, 7, 9] $2 \times 6 \times 5 \times 4$ or 240 OR [ends with 3 and starts with 5, 6, 7, 8, 9] $5 \times 6 \times 5 \times 1$ or 150 and [ends with 5, 7, 9 and starts with 6, 8 or two of 5, 7, 9] $4 \times 6 \times 5 \times 3$ or 360 or M1 for a partially correct method equivalent to one of the above two steps
6(b)	540	3	M2 for a fully correct method e.g. [starts with 5, 7 and ends with 2, 3 and 5 or 7] $2 \times 6 \times 5 \times 3 = 180$ and [starts with 6, 8, 9 and ends with 2, 3, 5, 7] $3 \times 6 \times 5 \times 4 = 360$ OR [ends with 2, 3 and starts with 5, 6, 7, 8, 9] $5 \times 6 \times 5 \times 2$ or 300 and [ends with 5, 7 and starts with 6, 8, 9 and 5 or 7] $2 \times 6 \times 5 \times 4$ or 240 or M1 for a partially correct method equivalent to one of the above two steps

Question	Answer	Marks	Guidance
7(a)	$\frac{\sin^2 x + (1 - \cos x)^2}{(1 - \cos x) \sin x}$ or $\frac{\sin^2 x}{(1 - \cos x) \sin x} + \frac{(1 - \cos x)^2}{(1 - \cos x) \sin x}$	M1	
	$\frac{\sin^2 x + 1 - 2 \cos x + \cos^2 x}{(1 - \cos x) \sin x}$	A1	OR $\frac{1 - \cos^2 x + (1 - \cos x)^2}{(1 - \cos x) \sin x}$
	$\frac{1 + 1 - 2 \cos x}{(1 - \cos x) \sin x}$ or $\frac{1 - \cos^2 x + 1 - 2 \cos x + \cos^2 x}{(1 - \cos x) \sin x}$	A1	OR $\frac{(1 - \cos x)(1 + \cos x) + (1 - \cos x)^2}{(1 - \cos x) \sin x}$
	Fully correct justification of given answer: $\frac{2(1 - \cos x)}{(1 - \cos x) \sin x} = 2 \operatorname{cosec} x$ or $\frac{2 - 2 \cos x}{(1 - \cos x) \sin x} = \frac{2}{\sin x} = 2 \operatorname{cosec} x$ or equivalent	A1	All steps correct and final step justified OR $\frac{1 + \cos x + 1 - \cos x}{\sin x} = 2 \operatorname{cosec} x$
	Alternative		
	$\frac{\sin x(1 + \cos x)}{(1 - \cos x)(1 + \cos x)} + \frac{(1 - \cos x) \sin x}{\sin x \sin x}$ or $\frac{\sin x(1 + \cos x)}{1 - \cos^2 x} + \frac{(1 - \cos x) \sin x}{\sin^2 x}$	(M1)	
	$\frac{\sin x + \sin x \cos x}{\sin^2 x} + \frac{\sin x - \cos x \sin x}{\sin^2 x}$	(A1)	
	$\frac{2 \sin x}{\sin^2 x}$	(A1)	
	Fully correct justification of given answer: $\frac{2}{\sin x} = 2 \operatorname{cosec} x$	(A1)	All steps correct and final step justified

Question	Answer	Marks	Guidance
7(b)	$3\sin^2 x - \sin x - 2 \quad [= 0] \text{ soi}$	B1	
	$(3\sin x + 2)(\sin x - 1) [= 0] \text{ oe}$	M1	
	$\sin x = -\frac{2}{3}, \sin x = 1$	A1	
	90 221.8 or 221.81[03...] rot to 2 or more dp 318.2 or 318.18[96...] rot to 2 or more dp	A1	and no extras in range If B1 M1 A0 A0 allow SC1 for 221.8 or 221.81[03...] rot to 2 or more dp and 318.2 or 318.18[96...] rot to 2 or more dp and no extras in range
8(a)	$2\pi rh + 2\pi r^2 + \pi r^2 [= 300] \text{ oe}$	M1	
	$h = \frac{300 - 3\pi r^2}{2\pi r} \text{ oe, isw}$	A1	
8(b)	$V = \pi r^2 \left(\text{their } \frac{300 - 3\pi r^2}{2\pi r} \right) + \frac{2}{3} \pi r^3 \text{ oe}$	M2	FT <i>their</i> h providing in terms of r and derived from a dimensionally correct equation in (a) ; M1 for $V = \pi r^2 \left(\text{their } \frac{300 - 3\pi r^2}{2\pi r} \right) + k\pi r^3,$ $k \neq \frac{2}{3} \text{ oe}$
	Correct completion to given answer: $150r - \frac{5}{6}\pi r^3 \text{ nfw}$	A1	
8(c)	Derivative of V : $150 - \frac{5}{2}\pi r^2 \text{ oe, soi, isw}$	B1	
	$\text{their } \left(150 - \frac{5}{2}\pi r^2 \right) = 0 \text{ and solves as far as } r = \dots$	M1	FT <i>their</i> $\frac{dV}{dr}$ providing that at least one term is correct
	$r = \sqrt{\frac{300}{5\pi}} \text{ oe or } 4.37 \text{ (cm)}$	A1	
	$150(\text{their } 4.37) - \frac{5}{6}\pi(\text{their } 4.37)^3$	M1	FT <i>their</i> 4.37
	437 or awrt 437 (cm ³) isw	A1	

Question	Answer	Marks	Guidance
9(a)	$\frac{1}{2}(\sqrt{5}-1)(\sqrt{5}+1)\sin A = \frac{2\sqrt{5}}{3}$	M1	OR $\frac{1}{2} \times (\sqrt{5}+1) \times \text{height} = \frac{2\sqrt{5}}{3}$ and $\sin A = (\text{their height}) \div (\sqrt{5}-1)$
	Simplifies to $\frac{1}{2} \times (5-1) \times \sin A = \frac{2\sqrt{5}}{3}$ oe	A1	OR $\sin A = \frac{4\sqrt{5}}{3(\sqrt{5}+1)} \div (\sqrt{5}-1)$ or $\sin A = \frac{5-\sqrt{5}}{3} \div (\sqrt{5}-1)$
	$\frac{\sqrt{5}}{3}$ isw	A1	
9(b)	$\cos A = \frac{2}{3}$ or exact equivalent	B2	B1 for $\cos A = \sqrt{1 - \text{their} \left(\frac{\sqrt{5}}{3}\right)^2}$ or for $\cos A = \cos \left(\sin^{-1} \frac{\sqrt{5}}{3}\right)$ or for a sketch 
	$(\sqrt{5}-1)^2 + (\sqrt{5}+1)^2 - 2(\sqrt{5}-1)(\sqrt{5}+1) \times \cos A$ soi	M1	
	$(\sqrt{5}-1)^2 + (\sqrt{5}+1)^2 - 2(\sqrt{5}-1)(\sqrt{5}+1) \times \frac{2}{3}$ soi	A1	
	$x = \sqrt{\frac{20}{3}}$ or $\frac{2}{3}\sqrt{15}$ or $2\sqrt{\frac{5}{3}}$ oe, isw	A1	

Question	Answer	Marks	Guidance
9(c)	$\frac{\text{their } x}{\text{their } \sin A} = \frac{\sqrt{5}+1}{\sin B}$ oe, soi or $\frac{1}{2} \times (\sqrt{5}-1) \times \text{their } x \times \sin B = \frac{2\sqrt{5}}{3}$ oe, soi	M1	FT <i>their x</i> and <i>their sinA</i>
	Correct expression $\frac{\sqrt{\frac{20}{3}}}{\frac{\sqrt{5}}{3}} = \frac{\sqrt{5}+1}{\sin B}$ oe or $\frac{1}{2} \times (\sqrt{5}-1) \times \sqrt{\frac{20}{3}} \times \sin B = \frac{2\sqrt{5}}{3}$ oe	A1	
	$\frac{\sqrt{15}+\sqrt{3}}{6}$ or $\frac{\sqrt{3}}{6}(\sqrt{5}+1)$ or $\frac{\sqrt{5}+1}{2\sqrt{3}}$ oe, isw	A1	
10(a)	$ar^2 = 4.5$ and $ar^5 = 15.1875$ soi	B1	
	Correctly eliminates one unknown using correct equations e.g $\left(\frac{4.5}{r^2}\right)r^5 = 15.1875$ or $\sqrt{\frac{4.5}{a}} = \sqrt[5]{\frac{15.1875}{a}}$ oe, soi	M1	
	$r = 1.5, a = 2$	A2	A1 for either
10(b)	$S_{15} = \frac{2(1-1.5^{15})}{1-1.5}$ and $S_{25} = \frac{2(1-1.5^{25})}{1-1.5}$ oe	B2	M1 FT <i>their a</i> and <i>r</i> for $S_{15} = \frac{2(1-1.5^{15})}{1-1.5}$ or $S_{25} = \frac{2(1-1.5^{25})}{1-1.5}$ oe
	Correct plan: $S_{25} - S_{15}$ oe attempted	M1	FT <i>their a</i> and <i>r</i>
	99 253	A1	

Question	Answer	Marks	Guidance
10(b)	Alternative 1		
	first term = $(their2)(their1.5)^{15}$ and an attempt at S_{10}	(M1)	FT <i>their a</i> and <i>r</i>
	Correct sum $S_{10} = \frac{875.7...(1-1.5^{10})}{1-1.5}$ oe	(B2)	M1 FT <i>their</i> first term and <i>r</i> for $S_{10} = \frac{their875.7...(1-their1.5^{10})}{1-their1.5}$
	99 253	(A1)	
	Alternative 2		
	Correct sum: $2(1.5)^{15} + 2(1.5)^{16} + 2(1.5)^{17} + 2(1.5)^{18} + 2(1.5)^{19} + 2(1.5)^{20} + 2(1.5)^{21} + 2(1.5)^{22} + 2(1.5)^{23} + 2(1.5)^{24}$ oe or $2(1.5)^{15}\{1 + 1.5 + (1.5)^2 + (1.5)^3 + (1.5)^4 + (1.5)^5 + (1.5)^6 + (1.5)^7 + (1.5)^8 + (1.5)^9\}$	(M3)	M2 FT <i>their a</i> and <i>r</i> for sum starting with $2(1.5)^{15}$ and ending with $2(1.5)^{24}$ with at most one omission or error or M1 FT <i>their a</i> and <i>r</i> for sum starting with $2(1.5)^{15}$ or ending with $2(1.5)^{24}$ with at most two omissions or errors
	99 253	(A1)	
11(a)	(-2, 2)	B2	B1 for one correct coordinate nfw M1 for $\overrightarrow{AC} = \frac{1}{3} \begin{pmatrix} 9 \\ -12 \end{pmatrix}$ or $\overrightarrow{CA} = \frac{1}{3} \begin{pmatrix} -9 \\ 12 \end{pmatrix}$ for $x = -5 + 3$ or $x = 4 - 6$ or for $y = 6 - 4$ or $y = -6 + 8$ or for $x + 5 = \frac{4-x}{2}$ or $6 - y = \frac{y+6}{2}$ or for $2 \left(\overrightarrow{OC} - \begin{pmatrix} -5 \\ 6 \end{pmatrix} \right) = \begin{pmatrix} 4 \\ -6 \end{pmatrix} - \overrightarrow{OC}$ oe

Question	Answer	Marks	Guidance
11(b)	$m_{AB} = \frac{-6-6}{4-(-5)}$ oe or $-\frac{12}{9}$ or $-\frac{4}{3}$	B1	
	$m_{CD} = \frac{3}{4}$	M1	FT $\frac{-1}{\text{their } m_{AB}}$
	$y - 2 = \frac{3}{4}(x + 2)$ oe or $y = \frac{3}{4}x + c$ and $2 = \left(\frac{3}{4}\right)(-2) + c$ oe soi	M1	FT <i>their</i> $(-2, 2)$ and $\frac{-1}{\text{their } m_{AB}}$
	$y = \frac{3}{4}x + \frac{7}{2}$ or equivalent in form $y = mx + c$	A1	
11(c)	$(x-4)^2 + (y+6)^2 = 125$ oe, soi	B1	
	Uses <i>their</i> $y = \frac{3}{4}x + \frac{7}{2}$ to eliminate one unknown	M1	if correct implies B1
	Correct equation in one unknown $[BD^2 =](x-4)^2 + \left(\frac{3}{4}x + \frac{7}{2} + 6\right)^2 = 125$ oe	A1	
	Writes in solvable form: $25x^2 + 100x - 300 = 0$ oe	A1	
	Factorises or solves a correct 3-term quadratic	A1	
	$(2, 5)$ and $(-6, -1)$	A1	If B1 , M0 award: SC2 for identifying one correct point by inspection from the length equation and testing it in the correct equation of <i>CD</i> and SC2 for identifying the second correct point by inspection from the length equation and testing it in the correct equation of <i>CD</i>

Question	Answer	Marks	Guidance
11(c)	Alternative		
	$[BC =]$ $\sqrt{(4 - (\text{their} - 2))^2 + (-6 - (\text{their} 2))^2}$	(B1)	FT their C
	$CD = \sqrt{125 - \text{their} 100}$	(M1)	FT their BC^2 providing $125 - \text{their } 100 > 0$
	$CD = 5$	(A1)	
	$\begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ or $\begin{pmatrix} -4 \\ -3 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ OR finds $25x^2 + 100x - 300 = 0$ oe	(A2)	A1 for $\overrightarrow{CD_1} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ or $\overrightarrow{CD_2} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}$ soi OR A1 for finds $(x + 2)^2 + \left(\frac{3}{4}x + \frac{7}{2} - 2\right)^2 = 25$
	$(2, 5)$ and $(-6, -1)$	(A1)	