UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
International General Certificate of Secondary Education

CANDIDATE NAME

CENTER NUMBER

CANDIDATE NUMBER

8849734721

ADDITIONAL MATHEMATICS (US) **0459/02**

Paper 2 **May/June 2013**

2 hours

Candidates answer on the Question Paper

Additional Materials: Electronic calculator
List of formulas and statistical tables (MF25)

READ THESE INSTRUCTIONS FIRST

Write your Center number, candidate number, and name on the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

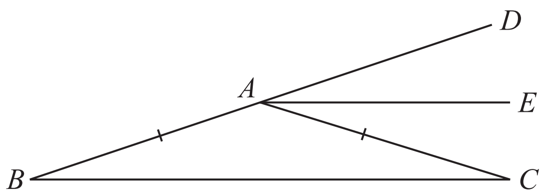
At the end of the examination, fasten all your work securely together.
The number of points is given in parentheses [ ] at the end of each question or part question.
The total number of points for this paper is 80.

This document consists of **16** printed pages.

DC (LEG/CGW) 66053/6

UNIVERSITY of CAMBRIDGE International Examinations

1



The diagram shows an isosceles triangle ABC with $AB = AC$. The line BA is extended to D and the line AE bisects angle DAC. Prove that AE is parallel to BC. [3]

2 **Without using a calculator**, express $\dfrac{(3+\sqrt{2})(2\sqrt{2}-1)}{6-\sqrt{2}}$ in the form $a + b\sqrt{2}$, where a and b are rational. [4]

3 P is the point (x, y) and S is the point $(3, 4)$.

(i) Write an expression for $(PS)^2$ in terms of x and y. [

P moves in such a way that the distance of P from S is equal to the distance of P from the line $x = 5$.

(ii) Find the equation of the parabola traced out by P in the form $y^2 = cx + dy$, where c and d are constants to be found. [3]

4 A tennis and badminton club has 169 members, male and female. The table below shows preference by the members for each of the two sports.

	Male	Female
Tennis	56	35
Badminton	48	30

(i) Find the probability that a member chosen at random is a female who prefers badminton. [1]

(ii) Determine if a preference for tennis is independent of being male or female. [3]

5 Solve the system of equations

$$x - 3y + 2 = 0,$$

$$x^2 + 4xy - 2y + 1 = 0.$$

[5]

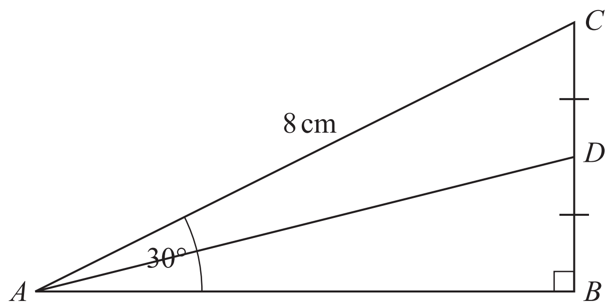
6 12 pairs of corresponding values of two variables, x and y, are plotted on a scatter diagram and a line of best fit is drawn. The sum of the x values is 72 and the sum of the y values is 120. Given below are four equations.

(a) $y = 2x + 2$ **(b)** $y = \dfrac{5x}{3}$ **(c)** $y = 10$ **(d)** $y = -2x + 22$

(i) For each of these four equations state whether it could or could not represent the line of best fit.

(ii) For each equation which is a possible line of best fit, state whether positive, negative or zero correlation is indicated. [5]

7



The diagram shows a triangle ABC in which $AC = 8\,\text{cm}$, angle $ABC = 90°$ and angle $BAC = 30°$. The point D is the mid-point of BC. **Without using a calculator**

(i) find the exact length of AD, [4]

(ii) show that the sine of angle ADC is $\sqrt{\frac{12}{13}}$. [2]

8 Solve the following equations.

(i) $e^{2x^2 - x} = 1$ [illegible]

(ii) $2\lg\sqrt{3x^2 - 14x + 15} = \lg 8 - \lg 2$ [4]

9 The table below refers to the probability distribution of a discrete random variable X.

X	0	2	5	n	12
Probability	$\frac{1}{16}$	$\frac{1}{2}$	p	$\frac{1}{8}$	$\frac{1}{16}$

(i) Given that X can only take five values, find p. [2]

(ii) Given that $\mathrm{E}(X) = 4$, find the value of n. [2]

(iii) Hence calculate $\mathrm{E}(X^2)$ and $\mathrm{Var}(X)$. [4]

10 The matrices **A**, **B** and **C** are given by

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{C} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

(i) Explain, in geometrical terms, the effect of each of the matrices **A**, **B** and **C** when regarded as a transformation of the plane. [3]

(ii) Find, as a single matrix, the matrix of the transformation obtained when **A** is followe
and explain its geometrical effect.

(iii) If the transformations represented by **B** and **C** are applied to any plane figure, determine whether or not it matters which transformation is applied first. [2]

11 A river, 180 m wide, flows at $3\,\text{ms}^{-1}$ between straight parallel banks. A canoeist, who can p at $5\,\text{ms}^{-1}$, crosses the river in her canoe.

(i) If the canoeist crosses to a point directly opposite her starting point, how long will the journey take her and at what angle to the bank must she steer? [5]

(ii) What is the shortest time she could take to cross the river and how far downstream w
this crossing take her?

12 Two complex numbers, z and w, are given by

$$z = 1 + 2\mathrm{i}, \qquad w = 1 - 3\mathrm{i}.$$

(i) Show that $\dfrac{w}{z} = -1 - \mathrm{i}$. [3]

(ii) Find the modulus and the argument of $\dfrac{w}{z}$. [2]

(iii) Write down and simplify an expression for $z + \overline{w}$, where $\overline{w}$ is the complex conjugate of w. [1]

(iv) Represent $z + \overline{w}$, $-w$ and w on an Argand diagram by the points P, Q and R respecti

(v) Write down the complex number represented by the mid-point of PR. [2]

Question 13 is printed on the next page.

13 The functions f and g are defined, for $x \in \mathbb{R}$, by

$$f(x) = \ln(x + 1) \quad \text{where} \quad x > a, \qquad\qquad g(x) = e^x - 1.$$

(i) State the value of a, given that this value is the least possible. [1]

(ii) Explain why f is a function. [1]

(iii) Show clearly that $fg(x) = x$ and explain what this result means. [3]

(iv) Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the axes below and explain how these graphs are related to one another. [3]