Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

2009383695

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/63**

Paper 6 (Extended) **October/November 2014**

1 hour 30 minutes

Candidates answer on the Question Paper

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO **NOT** WRITE IN ANY BARCODES.

Answer both parts **A** and **B**.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40.

This document consists of **12** printed pages.

Answer **both** parts **A** and **B**.

A INVESTIGATION THE END RESULT (20 marks)

You are advised to spend no more than 45 minutes on this part.

This investigation looks at some relationships between unit fractions.
A unit fraction is a fraction which has a numerator of 1.

1 The **difference** between the unit fractions $\frac{1}{4}$ and $\frac{1}{5}$ is

$$\frac{1}{4} - \frac{1}{5} = \frac{5-4}{20} = \frac{1}{20}.$$

The **product** of the unit fractions $\frac{1}{4}$ and $\frac{1}{5}$ is

$$\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}.$$

The difference and the product of the unit fractions $\frac{1}{4}$ and $\frac{1}{5}$ are the same.

(a) **(i)** Find the difference and the product of the unit fractions $\frac{1}{3}$ and $\frac{1}{4}$.

difference = ...

product = ...

(ii) Find the difference and the product of the unit fractions $\frac{1}{3}$ and $\frac{1}{5}$.

difference = ...

product = ...

(iii) Find another pair of unit fractions whose difference and product are the same.

.... and ....

(b) a and b are positive integers where $a < b$.

Write as a single algebraic fraction

(i) $\frac{1}{a} - \frac{1}{b}$,

....

(ii) $\frac{1}{a} \times \frac{1}{b}$.

....

(c) A pair of unit fractions has the same difference and product.

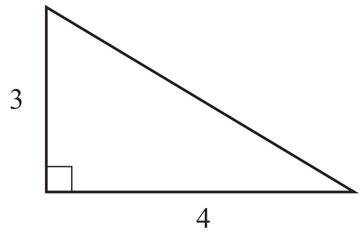
Complete the following statement.

When the larger fraction is $\frac{1}{n}$, the smaller fraction is ....

2 The **sum** of the unit fractions $\frac{1}{2}$ and $\frac{1}{4}$ is

$$\frac{1}{2} + \frac{1}{4} = \frac{4+2}{8} = \frac{6}{8} = \frac{3}{4}.$$

The numbers 3 and 4 form two sides of a right-angled triangle.



NOT TO SCALE

The hypotenuse of this triangle is $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$.

The set of integers (3, 4, 5) is called a Pythagorean Triple.

(a) **(i)** Find the sum of the unit fractions $\frac{1}{5}$ and $\frac{1}{7}$.

....

(ii) Using your answer to **part (a)(i)**, find a Pythagorean Triple that may be formed from the sum of $\frac{1}{5}$ and $\frac{1}{7}$.

(...................... , ... , ...)

(iii) Find the sum of $\frac{1}{9}$ and $\frac{1}{11}$.

....

(iv) Can a Pythagorean Triple be formed from your answer to **part (a)(iii)**?
Explain your answer.

....

(b) For some values of p and q, the sum of the unit fractions $\frac{1}{p}$ and $\frac{1}{q}$ can be used to form a Pythagorean Triple.

(i) Write as a single algebraic fraction $\frac{1}{p} + \frac{1}{q}$.

....

(ii) Complete the Pythagorean Triple in terms of p and q.

(.......................... , ... , $pq + 2$)

(iii) By applying Pythagoras' Theorem to your answer to **part (b)(ii)**, show that

$$p^2 + q^2 = 2pq + 4.$$

(iv) Find the two algebraic relationships between p and q so that $\frac{1}{p} + \frac{1}{q}$ can be used to form a Pythagorean Triple.

....

....

PART B STARTS ON PAGE 8.

B MODELLING **RESCUE MISSION (20 marks)**

You are advised to spend no more than 45 minutes on this part.

A rescue team is going to fly to a disaster area.
There must be at least 35 people in the team and they need at least 80 tonnes of equipment.

The objective of this task is to find the best way to organise the rescue mission.

Two planes, X and Y, are available to take the rescue team and their equipment.
Both planes must be back at the starting airport in less than 24 hours.

The maximum loads, flight times and total cost for each plane are shown in the table.

Plane	Maximum number of people per flight	Maximum mass of equipment (tonnes)	Maximum **return** flight time (hours)	Cost for **return** flight ($ thousand)
X	5	10	3	40
Y	7	20	4	65

Let x be the number of **return** flights made by plane X
and y be the number of **return** flights made by plane Y.

The rescue mission is to be modelled by five inequalities.

1 (a) (i) The total mass, in tonnes, of the equipment carried by the two planes is $10x + 20y$.

Explain why the mass of the equipment to be carried by the two planes can be shown by the inequality $10x + 20y \geqslant 80$.

....

(ii) Find, in terms of x and y, an inequality to show the total number of people carried by the two planes.

....

(iii) Find, in terms of x and y, an inequality to show the total flight time of the two planes.

....

(b) Complete each inequality below to show

(i) the greatest number of flights that plane *X* would be able to make,

$0 \leqslant x \leqslant$ ....

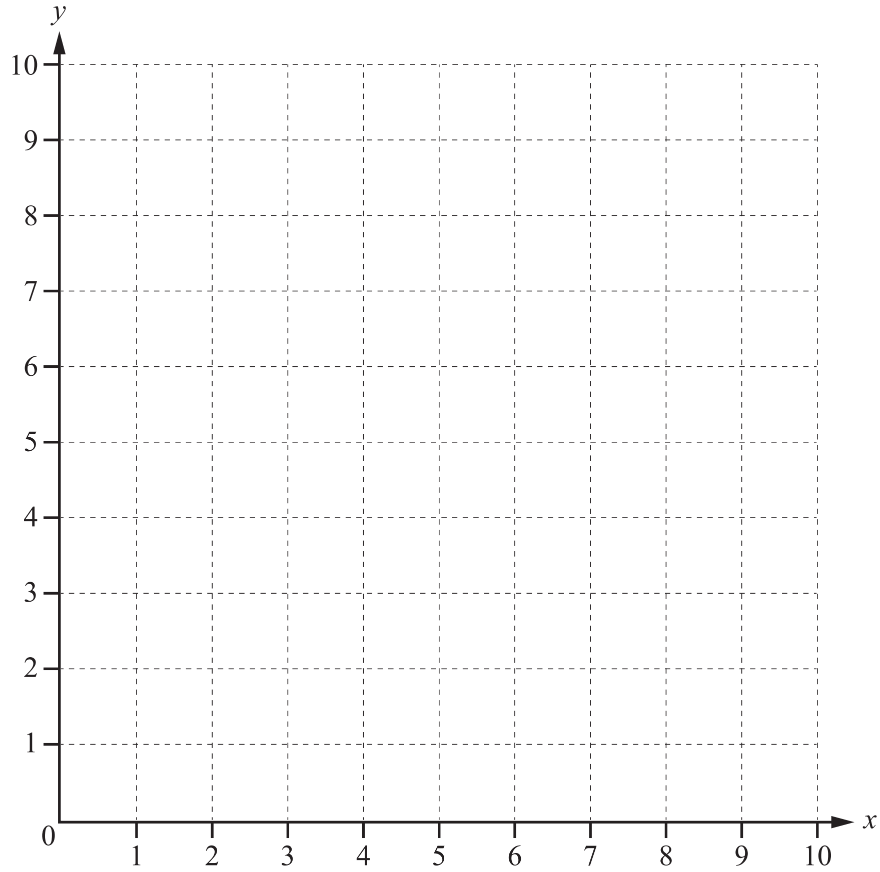
(ii) the greatest number of flights that plane *Y* would be able to make.

$0 \leqslant y \leqslant$ ....

(c) Write an expression, in terms of *x* and *y*, for the total cost of the flights, in thousands of dollars.

....

2 **(a)** On the grid, find the region satisfied by the inequalities from **question 1(a)** and **question 1(b)**.
Shade the **unwanted** region.



(b) Two of the lines cross when $x = 4\frac{2}{3}$ and $y = 1\frac{2}{3}$.

Explain why these values should not be used to find the minimum cost.

....

(c) When $x = 0$ and $y = 5$, the cost of the rescue mission is 325 thousand dollars.

Using your diagram, find the minimum cost of the rescue mission.
Write down the number of times each plane must fly.

plane *X* .... plane *Y* ....

minimum cost = ...thousand dollars

3 Plane *X* is not available to make more than 4 return flights.

How many times should each plane be used to minimise the cost?
What would be the **increase** in cost compared to your solution in **question 2**?

plane *X* .... plane *Y* ....

increase in cost = ...thousand dollars

4 Compare your solutions to **question 2(c)** and **question 3**.

Which is the better solution? Explain your answer.

....

....

....

5 A third plane, Z, is available to assist with the mission.
This plane must also be back at the starting airport in less than 24 hours.
The maximum loads, flight times and total cost for each plane are shown in the table.

Plane	Maximum number of people per flight	Maximum mass of equipment (tonnes)	Maximum **return** flight time (hours)	Cost for **return** flight ($ thousand)
X	5	10	3	40
Y	7	20	4	65
Z	4	8	2	50

Let x be the number of **return** flights made by plane X,
y be the number of **return** flights made by plane Y,
and z be the number of **return** flights made by plane Z.

This rescue mission is to be modelled by six inequalities.

(a) Find the six inequalities and an expression for the cost in terms of x, y and z.

Inequalities ....

....

....

....

....

....

Cost..

(b) Explain why the **method** of solution used in **question 2** is not appropriate for this model.

....

....