Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

7096774549

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/63**

Paper 6 (Extended) **May/June 2019**

1 hour 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO **NOT** WRITE IN ANY BARCODES.

Answer both parts **A** (Questions 1 to 3) and **B** (Questions 4 to 6).
You must show all the relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40.

This document consists of **16** printed pages.

DC (NH/CGW) 170705/2

The Investigation starts on page 3.

Answer both parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 to 3)

JUMPING FROGS (20 marks)

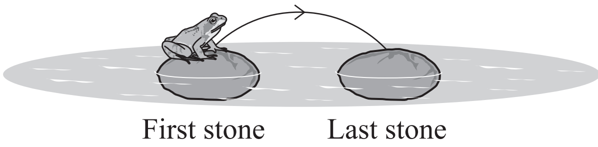
You are advised to spend no more than 45 minutes on this part.

This investigation looks at the number of different ways that a frog can jump between stones in a line.

The stones are always 1 unit apart.
The frog always jumps

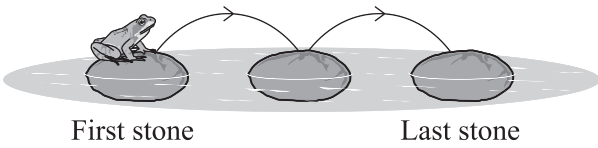
- from left to right
- from the first stone to the last stone.

The diagram shows a frog sitting on a stone in a pond.



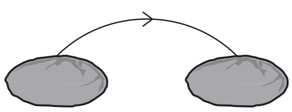
This frog has a jump length of **1 unit**. This is enough to move from one stone to the next stone.
There is only 1 way to jump between two stones.

There is only 1 way to jump between three stones.

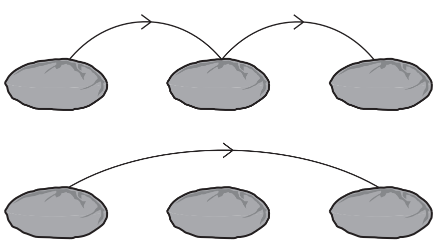


When there are more than three stones in a line, there is always only 1 way for this frog to jump from the first stone to the last stone.

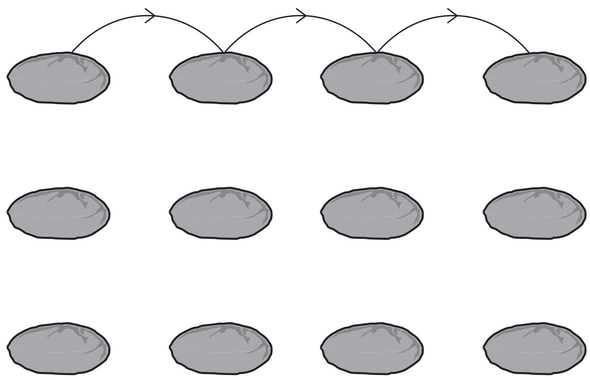
1 A different frog has a maximum jump length of **2 units**.
There is still only 1 way to jump between two stones.



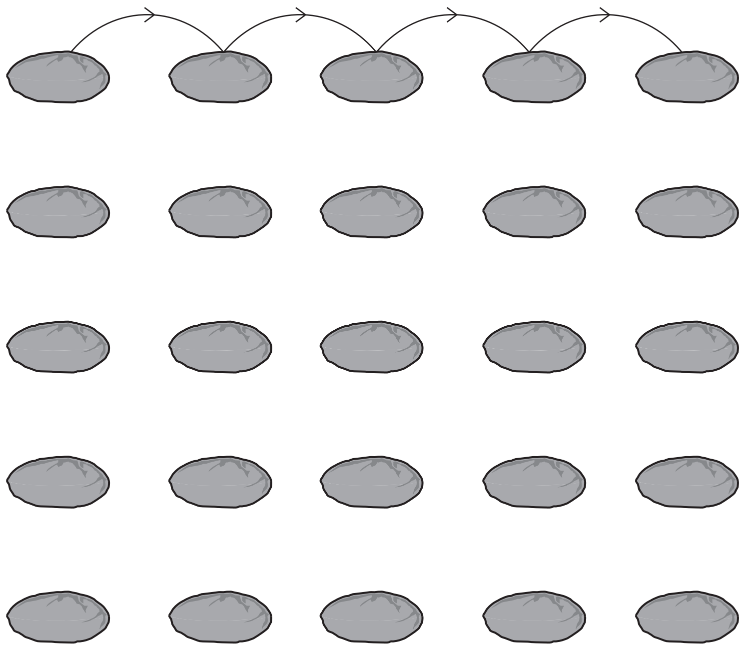
There are now 2 ways to jump between three stones.



(a) Complete the diagrams below to show the 3 ways that this frog can jump between four stones.



(b) Complete the diagrams below to show the 5 ways that this frog can jump between five stones.



(c) The table shows the number of ways to jump between stones when the maximum jump length is 2 units.

Number of stones	Number of ways
2	1
3	2
4	3
5	5
6	8
7	13
8	
9	

The numbers in the last column of the table form a sequence.

(i) Complete the table.

(ii) Write down the rule to find further terms in this sequence.

....

....

2 Another frog has a maximum jump length of **3 units**.

There is only 1 way for this frog to jump between two stones.

There are 2 ways to jump between three stones.

(a) There are 4 ways to jump between four stones.
These are the same 3 ways as in **question 1(a)** and 1 new way.

Draw the **new** way on the diagram below.

(b) There are the same 5 ways as in **question 1(b)** to jump between five stones and some **new** ways.

Draw diagrams to show the **new** ways below.

(c) The table shows the number of ways to jump between stones when the maximum jump length is 3 units.

Number of stones	Number of ways
2	1
3	2
4	4
5	
6	13
7	24
8	
9	

The numbers in the last column of the table form a sequence.

(i) Use your answer to **part (b)** to help you complete the table.

(ii) Write down the rule to find further terms in this sequence.

....

....

3 The table shows the number of ways to jump between 2 to 9 stones when the maximum jump length is 1 to 8 units.

Number of stones	Maximum jump length							
	1 unit	2 units	3 units	4 units	5 units	6 units	7 units	8 units
2	1	1	1	1	1	1	1	1
3	1	2	2	2	2	2	2	2
4	1	3	4	4	4	4	4	4
5	1	5		8	8	8	8	8
6	1	8	13	15	16	16	16	16
7	1	13	24	29	31	32	32	32
8	1			56	61	63	64	64
9	1				120	125	127	128

(a) Complete the table.

(b) The numbers in the column for maximum jump length of 4 units form a sequence.

Write down the rule for finding further terms in this sequence.

....

....

(c) The terms of the sequence for a maximum jump length of **8 units** are

1, 2, 4, 8, 16, 32, 64, 128

(i) Find an expression, in terms of x, for the number of ways of jumping between x stones, where $2 \leqslant x \leqslant 9$.

....

(ii) Show that this expression does not give the number of ways for $x = 10$.

(d) Complete the two rules for finding the number of ways of jumping between x stones when the maximum jump length is m units.

Rule 1

For $2 \leqslant x \leqslant$ ....

the number of ways = ...

Rule 2

For $x >$ ....

the number of ways is found by ....

(e) A frog sits on the first of 20 equally spaced stones.
The frog has a maximum jump length of 18 units.

Find the number of ways the frog can jump between the 20 stones.

....

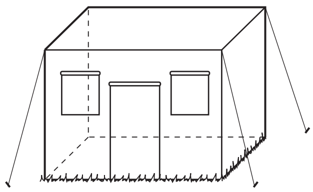
B MODELLING (QUESTIONS 4 to 6)

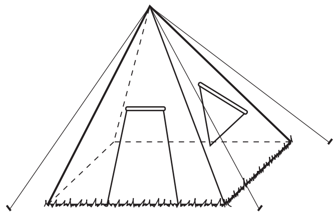
PLAY TENTS (20 marks)

You are advised to spend no more than 45 minutes on this part.

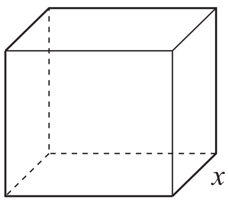
This task is about maximising the volume of a tent for a given area of material.

A company makes play tents for children.
They make them in the shape of cuboids or square-based pyramids.
The top and sides of the tents are made from material. No material is used for the base.


Cuboid tent


Pyramid tent

4 The cuboid tent has a square base of side x metres and height h metres.



(a) Write down the volume, V, of the cuboid in terms of h and x.

....

(b) Show that the total area, $A\,\mathrm{m}^2$, of material for the cuboid tent is

$$A = x^2 + 4hx.$$

(c) The company uses $8\,\text{m}^2$ of material to make each cuboid tent.

Use this information and **part (a)** and **part (b)** to show that a model for the volume is

$$V = \frac{(8 - x^2)x}{4}.$$

(d) **(i)** On the axes below, sketch the graph of the model $V = \frac{(8 - x^2)x}{4}$.

(ii) Find the maximum volume of a cuboid tent made from $8\,\text{m}^2$ of material.

....

(e) Customers want to buy tents with a height of at least 1 metre.

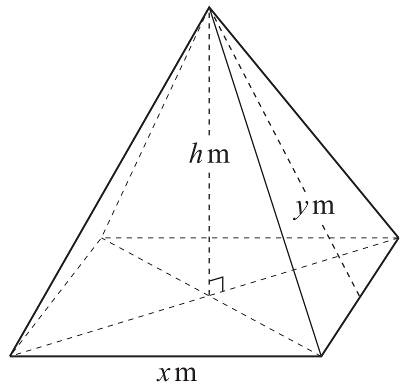
(i) Using your answer to **part (a)**, sketch, on the axes above, the graph of V against x when $h = 1$.

(ii) Find the volume of a cuboid tent made from $8\,\text{m}^2$ of material with a height of 1 metre.

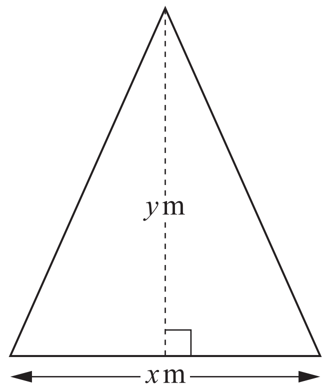
....

5 The pyramid tent also has a square base of side x metres and a height of h metres.

The distance between the top of the pyramid and the midpoint of each edge of the base is y metres.



The triangle below shows one of the faces of the pyramid.



(a) Find a model, in terms of x and y, for the **total** area, S, of the four triangular faces of the pyramid.
Give your answer in its simplest form.

....

(b) Find an expression for y^2 in terms of h and x.

(c) Use **part (a)** and **part (b)** to show that

$$h = \sqrt{\left(\frac{S^2}{4x^2} - \frac{x^2}{4}\right)}.$$

(d) The company uses $8\,\text{m}^2$ of material to make each tent.

Volume of a pyramid $= \frac{1}{3} \times$ base area $\times$ height

(i) Show that a model for the volume of the pyramid tent is

$$V = \frac{x^2}{3}\sqrt{\left(\frac{16}{x^2} - \frac{x^2}{4}\right)}.$$

(ii) On the axes below, sketch the graph of $V = \frac{x^2}{3}\sqrt{\left(\frac{16}{x^2} - \frac{x^2}{4}\right)}$.

(iii) Write down the maximum volume of a pyramid tent that uses $8\,\text{m}^2$ of material.

....

(iv) Customers want to buy tents with a height of at least 1 metre.

Sketch, on the axes on page 14, the graph of V against x when $h = 1$.

(v) Find the maximum volume of a pyramid tent made from $8\,\text{m}^2$ of material with a height of at least 1 metre.

....

Question 6 is printed on the next page.

6 The company decides that the cuboid tent and the pyramid tent should have the same volume.

Which type of tent will customers want to buy?
Give a reason for your answer.

Type of tent .... because ....

....