

# MATHEMATICS

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Paper 4024/11

Paper 11

## Key messages

To succeed in this paper, candidates need to be familiar with the whole syllabus, remember necessary formulae and show all their working.

Candidates should clearly show their answers in the appropriate answer space.

Accuracy in numerical work is essential.

## General comments

The paper gave opportunities and challenges for candidates to demonstrate their abilities.

Time did not appear to be an issue for candidates.

Candidates are reminded of the need for clarity. In some scripts, individual letters and figures were unclear and ambiguous.

Errors in numerical work hindered some candidates.

Candidates of all abilities scored well in **Questions 2(a) and (b), 5(a), 8(b), 10(a), 11(a), 13(b)(ii), 16(a), 17(a)(i) and (a)(ii), and 22(a).**

The questions with which candidates had most difficulty were **3(b), 4, 11(c), 12(b), 15(b) and (c), 19(b), 22(c) and 25(c)**

## Comments on specific questions

### **Question 1**

- (a) A good proportion of correct answers were seen. Frequently, however, candidates simply added one line to the diagram. This produced a pattern with just one line of symmetry, but not the line *AB*, as asked for in the question.
- (b) Again, a good proportion of correct answers were seen. Some candidates were still using the idea of one line of symmetry from part (a). A common error was to shade the top triangle.

### **Question 2**

- (a) Generally accurate work was seen with clearly set out working. This often included the insertion of correct brackets. A common error was adding the 5 and 1 to start with, leading to the answer 1.8. The second step of  $5 + 0.3$  sometimes led to 5.03.

- (b) Again, some accurate work was seen backed up with clearly set out working. The expected division of decimals was seen. Evaluation by means of conversion to fractions seemed equally popular. Candidates giving the answer 9 with no working may have been better advised to show some working.

Answers: (a) 5.3 (b) 90

### Question 3

- (a) Perimeter was well understood with accurate numerical work seen. Calculations were clearly set out, often with the correct use of brackets.
- (b) Many candidates used base  $\times$  height directly to obtain the correct answer. Errors here were the result of wrong ideas rather than inaccurate arithmetic. Weaker responses tended to use all three numbers given on the diagram. There were a number of speculative formulae based on these, such as  $\frac{1}{2}(a + b)h$ ,  $2(a + b)h$  and other combinations of  $a$ ,  $b$  and  $h$ .

Answers: (a) 29.2 (b) 38.7

### Question 4

Not many clear cut solutions using the expected inequality based on Pythagoras were seen.

There were a few solutions using the cosine rule accurately, reaching such as  $\cos \hat{P}QR = -0.1$  with a correct deduction following. It was clear that some candidates were unaware of the significance of a negative cosine, and were just as likely to select acute as their conclusion.

The wording of the question ruled out solutions by scale drawing.

Answer: obtuse angled

### Question 5

- (a) Much good work was seen with each step carefully set out. Transposition of inequalities always needs care, particularly if a step means changing the inequality sign. Some solutions gave  $y > 5$ , having lost the 'equal to' part.
- (b) A good number of candidates split the inequality into two separate parts and were able to obtain the correct set of integers. A number merely simplified the given inequality and left  $-2 \leq x \leq 2$  as their answer. In some cases,  $-4$  and  $-3$  were included, 0 omitted, and even  $-\frac{3}{2}$ ,  $-\frac{1}{2}$  included.

Answers: (a)  $\geq 5$  (b)  $-2, -1, 0, 1$

### Question 6

- (a) This part was generally well answered with the idea of ratio understood. The numbers 3, 4 and 5 led some candidates to believe they were dealing with a right-angled triangle. Attempts to use the sine rule or cosine rule were then seen.
- (b) The response to this question was fairly good. Many candidates expressed their ideas successfully in algebraic form. Correct equations were seen and solved. At this point, a good number of candidates gave their final answer as 12, not realising that this was the number of boys in the class.

Answers: (a)  $45^\circ$  (b) 27

### Question 7

Many found this question demanding. Some correctly used and applied the formula for arc length. Others thought that the formula for the area of a circle was required. As always, care is needed with the accuracy of numerical work. A large number of candidates omitted this question.

Answers:  $a = 10.05$   $b = 14 / 3$

### Question 8

- (a) The procedure required was understood and successfully used by many candidates. Attempting calculations without rounding first is unlikely ever to be successful for problems of this nature. Failure to obey the instruction 'by writing each number to one significant figure' resulted in long, complicated arithmetic for which no credit could be given. It is important to read the question carefully. Some candidates added, rather than multiplied, the numbers in the denominator.
- (b) There were many correct answers to this part. The conversion was achieved using careful long division as well as using equivalent fractions.

Answers: (a) 8 (b) 0.32

### Question 9

This question was answered quite well. There was some careful work shown, moving clearly from step to step, to a successful conclusion. Care is needed in dealing with plus and minus signs when transposing terms in an equation. Some candidates spoil their work with false cancelling at the end.

Answer:  $\frac{3y+4}{y+1}$

### Question 10

- (a) Mostly correct. Occasionally +4 or 14 were given as answers.
- (b)(i) Often correct, but the variety of wrong answers seen suggested that these candidates were not familiar with tables of this nature. 0914 was the most common wrong answer. Sometimes the gaps in the table were filled in, and one of these times given as the answer.
- (ii) The required subtraction was usually performed correctly. A common error was to include 1 hour from 10 – 09.

Answers: (a) –4 (b)(i) 0818 (ii) 33

### Question 11

- (a) There were many correct answers to this part. Sometimes the three figure bearing was unnecessarily embellished with South.
- (b) Most candidates measured the required angle accurately. A common error could have been measuring  $\hat{BAC}$  as  $45^\circ$ , because the answer  $225^\circ$  was sometimes given. There was also some confusion of ideas, with answers such as  $45^\circ$  and even 6 cm seen.
- (c) A good number of correct answers seen. Sometimes  $105^\circ$  was wrongly subtracted from  $360^\circ$

Answers: (a)  $180^\circ$  (b)  $220^\circ$  (c)  $285^\circ$

**Question 12**

- (a) This question was answered quite well. The expression was often left in its unsimplified form. A number of candidates quoted  $a + (n - 1)d$ . Some continued by explaining what the letters  $a$  and  $d$  represent, and giving their values. In some cases, there was no further work and this expression was given as the answer.
- (b) This proved demanding for most candidates and was often omitted. Many did not appear to understand the notation used in the question. Correct answers were infrequent.

Answers: (a)  $4n + 3$  (b) 5 and 29

**Question 13**

- (a) Generally good responses were seen. A common misunderstanding led to the equation  $25p = 250$ , followed by the answer  $p = 10$ .
- (b)(i) Mostly understood. A frequent incorrect answer was  $\frac{x}{5}$ .
- (ii) This part was well answered with many correct answers seen. Sometimes the correct procedure was used at the outset, leading to the multiplication of fractions, but the subsequent cancellations led to the wrong answer.

Answers: (a) 3 (b)(i)  $x^5$  (ii)  $\frac{2}{3a}$

**Question 14**

- (a)(i) Generally well done. Some gave 1.5 as the answer.
- (ii) A reasonable number of correct answers were seen. The frequency density was sometimes misread as 2.8. Some candidates gave their answer as 2.4, forgetting to multiply by 5.
- (b) There was a reasonable response indicating correct ideas concerning frequency density.

Answers: (a)(i) 15 (ii) 12 (b) column from 50 to 65 with frequency density 1.2.

**Question 15**

- (a) This question was answered quite well. 10 seemed to be the popular correct answer, with 13 also given. Substituting values in  $5r - 1$  and assessing the results also obtained correct answers. Answers such as 2 and 7 showed that not all the conditions given in the question had been considered.
- (b) The wording of the question allowed the value of  $s$  to be in eighths, e.g.  $\frac{1}{8}, \frac{3}{8}, \frac{17}{8}$ , solutions that were achieved by some candidates. The expected value, taking  $s$  to be an integer, 0, was seen on a number of occasions.
- (c) There were few correct answers to this part. Square roots of numbers from 50 to 63 were expected. Many candidates gave a decimal value between 7 and 8.

Answers: (a) e.g. 10 (b) e.g. 0 (c) e.g.  $\sqrt{50}$

**Question 16**

- (a) This was mostly correct, often with reference to alternate angles. A few candidates gave the answer on the diagram, leaving the answer space blank.
- (b) Again, usually correct with working shown.
- (c) A more challenging part with a reasonable response overall. Solutions usually accompanied by working and some explanation of the reasoning involved. The answer 95 was sometimes given instead of  $(180 - 95)$

Answers: (a)  $38^\circ$  (b)  $57^\circ$  (c)  $85^\circ$

**Question 17**

- (a) (i) Many correct answers.
- (ii) There were a good number of correct answers to this part, with clear working fully written out. Some mistakes were seen, leading to an answer such as  $10p - 13q$ , when candidates tried to write down the answer without showing any working.
- (b) Mostly correct. Misconceptions such as  $10xy$  were sometimes seen.

Answers: (a)(i)  $8t + 17$  (ii)  $2p + 13q$  (b)  $5x^2y(5xy - 3)$

**Question 18**

- (a) This question was well understood. The graph was generally read correctly, and the expression for the acceleration was properly formed and evaluated.
- (b) There were a few correct solutions seen but, on the whole, candidates found this part difficult. Appropriate formulae for the area of a triangle, rectangle or trapezium were used in a number of papers. Sometimes errors occurred in the computation of parts of the required area. A common misunderstanding was to give  $60 \times 8 - 60 \times 7.2$  as the required distance. Neither of these products was a relevant area.

Answers: (a) 0.12 (b) Blue boat, 36

**Question 19**

- (a) Most candidates understood that division and not multiplication was required. Standard form was reasonably well known. Candidates who wrote both numbers in full tended to be less successful than those who worked in powers of 10.
- (b) This part caused problems for many. Errors were made in arithmetic, such as  $1.67 \times 2 = 2.34$ . The question was frequently omitted

Answers: (a)  $2 \times 10^{-5}$  (b)  $2.99 \times 10^{-23}$

**Question 20**

- (a) There was a fair response to this question. Candidates realised the link with the method of completing the square, and proceeded accordingly. These usually achieved a complete answer. Sometimes  $(x - a)^2 + b$  was expanded correctly, but generally, candidates comparing terms usually managed to give only the value 7 for a.
- (b) What was being asked for was well understood, and many correct answers were seen. Those candidates who decided to solve the equation by applying the quadratic formula could not be given any credit. The question specifically asked for a solution by factorisation.

Answers: (a)  $a = 7$   $b = -9$  (b)  $\frac{2}{3}$   $-3$

### Question 21

- (a) (i) This part was answered fairly well. It was usually appreciated which x-coordinate and which y-coordinate was 0, but occasionally mixed up.
- (ii) A reasonable number of candidates obtained the correct value for the gradient. A few gave the reciprocal of the required answer.
- (b) This was mostly correct, with a few candidates subtracting the relevant x and y values instead of adding. Others just added the values and omitted to divide each by 2.

Answers: (a)(i) (0, 3) (2, 0) (ii)  $-\frac{3}{2}$  (b) (-1, 9)

### Question 22

- (a) This question was usually attempted and mostly correct. Sometimes an isosceles triangle was drawn.
- (b) (i) A reasonable number of correct perpendicular bisectors of AC were seen. Sometimes an angle bisector was drawn. Some candidates omitted this part.
- (ii) The required construction was generally correct and relevant to the problem. Again, some candidates omitted this part of the question.
- (c) The method of intersecting loci was usually understood by the few candidates getting this far through the question.

### Question 23

- (a) The concept of similar shapes appeared unfamiliar to most candidates. A few were able to reach the answer of 17 in the expected manner. Some attempted to find the radius of the beach ball. Answers of 51 and 459 were seen, using the factor  $\frac{1}{3}$ . The answer of 27 without any working seemed to stem from a speculative  $3^3$ .
- (b) Some very good work was seen in this part of the question with many correct answers. A few candidates used y inversely proportional to x but not its cube.

Answers: (a) 17 (b)  $\frac{72}{125}$

### Question 24

- (a) The tree diagram was often completed correctly.
- (b) (i) Many candidates understood the correct approach to this question and gave the expected answer.
- (ii) A reasonable number of correct responses were seen. Sometimes the correct method was spoilt by incorrect arithmetic. The most common error was to use only half of the complete expression for the required probability.

Answers: (a)  $\frac{3}{9}, \frac{6}{9}, \frac{4}{9}, \frac{5}{9}$  (b)(i)  $\frac{12}{90}$  (ii)  $\frac{48}{90}$

**Question 25**

- (a) While the general ideas of  $2 \times 2$  matrices appeared to be understood, care was needed to get all four elements of the answer correct. Many achieved 3 out of the 4. Some candidates struggled with the subtractions involving negative numbers.
- (b) Matrix multiplication was a more difficult process for quite a number of candidates. Again, the procedure ultimately relies on accurate basic number work.
- (c) The candidates who realised that  $\mathbf{A}^{-1}$  needed to be calculated usually obtained the correct matrix. Others who were working towards the inverse computed the determinant correctly but became mixed up with the order and signs of the elements in the adjoint matrix. A minority of candidates used a longer method based on first principles, but were rarely successful.

Answers: (a)  $\begin{pmatrix} 4 & -6 \\ -6 & 14 \end{pmatrix}$  (b)  $\begin{pmatrix} 11 & -7 \\ -14 & 18 \end{pmatrix}$  (c)  $\frac{1}{10} \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$

# MATHEMATICS

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Paper 4024/12

Paper 12

## Key messages

In showing all working for a question, candidates should be aware that the working space provided is adequate. It is unlikely to be necessary to use additional sheets of rough working paper.

Clarity in presentation is essential. The practice of working initially in pencil and then inking over should be discouraged for this reason.

## General comments

There were many well presented scripts of a good standard.

Candidates showed a good grasp of basic ideas across all areas of the syllabus. Many candidates would improve their marks in some areas by greater attention to accuracy when using directed numbers and performing calculations, particularly when using decimals. Candidates at all levels had difficulty with some aspects of the questions involving timetables and Venn diagrams.

The statistics examined in **Question 24** showed that, whereas the ideas required in parts **(a)** and **(b)** were generally well understood, candidates at all levels had difficulty in interpreting their significance.

## Comments on specific questions.

### Question 1

- (a) Candidates generally set out their working clearly. This question was nearly always attempted and there were a lot of correct answers. The common error was working as though  $12 + 8$  was in brackets.
- (b) Some candidates worked throughout in decimals, obtaining 0.3 correctly. A number of wrong answers, such as 0.03, 0.003 and 30 were seen, where either or both of the numbers in the question were converted to other values not equivalent to the originals such as converting 0.018 to  $\frac{18}{100}$ . Some candidates worked in fractions leading to the correct answer  $\frac{3}{10}$ . Evaluations leading to such as  $\frac{1.8}{6}$  were incomplete, as this quotient is neither a decimal number nor a fraction.

Answers: (a) 14 (b) 0.3

### Question 2

- (a) Choosing  $-7$  and  $2$  usually led to  $9$ , (or  $-9$ , equally acceptable). Some candidates picked these out correctly and gave the answer  $5$ . The lowest temperature was thought to be  $-6$  by some candidates, and some gave the difference between the first and last numbers in the unordered list. Some candidates did not notice the (positive)  $2$  so used  $0$  as the highest value.

- (b) Successful candidates ordered the data carefully. Occasionally there was doubt as to which value to choose,  $-3$ ,  $-2$ , or  $-2.5$ . After evaluating  $\frac{-3-2}{2} = -2.5$ , it was important not to drop the minus sign. The answer  $-4$  was seen, coming from the unordered data as given in the question. After ordering the data correctly, some candidates proceeded no further with this question. Some candidates worked out the mean.

Answers: (a) 9 (b)  $-2.5$

### Question 3

- (a) Most candidates understood what the question was asking, and gave a straightforward answer such as 0.8, 0.76 or 0.85. Occasionally, a correct fraction such as  $\frac{4}{5}$  was seen in this part leading to the incorrect decimal 1.25. Sometimes the question was misunderstood. The fraction  $\frac{7}{8} - \frac{3}{4}$  was evaluated and converted to a decimal. Some candidates stopped after converting both  $\frac{3}{4}$  and  $\frac{7}{8}$  to decimals. This and the next part of the question were sometimes omitted.
- (b) The transition from part (a) was usually managed successfully. Common correct answers were  $\frac{8}{10}$ ,  $\frac{4}{5}$ ,  $\frac{5}{6}$ ,  $\frac{6}{7}$  and  $\frac{13}{16}$ . In many cases, a correct decimal seen in part (a) was converted to a fraction. Final fractions had to be clear of decimals; quotients such as  $\frac{6.5}{8}$  were not creditworthy.

### Question 4

- (a) A lot of correct answers were seen. There were sometimes mistakes made when subtracting the appropriate times. When done formally using carrying figures, answers such as 1 hour 47 minutes were seen. The times were also treated as 4 digit numbers, leading to the answer 87 minutes. Candidates thinking of the time from 09 56 to 10 43 fared better. Some candidates checked more than just the first column of the timetable just to be sure. This question was well understood. The solution given where all the times in the first column were added together was the exception.
- (b) As well as the correct answer, there were a variety of other answers given that suggested that timetables of this nature were unfamiliar to some candidates. Such answers were 10 56, 11 33 and 12 03. The answer 11 25 was also seen. This came from working out the time taken from the City Hall to the Airport, 40 minutes, and subtracting it from 14 05 – 2 = 12 05.

Answers: (a) 47 (b) 11 03

### Question 5

- (a) The correct answer was often seen. The incorrect  $8.52 \times 10^5$  was a common wrong answer. Some misunderstanding of standard form was apparent, with answers such as  $85.2 \times 10^{-6}$  seen, and answers containing 852. Some candidates dropped the digit 2 in the working as well as in the final answer.
- (b) There were some clearly set out calculations leading to the correct answer. A number of candidates left  $0.5 \times 10^7$  as their final answer. This was sometimes incorrectly adjusted to  $5 \times 10^{-8}$ . An answer of  $0.5 \times 10^3$  was also seen. Some candidates did not reach 0.5 or 5, giving answers such as  $2 \times 10^3$  and  $2 \times 10^6$ . Getting both the correct  $a$  and the correct  $n$  in the standard form  $a \times 10^n$  was clearly a challenge for many candidates.

Answers: (a)  $8.52 \times 10^{-5}$  (b)  $5 \times 10^6$

### Question 6

- (a) Solutions with both numbers correct were the exception. Order of rotational symmetry 0 was common. Order of rotational symmetry 6, seemingly ignoring the shading in the diagram was also a common error. 3 or 6 lines of symmetry were frequently stated.
- (b) A number of correct solutions, as well as completing the shading, showed in addition the 2 lines of symmetry of the completed pattern. This question was usually attempted. A common error was a completed pattern having only one line of symmetry.

Answers: (a) Rotational symmetry of order 3 and 0 lines of symmetry



### Question 7

Most correct answers came from working that showed a confident grasp of the relevant algebraic ideas. A correct formula was established, followed by accurate substitution and evaluation. The type of proportion required was sometimes not understood, with the common error being to use direct proportion. As usual, the arithmetic that occurs in problems of this nature led to a loss of marks for some candidates. The constant  $\frac{24}{1600}$  was not always cancelled correctly. Converting  $\frac{3}{200}$  to a decimal was not always accurate, with such as 0.15 and 0.03 seen here.

Answer: 54

### Question 8

- (a) Usually correct. Equilateral or scalene were sometimes stated. Other shapes, such as rectangle and kite, were also given as answers. The answer  $\hat{A}TB = 64^\circ$  was seen.
- (b) Mostly correct, with working that showed correct deductions from the information given. When attempted via  $\hat{A}TB$ , the incorrect conclusion  $2 \times 52^\circ$  usually followed. Another common wrong answer was  $180 - 64 = 116$ . Some candidates stopped after writing  $90 - 64 = 26$ .

Answers: (a) Isosceles (b) 128

### Question 9

- (a) The addition of fractions was usually correctly handled and clearly set out, with both fractions converted to 28ths. Care has to be taken:  $4 + 21$  did not always come to 25 in answers seen. A minority of candidates worked in decimals.
- (b) Generally, the correct procedure was clearly set out, starting with the conversions to  $\frac{16}{3}$  and  $\frac{8}{5}$  followed by the appropriate change to  $\times$ . After a correct change, the numbers 24 and 80 were sometimes seen.  $\frac{3}{16} \times \frac{8}{5}$  was seen a few times, also giving rise to 24 and 80. There were many correct final answers giving the mixed number in its lowest terms. There was some uncertainty as to the required form of the final answer. Some gave  $\frac{10}{3}$  or 3.33. Some gave a choice between  $3\frac{1}{3}$  and  $\frac{10}{3}$ . 3.33 is a decimal number rather than a mixed number so could not gain full marks.

Some candidates had difficulty turning  $\frac{10}{3}$  into a mixed number, with answers such as  $1\frac{2}{3}$ ,  $1\frac{3}{1}$  and  $3\frac{2}{3}$  seen.

Answers: (a)  $\frac{25}{28}$  (b)  $3\frac{1}{3}$

### Question 10

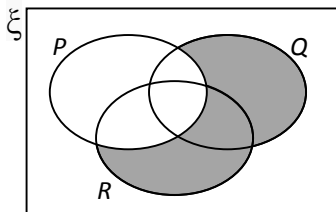
- (a) A good number of correct answers were seen, often with all the digits written out rather than in standard form. A common wrong answer was 405 000 000. 3 digit answers such as 406 and 4.06 were also common. Occasionally there was a choice between 406 and 406 000 000.
- (b) Usually candidates carried out the instruction in the question and attempted to write each number to 1 significant figure before attempting any calculation. Calculations before rounding were never successful. Common errors were the use of 9 instead of 10, and the use of 1 instead of 0.8. 4 and 41 were also seen instead of 40. Thus a common wrong answer was 4, coming from  $\frac{40}{10 \times 1}$ .

Answers: (a) 406 000 000 (b) 5

### Question 11

- (a) Usually attempted. Correct solutions were seen along with a variety of others. Candidates should shade firmly so that their intention is clear. In some cases, too many areas were shaded. If different shadings were seen, they were never explained, so the total area shaded was taken to be the answer.
- (b) Generally, successful candidates used a Venn diagram and their supporting working used an algebraic approach. Occasionally, the problem was solved without the use of a Venn diagram. Common wrong answers were 17 and 7. Some solutions made little progress beyond an attempted diagram with some entries.

Answers: (a)



(b) 12

### Question 12

- (a) There were many correct answers but also some poor responses to this question. Usually, candidates were aware that 6 products had to be evaluated. The answer was sometimes left as a matrix with two rows of 3 elements. Quite a number of candidates going further made errors in evaluating the products. Errors such as  $80 \times 0.8 = 6.4$  and  $40 \times 1.2 = 480$  were common.
- (b) This was usually attempted with many clear, accurate, explanations given. The main difficulty for some candidates was their inability to distinguish between the numbers of hot drinks sold and the revenue obtained. Some explanations made no reference to the days involved.

Answers: (a)  $\begin{pmatrix} 172 \\ 206 \end{pmatrix}$  (b) The revenue from the sale of hot drinks on Monday and Tuesday.

### Question 13

- (a) This was usually correct, with clearly set out working. Some answers of  $-13$  were seen. Other misunderstandings of directed numbers led to answers of  $-6$  or  $10$ . Some candidates arrived at  $17$  correctly, and then used  $-5$  again leading to final answers such as  $22$  and  $-3.4$ .
- (b) Candidates generally understood what was required and there were many correct answers. Quite a number of candidates arrived at the answer in the form  $\frac{x-2}{-3}$ . Starting with  $y = 2 - 3x$ , a first step is to make  $x$  the subject. A second step is then to interchange  $x$  and  $y$ . Sometimes candidates omitted this second step. In the main, errors occurred in transposing the equation. Candidates did not always choose the most direct way of doing this, and hence got tangled up with too many negative values. Some attempts were seen using flow diagrams. These generally were not successful.

Answers: (a) 17 (b)  $\frac{2-x}{3}$

### Question 14

- (a) Most candidates were able to write down the upper bound readily. A common wrong answer was  $35.4$ . Occasionally, candidates calculated the perimeter of the garden in this part of the question.
- (b) Some good work was seen here. There were many correct answers. Some candidates tried to obtain the answer by adjusting  $120$ . There were others who calculated the area of the garden.

Answers: (a) 35.5 (b) 118

### Question 15

- (a) Generally, the gradient was calculated accurately using the coordinates of two points on the line. Candidates read the diagram given in the question carefully. A few slips occurred, usually involving negative values, leading to answers such as  $-\frac{1}{2}$  and  $2$ . Reading the vertical and horizontal differences directly from the diagram was rarely seen. In some instances, just pairs of coordinates were given as the answer.
- (b) This was done well, with many fully correct answers, and many with at least one inequality correct. Finding  $c = \frac{1}{2}$  in the equation for  $L$  was a common error. Some candidates thought that the other two inequalities they were looking for were of the form  $x + y \leq k$ .

Answers: (a)  $\frac{1}{2}$  (b)  $x \geq 1$   $y \geq \frac{1}{2}x + 1$

### Question 16

- (a) This proved to be a straightforward calculation for many candidates. It is important to read the question carefully. There were some answers of  $140\%$ . Common errors included forgetting to divide by  $90$  and division by  $126$ .
- (b) Again, this was often correct, and again, it is important to read exactly what is being asked. In this case, there were a number of answers of  $\$318.75$ . Common misunderstandings seen in this calculation included the use of  $375 - 115$  and division by  $85$ . Also, candidates needed to work very carefully here to avoid making numerical errors.
- (c) (i) There seemed to be more correct answers here than in the previous parts. Care is always needed in dealing with the decimal point when multiplying. A number of candidates evaluated  $180 \div 1.25$ .

- (ii) A lot of correct answers were seen. Again, answers such as 4 and 40 indicated that division of decimals needed attention. Some candidates evaluated  $500 \times 1.25$  here.

Answers: (a) 40 (b) 56.25 (c)(i) 225 (ii) 400

#### Question 17

- (a) There was a good response to this question. Some incorrect vectors were seen, usually with – signs instead of + signs, or with the numbers 3 and 1 reversed.  $2 \times 2$  matrices were sometimes given as the answer.
- (b) Some candidates knew this matrix by memory but there were many answers where memory had failed. Some correct answers were accompanied by appropriate working, and in some instances the working showed an understanding of base vectors. This is to be encouraged in transformation geometry. There were a variety of incorrect matrices offered with no supporting working. Sometimes a column vector was given as the answer.
- (c) When the correct triangle was drawn, appropriate construction lines were often seen. There were a number of attempts that achieved the correct size and orientation, but showed triangle *D* out of position.

Answers: (a)  $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$  (b)  $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$  (c) Triangle with vertices  $(-1, 2)$ ,  $(1, 2)$ ,  $(1, 6)$

#### Question 18

- (a) Generally, the formula for the interior angle of a regular octagon was quoted correctly, leading successfully to  $135^\circ$ . Sometimes, division by 8 was not completed. The use of  $n = 4$  was also seen. Occasionally the approach via the internal angle was started, but usually left unfinished. Some candidates gave the angle sum of the regular octagon.
- (b)(i) Usually, correct answers in part (a) were successfully carried through to this part. Sometimes the work of part (a) was repeated. Candidates should understand that an internal angle of a polygon cannot be  $180^\circ$  or more.
- (ii) Again, correct work up to this point tended to remain on track, so there were a good number of correct answers. Some candidates were able to recover a method mark in this part.

Answer: (a) 135 (b)(i) 165 (ii) 24

#### Question 19

- (a)(i) This evaluation was often correct. Common wrong answers were  $\pm 6$ ,  $216^{\frac{1}{3}}$  and  $6^3$ .
- (ii) Sometimes  $16^{\frac{1}{2}}$  caused problems, being replaced by such as 16.5, 8 or  $\frac{1}{16^2}$ .  $16^0$  was frequently thought to be 0.
- (b) Candidates who simplified inside the bracket first generally fared better than those attempting to deal with the power  $-2$  as their first step. There were a good number of correct solutions. Some were left incomplete. Some candidates tried to deal with the power  $-2$  by adjusting the fraction and replacing the  $-2$  with  $\frac{1}{2}$ .

Answer: (a)(i) 6 (ii) 3 (b)  $\frac{16b^6}{a^2}$

**Question 20**

- (a) This aspect of the syllabus was well understood. The acceleration was usually correct. There were some answers of  $\frac{3v}{50}$ , and some candidates did not use  $v$ .
- (b) There were some accurate solutions using the complete area under the graph. Some candidates thought that the complete area was a trapezium with parallel sides of 100 and 50 and height  $3v$ . The common misunderstanding in this question was to ignore the area under the speed/time graph. Many candidates thought that the answer was simply  $\frac{2500}{100}$ .
- (c) Where part (b) was correct, this part was usually understood and effectively followed through. There were some accurate follow-throughs from incorrect work in part (b) but, otherwise, the response to this part of the question was poor.

Answers: (a)  $\frac{v}{25}$  (b) 10 (c) 108

**Question 21**

- (a) The branch for the first pen was usually completed correctly. The branches for the second pen were completed correctly by the majority of candidates. Some candidates became confused at this point. Probabilities expressed in tenths and eighths were seen as well as incorrect probabilities in ninths. There was some cancelling of fractions on the probability tree that led candidates to mis-read their figures later in the question.
- (b) This probability was generally understood and evaluated correctly. The correct product was occasionally cancelled incorrectly. Some additions of the two probabilities were seen.
- (c) The method for a combined event was generally well understood. Candidates often made correct deductions from their probability trees.

Answers: (a) (First pen)  $\frac{7}{10}$  (Second pen)  $\frac{7}{9}$   $\frac{3}{9}$   $\frac{6}{9}$  (b)  $\frac{1}{15}$  (c)  $\frac{7}{15}$

**Question 22**

- (a) The majority of candidates found their way to the correct right angled triangle and applied Pythagoras' theorem correctly. There were a number who got  $225 - 144$  to be 121, by subtracting whichever is the smaller digit from the larger in each place value column. The fact that it was a perfect square gave these candidates confidence to continue with 11. There were other misunderstandings such as  $15^2 - 12^2 = (15 - 12)^2$ . Answers such as  $15 - 12$ , and  $\frac{1}{2}BC$  were also seen.
- (b) Again, this part of the question was negotiated correctly by many candidates. For some candidates it did not seem obvious that the area contained two squares, so there were some puzzling answers given, even after  $AG$  had been obtained correctly. A common misunderstanding was applying the formula for the area of triangle  $ABE$ . This was thought to be  $\frac{1}{2} \times 15 \times 12$  by numbers of candidates.

Answer: (a) 9 (b) 279

**Question 23**

- (a) The instruction 'Expand and simplify' was well understood in the main and accurately carried out. A few candidates continued working after doing this, usually solving an equation.
- (b) Again, this part was mostly correct. Candidates were well versed in the procedure required. Both factorised and multiplied out denominators were seen in about equal measure.
- (c) There were many correct answers, usually achieved by way of factorisation. The problem was occasionally solved using the quadratic formula. One or two solutions were left at the factorisation stage without stating the expected answers. Most candidates managed to obtain a three term quadratic expression, but some did not progress beyond this point.

Answer: (a)  $2x^2 + 9x + 4$  (b)  $\frac{7x+6}{x(x+2)}$  (c) 2 or -5

**Question 24**

- (a) There were some accurate frequency polygons drawn. The heights were usually correct. These heights should be plotted at the mid-point of each interval. A common error was to plot them elsewhere. In a frequency polygon, points should be joined by straight lines. Some good work was spoilt by joining points with curved lines. In some cases, bar charts were drawn. Occasionally, candidates chose inappropriate scales, and some forgot to put a scale on the frequency axis.
- (b) This correct answer,  $1 < t \leq 1.5$ , was usually stated with the interval described accurately. There were some 8's given as the answer. Some candidates were unsure at this point, giving both  $1 < t \leq 1.5$  and 8 in the answer space.
- (c) Comparatively few candidates took the cue prompted by the previous part of the question to compare the modes of the two distributions. There were also relatively few answers that compared the ranges or noticed that the distribution of boys times was more evenly spread than the girls times. Candidates who based their answers on specific time intervals generally fared better than those trying to make generalisations.

**Question 25**

- (a) Most candidates were able to use  $360^\circ$  correctly in finding the required equation. Some candidates thought that the sum of two angles could be used to give  $180^\circ$ .
- (b) There were many correct solutions of the simultaneous equations. Candidates usually equalised coefficients using appropriate factors. It is important to remember to multiply all the terms in an equation by the factor chosen. Since the next step involved subtraction, care was needed in dealing with negative signs correctly. Those candidates using substitution were generally less successful. Errors were usually made in simplifying the resulting equation.
- (c) Correct work in part (b) was usually followed by the correct answer here. Some candidates quoted an expression or letter here rather than the actual size asked for.

Answer: (a) Accurate work with  $360^\circ$  seen (b) 20, 35 (c)  $65^\circ$

# MATHEMATICS

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Paper 4024/21

Paper 21

## Key messages

To succeed in this paper, candidates need to have completed full syllabus coverage and remember necessary formulae.

All working must be clearly shown. This is especially important since the use of a calculator on this paper can lead to answers being written with no working.

Candidates are advised to check that their calculators are set in degree mode before evaluating trigonometric expressions.

It is also important to ensure that work is, as far as possible, written clearly in pen; it is not advisable for candidates to write in pencil and then over-write it with ink.

## General Comments

The paper provided challenges and opportunities for all candidates to demonstrate their knowledge. This was evidenced by the range of marks that the candidates obtained.

It was pleasing to see the vast majority of candidates familiar with the quadratic equation formula and the cosine rule, as well as showing their understanding of what was required to produce a cumulative frequency graph. On the other hand, the response by candidates to the vector question was poor.

Candidates on the whole were aware of the need to work with four figures when the final answer is to be given to three significant figures, and seemed more aware than previously that premature approximation of values in working can lead to loss of accuracy marks.

## Comments on Specific Questions

### **Section A**

#### **Question 1**

- (a) Fully correct answers were seen only rarely. The majority of candidates chose the more difficult option of using the denominator of  $(x - 4)^3$  rather than the easier choice of  $(x - 4)^2$ . The problem in choosing the former option for many candidates then was that they were left with the term  $\dots -2(x - 4)^2$  in the numerator and this is where many either did not expand the brackets correctly, e.g.  $-2(x^2 - 4^2)$  was common, or they failed to multiply by the  $-2$  correctly after a correct expansion was seen. Candidates must remember to use brackets for both pairs of factors in  $(x - 4)^2(x - 4)$ . It was common to see  $(x - 4)^2 x - 4$ , however.
- (b) A good proportion of candidates obtained the correct solutions here and the method of solving simultaneous equations seems to be well understood. Many chose to eliminate the  $x$  terms first of all and reached  $y = -3$  successfully, but then made errors when substituting this value into an equation to find the value of  $x$ .
- (c) This was another question where there were a lot of correct answers seen. Some candidates tried to factorise too early, after reaching  $x^2 - 5x = 6$  doing  $x(x - 5) = 6$ . A good number reached the correct quadratic equation but then were unable to come up with the correct factors and either gave  $(x + 6)(x - 1)$  or  $(x + 3)(x - 2)$  for the factors. A small number of candidates tried to solve the

equation by using the quadratic formula and the common error here was the use of a short division line and not a full line. It was common to see  $5 \pm \frac{\sqrt{49}}{2}$  leading to incorrect answers being obtained.

- (d) Many candidates knew how to factorise the difference of two squares in the numerator, but could not successfully obtain the factors for the denominator. There were a number of candidates who divided the denominator by 2 reaching  $y^2 - y/2 - 15/2$  and then tried to factorise this. Candidates needed to concentrate on the  $2x^2$  and  $-15$ , and the pairs of factors required to produce these terms.

Answers: (a)  $\frac{8-x}{(x-4)^2}$  (b)  $x = 2.5$   $y = -3$  (c)  $x = 6$  or  $-1$  (d)  $\frac{y-3}{2y+5}$

## Question 2

- (a) (i) Not many correct answers were given. It was common to see  $\{ \}$  or  $\Phi$  instead of 0 or none.  
(ii) This was fairly well done  
(iii) This was poorly answered. Many did not attempt this part.  
(iv) Fairly well answered. Some candidates put  $\{ 5 \}$ , which was acceptable.
- (b) All parts were poorly answered with many blank responses. Candidates tried to use a Venn diagram in part (b) but there was much confusion as to where to place the values of the sub-sets.

Answers: (a)(i) 0 or None (ii) 7, 8, 11, 13, 14 (iii) 3/11 or 0.27 (iv) 5 (b)(i) 3 (ii) 11 (iii) 18

## Question 3

This was a question where many candidates usually did well.

- (a) (i) Many found the profit per bottle first, and then were usually successful in obtaining the percentage profit.  
(ii) Many found the profit for selling the 45 bottles at full price, but could not always find the profit from selling the remaining 15 bottles at 20% discount. If not fully correct here, then most earned the method mark.  
(iii) This was the most poorly answered part of this question. By far the most common error was for candidates to calculate 15% of \$240 and then subtract this from \$240 to give the answer of \$204. A few candidates decided to add the \$36, getting \$276 for their answer.  
(iv) It was pleasing to see the vast majority knew how to calculate the simple interest and this part was done well. A small number spoiled their attempt by using 100 twice in the denominator of the formula. This arises from candidates knowing that there is 100 in the denominator but then they multiply by another 100 when writing 4.5% as  $4.5 / 100$ . A small number confused simple interest with compound interest and mistakenly calculated this instead.
- (b)(i) A very well answered part question; nearly all candidates were successful here. Only a few did the incorrect calculation  $250 / 0.64$ .  
(ii) There was more confusion in this part. Some candidates simply had the conversion fraction incorrect, so  $0.64 / 0.78$  was a common mistake leading to €0.82 as the answer. Others just did  $0.78 - 0.64$ , giving €0.14, or, thinking this answer was too low, simply added another euro giving €1.14 for their answer. There were many fully correct answers seen as well.

Answers: (a)(i) 37.5 (ii) 73.50 (iii) 208.70 (iv) 2837.50 (b)(i) 160 (ii) 1.21875 to 1.22

#### Question 4

- (a) This part question was not well answered. There was an incorrect assumption that candidates seemed to make, namely, that  $AC$  was parallel to  $ED$  and that therefore alternate angles were present. It was pleasing to see that many worked out that angle  $ACD$  had to be  $90^\circ$  from the angle subtended at the circumference in a semicircle, but many candidates did not know that angles in the same segment are equal.
- (i) Following from the above, a great many candidates correctly evaluated angle  $CAD$  to be  $18^\circ$ , but then the angle  $ADE$  was given to be  $18^\circ$  also as their answer, presumably from alternate angles being equal.
- (ii) Similarly, angle  $CED$  was often given as  $24^\circ$ .
- (iii) Many candidates did not attempt this part. This was surprising since angle  $FCD$ ,  $66^\circ$  was readily obtained from  $90^\circ - 24^\circ$  and then angle  $CFD$ ,  $42^\circ$  should have followed from the angle sum of a triangle.
- (iv) Again, many did not attempt this part. Only a handful of candidates had this correct.
- (b)(i) This part was generally well answered with a good number of fully correct answers seen, from candidates choosing to use the correct trig ratio of  $AD = 4.5 / \cos 72^\circ$ . A smaller number chose to use  $AD = 4.5 / \sin 18^\circ$  as the alternative correct path, while a few chose the long route by first of all finding  $AC$  and then employing Pythagoras' theorem to obtain  $AD$ . However, some did lose the accuracy here from either premature rounding or truncating of figures in the intermediate stages of the calculations.
- (ii) This part was not so well answered. One common error from candidates who had previously found angle  $ADE = 18^\circ$ , was then to use  $DE = 4.5 \tan 66^\circ$ , which assumes angle  $CDE$  to be  $90^\circ$ . The other common mistake made by others was to assume that  $EC$  was a diameter as well, presumably from the fact that it intersected with  $AD$  quite close to the centre of the circle. So it was common to see Pythagoras' theorem being used with  $DE^2 = 14.56^2 - 4.5^2$  and giving the answer of  $DE = 13.85$ .

Answers: (a)(i)  $24^\circ$  (ii)  $18^\circ$  (iii)  $42^\circ$  (iv)  $108^\circ$  (b)(i) 14.56 to 14.6 (ii) 13.3 to 13.304

#### Question 5

- (a)(i) Very well answered. Nearly all candidates scored here.
- (ii) Many did score full marks here, but others lost marks for omitting brackets around the pairs of terms. So it was common to see  $n + 1 \times n + 6 - n \times n + 7$ . Many did gain a mark for correctly expanding the  $(n + 1)(n + 6)$  to  $n^2 + 7n + 6$ , but it was a common error to see  $-n(n + 7)$  given as  $-n^2 + 7n$ .
- (b)(i) Many candidates were able only to complete the cross with the correct terms but did not find the sum. Some candidates made no attempt here.
- (ii) This part was generally answered correctly by those able to answer part (i)

Answers: (a)(i)  $n + 6, n + 7$  (ii)  $n^2 + 7n + 6 - n^2 - 7n = 6$  (b)(i)  $5n + 50$  or  $5(n + 10)$  (ii) 56, 65, 66, 67, 76

#### Question 6

- (a)(i) Many scored full marks here. However, there were a number who thought that the volume of a cylinder was  $\frac{1}{3} \pi r^2 h$  or  $\pi r h$  or  $2\pi r h$ . A few gave the correct method but then did not evaluate correctly.
- (ii)(a) Only a small percentage of candidates were able to obtain the correct answers here. A good many did not seem to realise that they needed to double the radius and then multiply by 3 to get the length and by 2 for the width. It was common to see answers of 4.8 and 3.2 respectively.

- (b) Some candidates knew how to find the total volume of the box and gave this as their answer, mistakenly thinking that this represented the volume of the empty space in the box, not realising that they had to subtract the volume of the candles from this.
- (b)(i) Many candidates did  $17.8 \times 12.7 = 226.06$  and then subtracted 0.5, giving the answer 225.56. Others used the upper bounds of 17.85 and 12.75 instead.
- (ii) Not being able to obtain the lower bounds for either the length or the width of the frame proved to be a hindrance for the majority of candidates here. Some found the area of the frame using  $18 \text{ cm} \times 13 \text{ cm} = 234 \text{ cm}^2$  and said 'No, because the area of the frame is bigger than the area of the photo, so some space is left'.

Answers: (a)(i) 60.28 to 60.35 (ii)(a) length 9.6, width 6.4 (b) 98.7 to 99.2 (b)(i) 224.5 or 225 (ii) No, frame could measure 17.5 cm by 12.5 cm

## Section B

### Question 7

- (a) Very well answered. Usually both values were correct, but if not, then it was common to see  $-3.5$  as the correct value.
- (b) Generally candidates plotted the points accurately. The drawing of the curve was more mixed, however. Candidates should concentrate on making the curves more rounded at the maximum and minimum values and in between there should be less of a straight line appearance and more of a gradual curvature. This would then give the curves a smooth appearance.
- (c) Apart from the fully correct answers, some candidates gave only two correct solutions even though their graphs showed that there should be three. There was some misreading of the scale on the x-axis, but generally this was good. A few omitted to put the minus sign on the negative solution. A substantial number of candidates did not attempt this part.
- (d) It was pleasing to see many correct tangents. Candidates should remember that the tangent must only touch the curve at the point of contact, however, and some failed to appreciate this. Candidates whose curves were not drawn smoothly enough at the point  $(-2, 3)$ , often gave an answer that was too steep a gradient.
- (e)(i) This was generally not done correctly. Calculating the gradient proved difficult for most. However some had a correct figure but failed to put the minus sign in, giving a positive gradient of 4. More were successful in realising that the intercept was 5 and a fair number of candidates had partial success here.
- (ii) Very few candidates were successful here. The common problem was an apparent failure to realise that they had to equate the equation of the given curve with the equation of the line  $AB$  and then rearrange to find  $C$  and  $D$ . By far the most common error was to substitute the value  $x = 1.7$  into the equation of the cubic curve, which led candidates nowhere.

Answers: (a)  $-3.5, 5.5$  (c)  $x = -2.7$  to  $-2.6, 0.3$  to  $0.4, 2.2$  to  $2.3$  (d) 2 to 3 (e)(i)  $y = -4x + 5$   
(ii)  $C = 1, D = -4$

### Question 8

- (a) A fairly well answered question with nearly all candidates scoring at least the 1 mark here for knowing that they had to divide by the total frequency of 80. Some candidates were unsure whether or not to multiply the frequencies by either the mid-points or the end-points and gave a mixture of these in their calculations for  $\bar{fx}$ . The most common error was to multiply the class width of 10 by the frequencies, so to see  $800/80$  was very common.
- (b)(i) The vast majority of candidates had this correct.

- (ii) It was pleasing to see so many candidates able to draw very good cumulative frequency curves and most scored full marks for this part of the question.
- (iii) These two parts were not so well answered. More candidates had the median correct than the interquartile range and there was quite a proportion of candidates not attempting either part.
- (c) This was not well done by most candidates. The most common errors were either to use the product  $5/80 \times 4/79$  or the product  $5/25 \times 5/25$ .

Answers: (a) 32.25 or 32.75 (b)(i) 4, 16, 32, 55, 75, 80 (iii)(a) 33 to 35 (b) 18 to 20 (c)  $1/30$  or 0.0333

#### Question 9

- (a) This part was well answered by a good proportion of the candidates, showing that the cosine rule was well understood by the majority. However, a small number spoiled their attempt by using  $\sin 115^\circ$  instead of  $\cos 115^\circ$ . The common error was to think that Pythagoras' theorem could be used, even though the angle  $ABC$  was  $115^\circ$  and so the answer 209.27... was often seen.
- (b) This part was also quite well answered, with many candidates able to use the sine rule correctly to obtain the correct answer. Weaker responses omitted the  $\sin 115^\circ$  and just did the calculation  $\frac{1}{2} \times 130 \times 164 = 10660$ . There were a handful of attempts at using Hero's formula, but unfortunately they were usually unsuccessful for two reasons. Either candidates divided by 3 and not 2 in finding the semi-perimeter, or they did  $(s + a)(s + b)(s + c)$ .
- (c) Not many candidates knew what to do to reach the correct answer. There was much confusion shown, with some candidates simply using the figures 3.25 and 5 and doing calculations involving either multiplication or division with them and ignoring their answer from part (b) completely. Of those who did use the latter, some just divided this by 3.25 and then divided by 5 again. A common error, even among those who knew how to proceed, was to use mixed units in their calculation. There were many instances where candidates divided their total number of grams by 5 and not by 5000. Of those who did all of the calculations correctly and reached the figure 6.279 for the number of bags required, some did not realise that this answer needed to be rounded, or incorrectly rounded down.
- (d) There were many correct answers seen. Unfortunately there were quite a number of others incorrectly choosing to use either  $130 \times \sin 18.5^\circ$  or  $130 \times \cos 18.5^\circ$  in their calculation.
- (e) Candidates who realised that the shortest distance involved the perpendicular line from point C to the road XY, and that this gave them a right-angled triangle with an angle of  $65^\circ$ , usually went on to obtain the correct answer by employing correct trigonometric ratios. There were many candidates though who thought that the shortest distance was either  $BC$  or  $AC$ .

Answers: (a) 248.6 to 249 (b) 9660 or 9661.2(...) (c) 7 (d) 43.49 to 43.5 (e) 148.6 to 149

#### Question 10

- (a) (i) Very poorly answered with only a small percentage able to obtain the correct answer, from knowing that they had to employ Pythagoras' theorem here.
- (ii) Very poorly answered. Candidates did not appreciate that the vector  $\mathbf{p} - \mathbf{q}$  involved making movements of three units to the right and three units down, and then drawing the diagonal line to represent the vector. Candidates who did reach this stage very often did not put the arrow on the line to indicate the vector direction, so did not give a fully correct answer. Many candidates drew two separate lines of three units each, not always connected, and labelled these  $\mathbf{p}$  and  $\mathbf{q}$ . There were also many who did not attempt this part.
- (iii) Another very poorly answered part question. Very few realised that the method involved solving two simultaneous equations in  $a$  and  $b$  formed from the given vectors  $\mathbf{p}$ ,  $\mathbf{q}$  and  $\mathbf{r}$ . Consequently, there were only a few correct answers seen.
- (b) (i) There was more success on this part. If not giving fully correct answers, then most candidates did earn partial credit, usually for stating that the transformation was enlargement and for giving the correct centre of enlargement. The common error was to give the scale factor as 2 and not  $-2$ .

There were relatively very few candidates spoiling their answers by mentioning a second transformation.

**(ii)(a)** This part was also quite well answered. Multiplication of the coordinates of the vertices of triangle A by the transforming matrix was well carried out.

**(b)** There were many candidates who obtained partial credit here. This was for knowing that the transformation was a stretch, but they then either did not go any further with their answer, or gave the scale factor incorrectly as a  $\frac{1}{2}$ , or said that the invariant line was the y-axis. Candidates need to remember that the correct description for the invariant line is, 'x-axis is invariant' and not just 'x is invariant'.

Answers: **(a)(i)** 3.16 to 3.163 or  $\sqrt{10}$  **(iii)**  $a = 2$ ,  $b = 3$  **(b)(i)** Enlargement, Scale factor – 2, Centre (3, 1)  
**(ii)(a)** (5, 4), (7, 4), (5, 6) **(b)** Stretch, Factor 2, x-axis invariant

### Question 11

**(a)** Very well answered with most candidates correct here. A few candidates gave both of the expressions for the time for the first part of the journey and the second part as their answer.

**(b)** It was pleasing to see candidates exhibiting the necessary skills in order to manipulate this successfully. Some candidates did attempt the formation of the initial equation and did correctly give  $80/(x - 5)$  as the time for the second part of the journey, but then sometimes spoiled the equation by equating to 150 minutes instead of 2.5 hours. There were many other candidates who, seeing the quadratic equation given in the question, immediately tried to solve it using the quadratic formula. Either they misread the question, or did not know how to proceed further along the correct route and form the required equation.

**(c)** A very well answered part question, with candidates showing that they were well-versed in the use of the quadratic formula. What spoiled the answers for some was the use of a short dividing line in the formula. Candidates need to remember that it is the whole of  $-b \pm \sqrt{b^2 - 4ac}$  that is divided by  $2a$  and not just the part of the formula involving the square root. Others spoiled their otherwise correct answer, by not giving them to the required 2 decimal places.

**(d)** Quite well answered; many chose the correct positive solution but did not always give an acceptable reason. Some candidates just said that 'the other speed would be too slow' or '2.69 is not quick enough' or 'because it is negative'. Candidates should refer to the fact that the other solution would give a negative value for the speed for the second part of the journey.

**(e)** Not well answered. Candidates who knew how to proceed reached 0.191 (hrs), but then prematurely rounded this to 0.2, thus leading to their incorrect answer of 12 minutes. A few others correctly reached 11.448 minutes, but some then wrongly rounded this to 12 also, instead of 11.

Answers: **(a)**  $100/x$  **(b)**  $x^2 - 77x + 200 = 0$  **(c)** 74.31 and 2.69 **(d)** 74.31 since 2.69 would give a negative speed value for second part **(e)** 11

# MATHEMATICS

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Paper 4024/22

Paper 22

## Key messages

To succeed in this paper, candidates need to have completed full syllabus coverage and remember necessary formulae.

All working must be clearly shown. This is especially important since the use of a calculator on this paper can lead to answers being written with no working.

Candidates are advised to check that their calculators are set in degree mode before evaluating trigonometric expressions.

It is also important to ensure that work is, as far as possible, written clearly in pen; it is not advisable for candidates to write in pencil and then over-write it with ink.

## General Comments

Most candidates seemed to have enough time to complete the paper and it appeared that lack of knowledge was the cause of blank answer spaces rather than lack of time.

Premature approximation was a problem in several questions for a significant number of candidates, often resulting in inaccurate final answers and loss of accuracy marks

The questions which proved most challenging included **Question 4(b)**, **Question 7(b)(ii)**, **Question 8(b)(ii)**, **Question 9(e)**, **Question 10(e)** and **(f)**, **Question 11(a)(i)** and **(ii)** and **Question 11(b)(ii)**.

On the other hand there were several questions which most candidates could tackle with confidence. The Algebra and Trigonometry questions were handled well as were parts **(b)** and **(c)** of **Question 9**.

## Comments on Specific Questions

### **Section A**

#### **Question 1**

- (a)** The majority of candidates scored the available mark for the interpretation of the scale. Occasionally candidates simply measured the distance of 7 cm and gave this as their response.
- (b)** Candidates were usually accurate in their constructions but sometimes it was difficult to see their very faint construction lines. There were a few who reversed their 5 cm/6 cm arc lengths, and/or  $D$  was quite often seen on the east of  $AB$  in almost all cases candidates did label their point  $D$ . A few attempted to locate the correct position of  $D$  from measurement alone without the use of arcs.
- (c)** Although there was an evident lack of understanding of bearings, many correct answers were stated. The most common error was to take the bearing to be angle  $ABC$ .
- (d)(i)** The perpendicular bisector was in the majority of cases accurately drawn although occasionally candidates believed that it had to pass through point  $B$ . The circle of radius 4.5 cm was usually drawn accurately but a significant number only drew small arcs rather than the full locus to position  $P$  and  $Q$ . They were generally located correctly but sometimes one point was correct and the other was positioned where the line  $AC$  and the perpendicular bisector met. The latter happened when

the perpendicular bisector was not produced far enough to meet the circle at the upper of the two points.

- (ii) Candidates had relatively little success with this part and responses again reflected a lack of knowledge of bearings. Many attempted to give the bearing of the line CA after marking their  $P$  or  $Q$  on that line

Answers: (a) 139 (c) 103 (d)(ii) 249

### Question 2

- (a) (i) Most candidates were able to substitute correctly and evaluate the expression. A common incorrect response was  $f = 6 \times 64 + 1 = 385$  from dividing only the  $-d$  term by 4.
- (ii) A majority of candidates were able to rearrange the formula correctly. Most errors that did occur were a result of mishandling the square root, usually giving the square root of the numerator only. Those who did not score in full almost always got partial credit for a correct first step. Some got to the correct expression for  $c^2$  but then cancelled the  $6/4$  to  $3/2$  without considering the  $d$  term. Others sometimes considered this part as a continuation of part (a)(ii) by substituting  $f = 97$  into the given formula and attempting to solve it numerically.
- (b) Candidates showed good ability in this topic and were able to solve the inequality correctly. Most arrived at an inequality or equation containing 2, although occasionally 2 was seen alone. The response  $2 \leq x$  occurred just as often as  $x \geq 2$ .
- (c) This part was almost always correct; candidates recognised that the expression was a difference of two squares. Sometimes the expression was written in the form  $(5x + 3)(-5x - 3)$ .
- (d) A lot of candidates successfully arrived at the correctly factorised response. In the scripts of those who did not, there was usually evidence of a correct partial factorisation,  $8p(x - 2y)$  being most commonly seen. A number of candidates made sign errors and ended with  $(8x + 3q)(x - 2y)$  or alternatively  $(8p - 3q)(x + 2y)$ .
- (e) This was another well-attempted algebraic question. The majority of candidates knew the quadratic formula correctly and were able to substitute the coefficients accurately, with most realising the need to give their solutions to the specified two decimal places. As is usually the case, the main source of error was either an incomplete dividing line or a wrong identification of the coefficients. Very few attempts to solve the quadratic using the completing the square method were seen this year.

Answers: (a)(i) 97 (ii)  $\sqrt{(4f + d)/6}$  (b)  $x - 2$  (c)  $(3 + 5x)(3 - 5x)$  (d)  $(8p - 3q)(x - 2y)$   
(e) 1.12 and  $-232$

### Question 3

Candidates performed well on this everyday arithmetic question.

- (a) (i) Most candidates realised that they had to evaluate the hourly pay and the bonus and sum the two amounts. A small number had difficulty with calculating the percentage. A more common mistake resulted from misinterpretation with candidates believing it necessary to add the \$2450. The outcome showed itself in calculations such as
- $$15/100 \times 2450 = 367.5 \text{ and } 5.2 \times 32 = 166.4, \text{ followed by } 166.4 + 2450 + 367.5 = 2983.9$$
- $$\text{and } 15/100 \times 2450 + 24.50 = 367.5 + 2450 = 2817.50.$$
- (ii) This was the part which was found most difficult. A few candidates used values from part (i) but many more just found the pay for the 28 hours work and subtracted this from \$409.60, giving \$264.00. This was then written as their final answer. Some recognised that they needed to involve a percentage but went wrong by multiplying \$264 by 1.15.

- (b)(i) A large proportion of candidates knew how to handle simple interest and gave the correct outcome. Quite regularly occurring wrong answers resulted from either taking the interest to be \$920 instead of \$120, using a period of 1 year instead of 4, or forgetting the 100 and arriving at 0.0375.
- (ii) Again there was success here with candidates taking the direct route and evaluating  $7 / 16 \times 920$ . Others calculated holiday and computer allocations and subtracted each of them from 920. Some candidates did not read the question properly and offered all three values. A small number divided \$920 by 4, 5 and 7 to get the apportionments.

Answers: (a)(i) 533.90 (ii) 1760 (b)(ii) 3.75 (ii) 402.50

#### Question 4

- (a) Both parts were very well answered but a significant number of candidates did not simplify their fractions. A fairly common wrong response in the second part was  $1/2$ . Another recurring error was to put a denominator of 7, which may have been due to a failure to notice that there was no ball numbered 1.
- (b)(i) Many candidates did not appreciate the rubric “without replacement”. The consequence was a digram which was often incorrect, with 35 or even all 36 cells completed. It was very rare to see incorrect figures in the boxes. There were some candidates who left the whole of the first column blank, rather than the diagonal. Sometimes, although rarely, there were no entries below the diagonal.
- (ii) A large number gained partial credit for correctly following through from their incorrectly completed grid. However, a surprising number used a denominator of 36 even though they had only made 30 or 35 entries in the grid. A few simply quoted the number of entries that were odd or were less than 3 instead of the probabilities of these events. A rarer error was probabilities with denominators of 49, the total number of boxes in the table.

Answers: (a)(i)  $1/3$  (ii)  $2/3$  (b)(ii)(a)  $3/5$  (b)  $4/15$

#### Question 5

- (a) This part was generally done very well. The majority started from  $\cos 35 = 64/AB$ . A much smaller group used the sine rule. Obtaining  $AB$  in an explicit form proved problematic to some and in other cases premature approximation spoiled otherwise good work. The most common mistake involved the use of an incorrect trigonometric ratio, while others went applied longer approaches involving trigonometry and Pythagoras.
- (b) Most candidates seemed to know that they should use the cosine rule in this part and most gave a correct statement with the appropriate numerical values inserted. A significant number had problems with  $\cos 125$  being negative and ended up with 67.9... . The other error, which occurred occasionally, was to combine the terms wrongly and evaluate  $(64^2 + 80^2 - 2 \times 64 \times 80) \times \cos 125$ . A few candidates assumed that triangle  $ACD$  was isosceles and calculated  $AD$  from  $64 \times \sin 62.5 + 80 \times \sin 62.5$ .
- (c) Again there was a high success rate in this part with most candidates sensibly using the sine rule directly from their previous answer. A few applied the cosine rule again.
- (d) Most candidates recognised the shape as a trapezium and used the appropriate formula. Errors were sometimes made in cancelling but usually the expression was computed correctly. However  $40 \times 65 = 2600$  was seen a few times.

Answers: (a) 78.1 (b) 128 (c) 24.2 (d) 2900

#### Question 6

- (a) Almost all candidates tried to use the difference of 6 but this often resulted in a formula containing  $n - 6$  or just  $6n$  rather than  $-6n$ . There were a few who gave  $17 - 6n$  as their response, with the first term being linked to  $n = 0$ . Many tried to apply the arithmetic progression formula and a common term in the working was  $17 + (n - 1) \times 6$ .

- (b)(i) There were many who found the four required terms correctly and any wrong values were a result of computational errors. It was the first term that was most often wrong, usually calculated as 3. A small number worked out the first term but then used this as  $n$  to work out the next term and repeated the process.
- (ii) Many candidates failed to do what the the question asked and form an equation in  $n$ . Often  $S_n$  was equated to  $6 \times (5n - 12)$  but sadly many did not substitute  $n^2 + 3n$  for  $S_n$ . There were some who wrote this equation correctly but made a mistake when writing it in the form  $ax^2 + bx + c = 0$ . There was a significant number who opted to try to find the vaulues by trial and error. The value 3 was quite often a result of this approach but the 24 was not found this way.

Answers: (a)  $23 - 6n$  (b)(ii) 4, 10, 18, 28 (ii) 3 and 24

## Section B

### Question 7

- (a)(i) Candidates who used the direct method of seeing that 1600 votes corresponded to  $1/6$  of the total usually found the correct answer. Problems arose when the number of votes linked to one degree was calculated separately. The value 26.6 was often used, and slightly inaccurate vote numbers were subsequently found. A large minority worked out the number for each individual sector and added them up. Those that chose this method often made arithmetic slips.
- (ii) Again, most candidates got the correct answer for this part. A common error was to give the number of votes received by candidate D and not the fraction of votes. A few candidates worked out the fraction for one of the other sectors. Sometimes an angle was given as the answer.
- (iii) The method was clearly understood but candidate A was occassionally paired with candidate B or D rather than with C.
- (b)(i) Many candidates were successful in computing the mean. A common error in working out the numerator was to multiply the width of the interval, rather than the midpoint, by the frequency. Others who knew the correct method made arithmetic slips in evaluating the numerator or the midpoints. Most candidates knew that they had to divide the result by 150.
- (ii) The histogram was problematic for many candidates. About half of them plotted frequency, rather than frequency density, against age and so the resulting diagram had only the second and third heights correct. The bar widths were nearly always right but occasionally a frequency ploygon was seen.
- (iii) A common wrong answer was  $33 + 24 = 57$  achieved by adding the frequencies of the last two intervals. Candidates had been expected to evaluate  $33/2 + 24$  and then give an integer answer.

Answers: (a)(i) 9600 (ii)  $11/60$  (iii) 1440 (b)(i) 40.1 (iii) 38 to 41

### Question 8

This was one of the less popular of the optional questions.

- (a)(i) A significant number of candidates added the vectors while a few simply multiplied corresponding elements. Many did subtract the first from the second but there were slips associated with working out the negative value.
- (ii) A number of candidates clearly did not understand the modulus symbol. However, overall, the question was answered competently using Pythagoras' theorem.
- (iii) There were surprisingly few correct equations found. Many knew how to find the gradient but, when attempting to give the equation, a significant number omitted the  $x$  or the negative sign. Some candidates who achieved the correct gradient mixed up various  $x$  and  $y$  values when substituting into their equation and reached an incorrect intercept.

- (iv) This part was sometimes misread with  $R$  being taken as the midpoint. The working was often confused and responses occasionally given as vectors.
- (b)(i) Most candidates who attempted the first part got it correct. Occasionally signs were reversed or the answer  $\mathbf{a} + \mathbf{b}$  was given. Marks were lost in the final two parts because of a failure to simplify. The answer to (c) was occasionally given as  $3(\mathbf{b} - \mathbf{a})$ .
- (ii) Candidates who had been successful in the previous parts usually scored here although a minority of them wrote  $1 : 3$  instead of  $1 : 4$ . This presumably resulted from  $AC$  being 3 times  $QA$ . Many candidates found the second part a problem and quite long and involved working was often a feature. Common wrong answers were  $1 : 16$  from candidates who recognised that the magnification of the areas of the triangles requires the squaring of the previous answer, but did not realise that they then had a further step to take to get to the required ratio. Also seen was  $1 : 9$ , arrived at by squaring the ratio of  $AC$  to  $OA$ . Many candidates omitted this part altogether or gave answers involving the vectors.

Answers: (a)(i) 4 (ii) 6.40 (iii)  $y = -1.25x + 7$  (iv)  $(12, -8)$  (b)(i)(a)  $\mathbf{b} - \mathbf{a}$  (b)  $3\mathbf{a}$  (c)  $4(\mathbf{b} - \mathbf{a})$   
(ii)(a)  $1 : 4$  (b)  $1 : 15$

### Question 9

- (a) Most candidates were well aware of the need to use Pythagoras' theorem but many failed to show a value to at least two decimal places in order to demonstrate that the given 16.2 is correct to one decimal place.
- (b) Most, although not all, saw to include the area of the base. The main error was in the calculation of the curved surface area, where many used 15 rather than 16.2 for the slant height. Success was generally high in reaching the correct solution.
- (c) This proved straightforward for almost all candidates. Very occasionally  $1/3$  become  $1/2$  and/or 16.2 was used instead of 15 for the height.
- (d) Those candidates who showed understanding of both density and units generally went on to earn both marks. Many candidates, though understanding the idea of density, were confused by the changes in units and often gained just one mark.
- (e) This part, and particularly (ii), was a significant challenge to candidates.
- (i) Few candidates equated the ratio of the masses to the ratio of the cubes of corresponding sides, with most adopting the wrong argument that if it is double the mass it must be double the height. The answer 30 was therefore very common.
- (ii) A mixture of methods were seen here some attempting to use ratios and scale factors and this method was often at least partially, successful. Many, however, reverted to finding the surface area as they did in part (b). Unfortunately the new radius and slant height values were rarely found correctly. Premature approximation also caused difficulties, with rounded values for the lengths leading to inaccurate values for the surface area.

Answers: (a) 16.155 found (b) 418 (c) 565 (d) 317 (e)(i) 18.9 (ii) 664

### Question 10

- (a) Some candidates started with the given expression and attempted to manipulate it in some way. Those who started by writing an expression for the width in terms of  $x$  almost always went on to achieve full marks.
- (b) Most candidates correctly completed the table; incorrect values were rarely seen.
- (c) Accurate graphical plots and reasonably smooth curves were frequently seen, with their quality being better than in the past. The scales used were nearly always as instructed although the horizontal one was occasionally changed to 4 small squares (instead of 5) to represent 1 m. Only a very small number of candidates reversed the axes.

- (d) A high percentage of candidates realised the requirement to read off from the horizontal 40 line but a significant number of these did not produce the line far enough to give the two lengths needed. Often only the smaller one was given together with another close to it. There were also quite regular mistakes made in re-interpreting the horizontal scale, particularly by those using the changed axes.
- (e) (i) The response of 28.5 was common, with candidates reading the least value in the table and not taking into account that the graph dipped lower than this value.
- (ii) Few candidates showed insight into this part of the question and virtually all candidates ignored the instruction "correct to the nearest metre". Correct responses were rare.
- (f) This proved beyond the understanding of most candidates.

Answers: (b) 41.6 and 45 (d) 3 and 17 (e)(i) 27.5 to 28.4 (ii) 7 by 7 (f) 10 by 10

### Question 11

- (a) (i) Although rarely scoring three marks, candidates often picked up one or two marks for partial solutions. Most appreciated that  $\angle AEC = \angle EBF = 90^\circ$  but relatively few explained why. In questions of this type it is important for candidates to give reasons for their statements. Those who stated  $AC = EF$  generally gave the explanation that this was because of equal diameters. Candidates who stated  $EB = EC$  generally gave the reason (equal radii). Many candidates simply wrote as many statements as possible about the triangles. It was rare to find candidates who chose the correct congruency condition. Many who utilised the condition  $\angle AEC = \angle EBF = 90^\circ$  and the previously stated line equalities quoted SAS when it should have been the RHS condition.
- (ii) The vast majority identified the triangle  $BDF$  correctly.
- (iii) A lot of candidates stated and/or labelled  $\angle EBF$  and  $\angle DFB$  as both being equal to  $90^\circ$ . Apart from that there was often little in the way of an adequate explanation. Many quoted random facts, some of which were correct, like angles in the same segment being equal, but not forming a complete and acceptable explanation why the two lines were parallel. A significant number believed that just saying "ES and DF are chords" was sufficient.
- (iv) There were many correct answers to this part. A fairly common wrong response was  $90^\circ$ .
- (b) (i) Most candidates applied the appropriate area formula where two sides and the included angle are given. Unfortunately premature approximation was often seen and the accuracy mark was quite often not gained as a result. A few drew a perpendicular from PQ to O and used trigonometry to find the area of one of the right angled triangles and double it.
- (ii) It was common to see correct expressions for the areas of the major and minor sectors OPQ with the values 32.11 and 18.15 being regularly seen. Quite often one of these, usually the former, was given as the answer. Identifying which combination of areas produced the major segment was the stumbling block for a lot of candidates with only a relative few realising that they needed to add their triangle area to that of the major sector.

Answers: (a)(i) BFD (iv) 120 (b)(i) 6.13 (ii) 38.2