CAMBRIDGE INTERNATIONAL EXAMINATIONS

International General Certificate of Secondary Education

MARK SCHEME for the May/June 2014 series

0459 ADDITIONAL MATHEMATICS (BES)

0459/01 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

1		$y = (x+2)(x-1)(x-3)$ (x^2+x-2) or (x^2-4x+3) or (x^2-x-6) $y = x^3 - 2x^2 - 5x + 6$	**B1** **M1** **A1**	 **ft** *their* brackets cao
2		$\frac{x^2-2x-3}{x^2-5x+6} \times \frac{1}{x^2-1}$ $= \frac{(x-3)(x+1)}{(x-2)(x-3)} \times \frac{1}{(x+1)(x-1)}$ $= \frac{1}{(x-2)(x-1)}$ isw	**M1** **M1** **A1** **A1**	x^2-1 or $(x+1)(x-1)$ in denom of 2-layered fraction attempt factorise ⩾ 2 quad expns correctly factorise ⩾ 2 quad expns
3	**(i)**	$\frac{\text{No. of J \& F}}{\text{Total}}$ $= \frac{97}{132}$ or 0.735 (3 sf)	**M1** **A1**	attempted
	(ii)	Attempt P(F) and P($F \mid J$) P(F) = $\frac{72}{132}$ P($F \mid J$) = $\frac{30}{55}$ one correct Both = $\frac{6}{11}$ Because these are equal, A & B are indep	**M1** **A1** **A1** **A1**	attempt P(J) and P($J \mid F$) P(J) = $\frac{55}{132}$ P($J \mid F$) = $\frac{30}{72}$ Both = $\frac{5}{12}$ Because these are equal, A & B are indep
4	**(i)**	$\mathbf{A} = (2.25 \;\; 2.35)$ $\mathbf{C} = \begin{pmatrix} 8500 \\ 9400 \end{pmatrix}$	**B1** **B1**	
	(ii)	$\mathbf{A} = (2.25 \;\; 2.35) \begin{pmatrix} 0.020 & 0 \\ 0 & 0.018 \end{pmatrix} \begin{pmatrix} 8500 \\ 9400 \end{pmatrix}$ $= (0.045 \;\; 0.0423) \begin{pmatrix} 8500 \\ 9400 \end{pmatrix}$ or $(2.25 \;\; 2.35) \begin{pmatrix} 170 \\ 169.2 \end{pmatrix}$ $= 780.12$ or 780 (3 sf)	**M1** **A1** **A1**	first pair conformable and their product of correct shape correct figures and shape in first product dep all three mats conformable

5	**(i)**	$2 + 2\text{i}$ or $2(1 + \text{i})$	**B1**	or $2\sqrt{2}\text{cis}\frac{\pi}{4}$
	(ii)	$\sqrt{2}\,\text{i}$ or $0 + \sqrt{2}\,\text{i}$	**B2**	**B2** for $\sqrt{2}\,\text{i}$; **B1** for $k\text{i}$ ($k \neq \sqrt{2}$ or 0)
	(iii)	$\frac{2 + 2\text{i} + \sqrt{2}\text{i}}{2}$	**M1**	**ft** *their* **(i)** and **(ii)**
		$= 1 + \frac{2 + \sqrt{2}}{2}\text{i}$ or $1 + 1.71\text{i}$ (3 sf)	**A1**	
6	**(i)**	Centre $(-4, 3)$ stated or implied	**B1**	
		$(x + 4)^2 + (y - 3)^2$	**M1**	
		$= 5$	**M1**	dep **M1**
		$x^2 + 8x + 16 + y^2 - 6y + 9 = 5$		
		$x^2 + y^2 + 8x - 6y + 20 = 0$	**A1**	
	(ii)	grad $= -\frac{1}{2}$	**B1**	seen or implied
		$y - 1 = -\frac{1}{2}(x + 5)$ oe	**M1**	or $-4 = -1 \times 1 + c$ **ft** their gradient
		$y = -\frac{1}{2}x - \frac{3}{2}$ oe	**A1**	cao
7	**(i)**	$\text{P}(X = 2) = \frac{2}{5} \times \frac{1}{4}$ $(= \frac{1}{10}$ oe$)$	**M1**	
		$\text{P}(X = 3) = \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} + \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3}$ $(= \frac{1}{5}$ oe$)$	**M1**	correct products for any two of $\text{P}(X = 2)$, $\text{P}(X = 3)$, $\text{P}(X = 4)$ **M1M1**
		$\text{P}(X = 4) = 1 - (\frac{1}{10} + \frac{1}{5} + \frac{2}{5})$ $(= \frac{3}{10}$ oe$)$	**M1**	$1 - (\frac{2}{5}$ + sum of two attempted $\text{P}(X = n))$ **M1**
		$\frac{1}{10}, \frac{1}{5}, \frac{3}{10}$	**A1**	all three probs correct
	(ii)	Σxp attempted	**M1**	
		$= 4$	**A1ft**	
		$\Sigma x^2 p$ attempted (= 17)	**M1**	
		$-$ "4"^{2}	**M1**	dep previous **M1** and +ve result
		$= 1$	**A1ft**	
8		$\left(1 + \sqrt{2}\right)^2 = 3 + 2\sqrt{2}$	**B1**	or $(1 - \sqrt{2})^2 = 3 - 2\sqrt{2}$
		$\frac{4 - \sqrt{2}}{3 + 2\sqrt{2}} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}}$	**M1**	Mult numerator and denominator by $3 - 2\sqrt{2}$ or $(1 - \sqrt{2})^2$
		$16 - 11\sqrt{2}$	**A1** **A1**	**A1** for numerator $= 16 - 11\sqrt{2}$ **A1** for denominator = 1 oe

9		Eliminate x or y $4x^2 - x - 5 = 0$ or $4y^2 - 45y = 0$ Factorise quadratic $x = \frac{5}{4}$ and -1 $y = \frac{45}{4}$ and 0	**M1** **A1** **M1** **A1** **A1**	
10	**(a)**	$\sin 2A = -0.5$ or $\sin 330^\circ$ or $\sin 210^\circ$ 105° 165°	**M1** **A1** **A1**	 answer(s) only do(es) not score
	(b) (i)	$\sin P = \frac{4}{5}$ or $\cos Q = \frac{12}{13}$ $\textit{their}\,\frac{4}{5} \times \textit{their}\,\frac{12}{13} + \frac{3}{5} \times \frac{5}{13}$ $\frac{63}{65}$	**B1** **M1** **A1**	 answer only does not score
	(ii)	$\frac{\frac{5}{12} + 2}{1 - \frac{5}{12} \times 2}$ $\frac{29}{2}$ oe	**M1** **A1**	 answer only does not score
11	**(i)**	-2	**B1**	
	(ii)	$f \geqslant -9$	**B1**	allow $y \geqslant -3$ or $[-3, \infty)$
	(iii)	$\sqrt{y+9}$ seen $\sqrt{x+9}$ seen $f^{-1}(x) = -2 + \sqrt{x+9}$ oe	**M1** **M1** **A1**	may be implied by next mark interchanging x and y
	(iv)	Correct domain Correct use of mod Cusp at (1, 0)	**B1** **B1** **B1**	 Allow unlabelled cusp on +ve x-axis

12	**(i)**	$-4\mathbf{i} + 2\mathbf{j}$ $2\sqrt{5}$ or $\sqrt{20}$ or 4.47 (3 sf)	**M1** **A1**	May be implied by answer
	(ii)	$12\mathbf{i} - 6\mathbf{j}$ $k = -\frac{1}{3}$	**M1** **A1**	
	(iii)	They are collinear or equivalent	**B1**	or They all lie on the same straight line.
	(iv)	$\overrightarrow{OD} = 4\mathbf{i} - 2\mathbf{j}$ $\overrightarrow{OE}$ = (*their* $\overrightarrow{OD}$) $1.6\mathbf{i} - 0.8\mathbf{j}$	**B1** **M1** **A1**	
13	**(a)**	Correct change of base to $\frac{\log_b c}{\log_b a}$ $\log_b \left(a^2\right)^{3/2}$ $\log_b a^3 - \log_b c$ $\log b\left(\frac{a^3}{c}\right)$	**B1** **B1** **M1** **A1**	
	(b)	$3^{2x} - 3 \times 3^x - 4 = 0$ $(3^x + 1)(3^x - 4)$ $3^x = 4$ 1.26(18……) or $\log_3 4$ only	**M1** **M1** **A1** **A1**	allow substituted letters for 3^x ignore other soln, if given, for this **A1** $3^x = -1$must be discarded for last **A1**