Cambridge IGCSE

Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME			
CENTER NUMBER		CANDIDATE NUMBER	

2128538704

MATHEMATICS (US) **0444/41**

Paper 4 (Extended) **May/June 2019**

2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Geometrical instruments
Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your center number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.
If work is needed for any question it must be shown in the space provided.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant digits.
Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

The number of points is given in parentheses [ ] at the end of each question or part question.
The total of the points for this paper is 130.

Write your calculator model in the box below.

Formula List

For the equation $ax^2 + bx + c = 0$ $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Lateral surface area, A, of cylinder of radius r, height h. $\quad A = 2\pi rh$

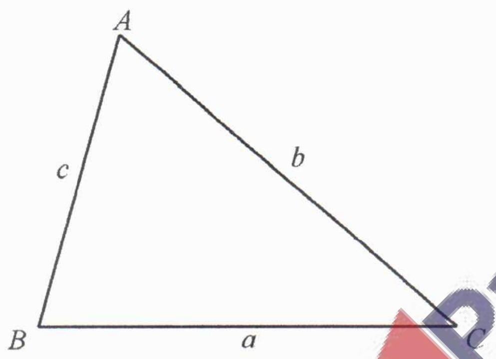
Lateral surface area, A, of cone of radius r, sloping edge l. $\quad A = \pi rl$

Surface area, A, of sphere of radius r. $\quad A = 4\pi r^2$

Volume, V, of pyramid, base area A, height h. $\quad V = \frac{1}{3}Ah$

Volume, V, of cone of radius r, height h. $\quad V = \frac{1}{3}\pi r^2 h$

Volume, V, of sphere of radius r. $\quad V = \frac{4}{3}\pi r^3$

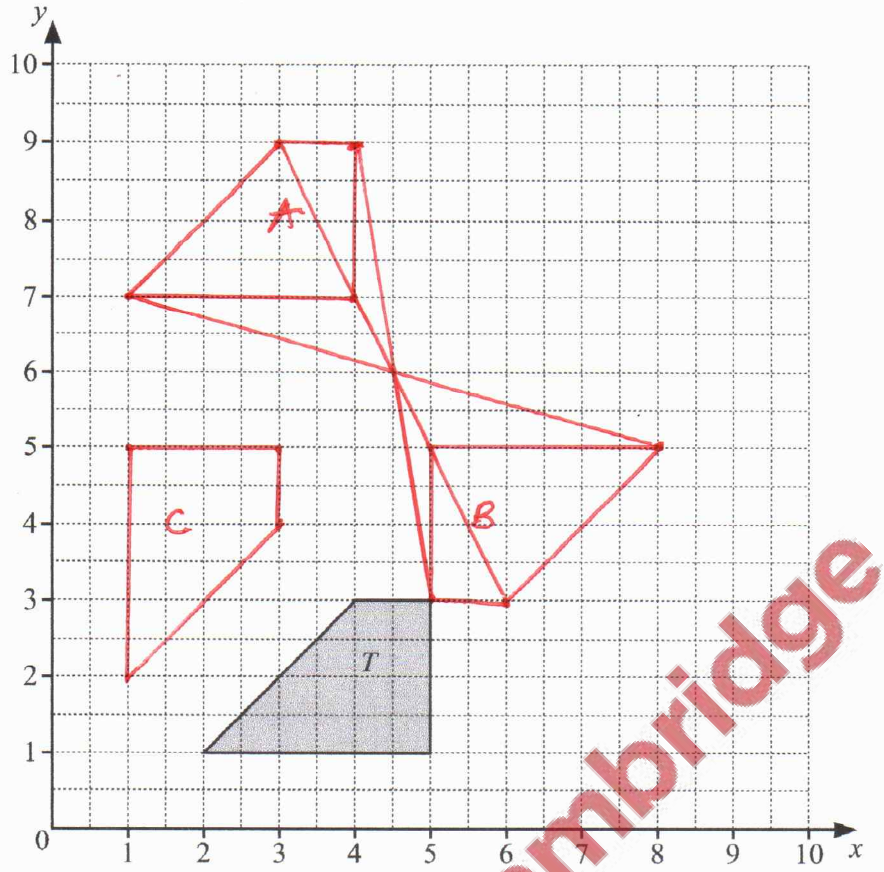


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area} = \frac{1}{2}bc\sin A$$

1



(a) **(i)** Translate shape T by the vector $\begin{pmatrix} -1 \\ 6 \end{pmatrix}$.

Label the image A. [2]

(ii) Rotate shape T about the point (5, 3) through 180°.
Label the image B. [2]

(iii) Describe fully the **single** transformation that maps shape A onto shape B.

Rotation 180°, Centre (4.5, 6)

It is enlargement by scale factor 1, Centre (4.5, 6) [3]

(b) **(i)** Reflect shape T in the line $y = x$. [2]
Label the image C.

(ii) Shape C can be mapped onto shape A by a rotation about the point (1, 7) followed by a reflection.

Write down

(a) the angle of rotation,

90° anticlockwise [1]

(b) the equation of the line of reflection.

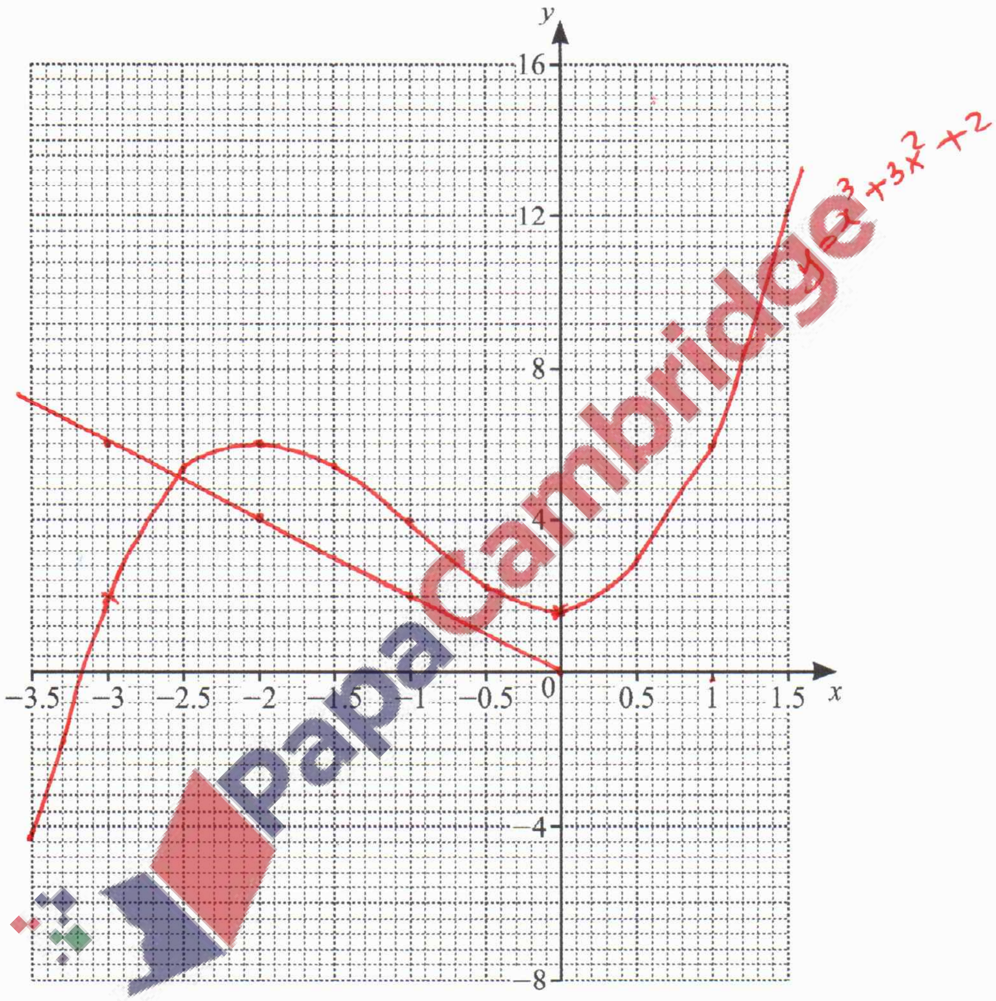
$x = 3.5$ [1]

2 The table shows some values for $y = x^3 + 3x^2 + 2$.

x	−3.5	−3	−2.5	−2	−1.5	−1	−0.5	0	0.5	1	1.5
y	−4.1	2	5.1	6	5.4	4	2.6	2	2.9	6	12.1

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = x^3 + 3x^2 + 2$ for $-3.5 \leqslant x \leqslant 1.5$.



[4]

(c) Use your graph to solve the equation $x^3 + 3x^2 + 2 = 0$ for $-3.5 \leqslant x \leqslant 1.5$.

$y = x^3 + 3x^2 + 2$
$0 = x^3 + 3x^2 + 2$ $\quad y = 0$

$x =$ .... -3.3 .... [1]

(d) By drawing a suitable straight line, solve the equation $x^3 + 3x^2 + 2x + 2 = 0$ for $-3.5 \leqslant x \leqslant 1.5$.

x	0	1	2	3
y	0	-2	-4	-6

$y = x^3 + 3x^2 + 2$
$0 = x^3 + 3x^2 + 2x + 2$
$y \quad 0 \quad 0 \; -2x$

$y = -2x$

x	-3	-2	-1
	6	4	2

$x =$ .... -2.55 .... [2]

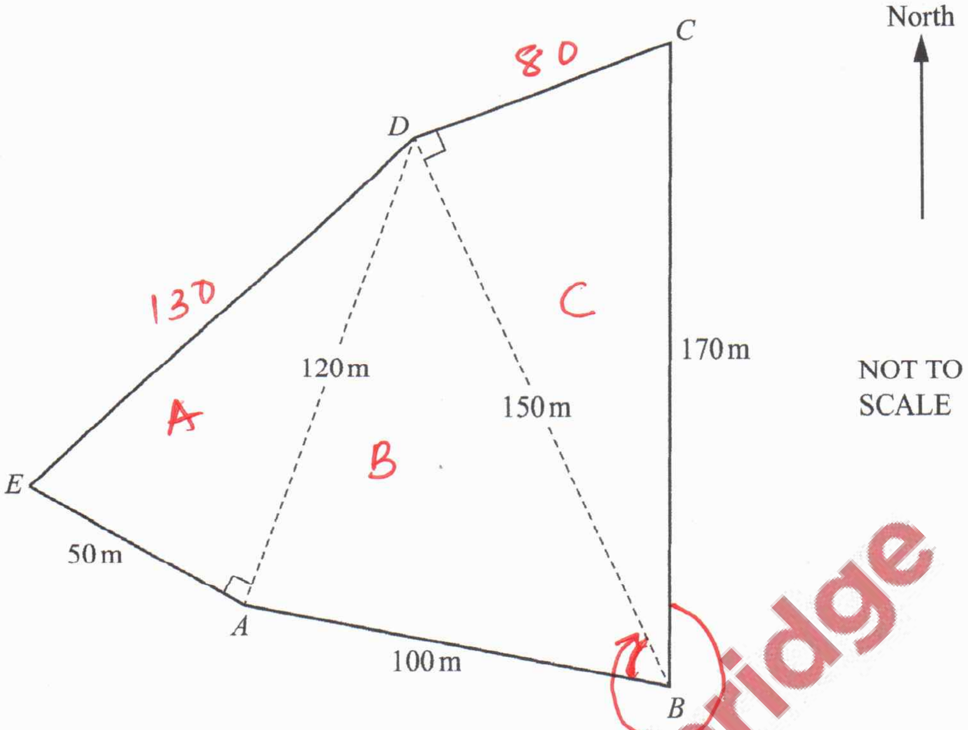
(e) For $-3.5 \leqslant x \leqslant 1.5$, the equation $x^3 + 3x^2 + 2 = k$ has three solutions and k is an integer.

Write down a possible value of k.

$x^3 + 3x^2 + 2 = k$
$y = 0$

$k =$ .... 3, 4 .... [1]

3



The diagram shows a field *ABCDE*.

(a) Calculate the perimeter of the field *ABCDE*.

$ED^2 = EA^2 + AD^2$
$= 50^2 + 120^2$
$= \sqrt{2500 + 14400}$
$= \sqrt{16{,}900}$
$= 130$

$DC^2 = 170^2 - 150^2$
$= 28900 - 22500$
$= \sqrt{6400} = 80$

Perimeter $= 100 + 50 + 130 + 80 + 170$
$= 530$

530 .... m [4]

(b) Calculate angle *ABD*.

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\cos B = \dfrac{150^2 + 100^2 - 120^2}{2 \times 150 \times 100}$

$\cos B = \dfrac{22500 + 10{,}000 - 14400}{30{,}000}$

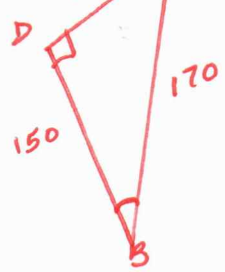
$\cos B = \dfrac{18100}{30{,}000}$

$\cos^{-1} B = 0.603\dot{3}$

$B = 52.89°$

$\approx 52.9°$

Angle *ABD* = ... [4]



(c) **(i)** Calculate angle CBD.

$\cos CBD = \frac{150}{170}$

$\cos^{-1}(CBD) = 0.882$
≈ 28.072

Angle CBD = ...28.1°...... [2]

(ii) The point C is due north of the point B.

Find the bearing of D from B.

$360° - 28.1°$
$= 331.9°$

......331.9°...... [2]

(d) Calculate the area of the field $ABCDE$.
Give your answer in hectares.
[1 hectare = $10\,000\,\text{m}^2$]

Area of A = $\frac{1}{2} \times 50 \times \cancel{120}^{60} = 3000\ \text{cm}^2$

Area of B = $\frac{1}{2} \times \cancel{100}^{50} \times 150 \sin 52.9 = 5981.879\ \text{cm}^2$

Area of C = $\frac{1}{2} \times \cancel{80}^{40} \times 150 = 6000\ \text{cm}^2$

14981.879

1 ha = $10{,}000\ \text{m}^2$
? = 14982

$\frac{14982}{10{,}000} = 1.4982$ hectares

......1.4982...... hectares [4]

4 (a) The test scores of 14 students are shown below.

21 21 23 26 25 21 22 20 21 23 23 27 24 21

(i) Find the range, mode, median, and mean of the test scores.

Range = Highest − lowest values
= 27 − 20
= 7

Mode = Most appearing number. (21)

Median =
~~20~~, ~~21~~, ~~21~~, ~~21~~, ~~21~~, ~~21~~, 22, 23, ~~23~~, ~~23~~, ~~24~~,
~~25~~, ~~26~~, ~~27~~

$\frac{22+23}{2} = 22.5$

Mean = $\frac{21+21+23+26+25+21+22+20+21+23+23+24+21+27}{14}$

$= \frac{318}{14} = 22.71$

Range = ...7..........

Mode = ...21..........

Median = ...22.5..........

Mean = ...22.7.......... [6]

(ii) A student is chosen at random.

Find the probability that this student has a test score of more than 24.

P(More than 24) = 3/14

25, 26, 27

..........3/14.......... [1]

(b) Petra records the score in each test she takes.

The mean of the first n scores is x.
The mean of the first $(n-1)$ scores is $(x+1)$.

Find the nth score in terms of n and x.
Give your answer in its simplest form.

(n+1)(x+1)
n(x+1) +1(x+1)
nx − n + x + 1

First score = nx
nx − (n−1)(x+1)
~~nx~~ − ~~nx~~ − n + x + 1
−n + x + 1
= x − n + 1

..........x − n + 1.......... [3]

(c) During one year the midday temperatures, t°C, in Zedford were recorded.
The table shows the results.

F. d   5   17   20   24   1

Temperature (t°C)	$0 < t \leqslant 10$	$10 < t \leqslant 15$	$15 < t \leqslant 20$	$20 < t \leqslant 25$	$25 < t \leqslant 35$
Number of days	50	85	100	120	10

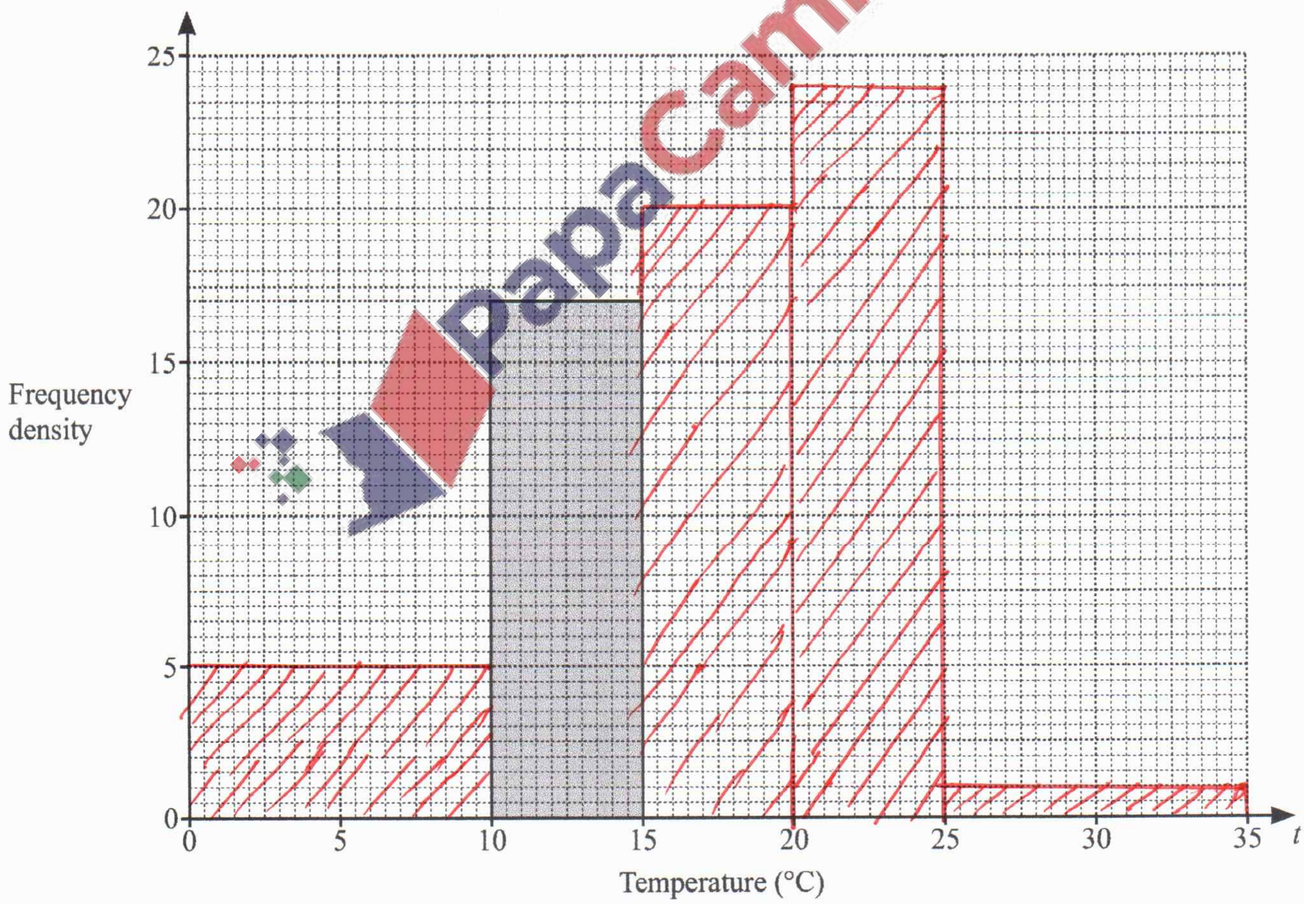
(i) Calculate an estimate of the mean.

Temp	Frequency	Midpoint x	FX
$0<t\leq 10$	50	5	250
$10<t\leq 15$	85	12.5	1062.5
$15<t\leq 20$	100	17.5	1750
$20<t\leq 25$	120	22.5	2700
$25<t\leq 35$	10	30	300

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{6062.5}{365} = 16.609 = 16.6$$

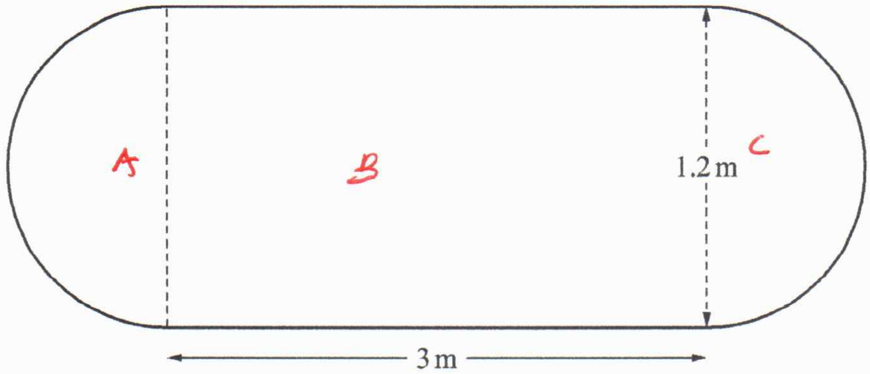
16.6 .... °C [4]

(ii) Complete the histogram to show the information in the table.



[4]

5



NOT TO SCALE

The diagram shows the surface of a garden pond, made from a rectangle and two semicircles.
The rectangle measures 3 m by 1.2 m.

(a) Calculate the area of this surface.

Area of A $= \frac{1}{2}\pi r^2$
$= \frac{1}{2} \times \pi \times 0.6 \times 0.6$
$= 0.565 m^2$

Area of B = L x W
$= 3 \times 1.2$
$= 3.6 m^2$

Area of C $= \frac{1}{2}\pi r^2$
$= \frac{1}{2} \times \pi \times 0.6 \times 0.6$
$= 0.565 m^2$

Total area $= 0.565 m^2 + 3.6 m^2 + 0.565 m^2$
$= 4.73 m^2$

4.73 m² .... m^2 [3]

(b) The pond is a prism and the water in the pond has a depth of 20 cm.

Calculate the number of liters of water in the pond.

$$\frac{4.73 \times 100 \times 100 \times 20}{1000}$$

$$= \frac{946{,}000}{1000}$$

$$= \underline{946 \text{ Litres}}$$

946 liters [3]

(c) After a rainfall, the number of liters of water in the pond is 1007.

Calculate the increase in the depth of water in the pond.
Give your answer in centimeters.

Increase $= 1007 - 946$

$= \underline{61}$

$$\frac{61}{946} \times 20$$

$= 1.2896$

$= \underline{1.29 \text{ CM}}$

1.29 cm [3]

6 (a) (i) $s = ut + \frac{1}{2}at^2$

Find s when $t = 26.5$, $u = 104.3$ and $a = -2.2$.
Give your answer in scientific notation, correct to 4 significant figures.

$S = ut + \frac{1}{2}at^2$
$= (104.3 \times 26.5) + \frac{1}{2} \times -2.2 \times (26.5)^2$
$= 2{,}763.95 + (-772.475)$
$= 1991.475$
$= 1991 \quad = 1.991 \times 10^3$

$s =$ 1.991×10^3 [4]

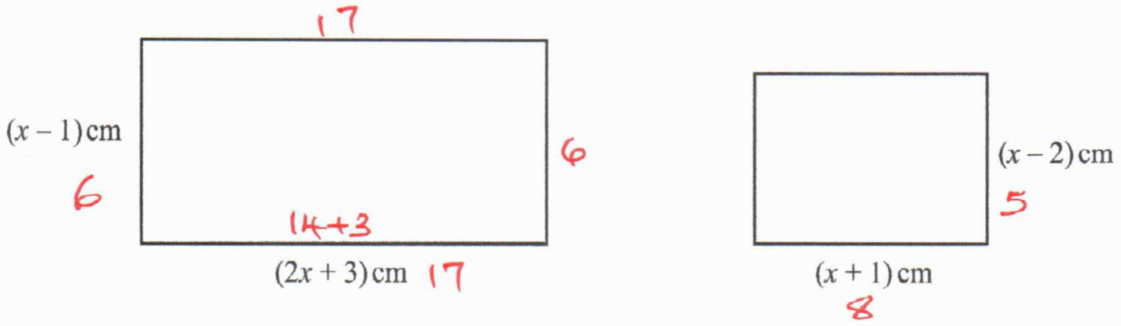
(ii) $s = ut + \frac{1}{2}at^2$

Solve for a.

$S = ut + \frac{1}{2}at^2$
$2(S - ut) = \frac{1}{\cancel{2}}at^2 \times 2$
$\frac{2(s-ut)}{t^2} = \frac{at^2}{\cancel{t^3}}$
$a = \frac{2(s-ut)}{t^2}$

$a =$ $\frac{2(s-ut)}{t^2}$ [3]

(b)



NOT TO SCALE

The difference between the areas of the two rectangles is $62\,\text{cm}^2$.

(i) Show that $x^2 + 2x - 63 = 0$.

$(2x+3)(x-1) \quad - (x+1)(x-2)$

$2x(x-1) + 3(x-1) \quad - x(x-2)1(x-2)$

$2x^2 - 2x + 3x - 3 \quad - x^2 - 2x + x - 2$

$= 2x^2 + x - 3 \quad - x^2 - x - 2$

$= 2x^2 - x^2 + x - (-x) - 3 - (-2) = 63$

$= x^2 + 2x = 62 + 1$

$x^2 + 2x - 63 = 0$

[3]

(ii) Factor $x^2 + 2x - 63$.

$P = -63 \quad (9, -7)$

$S = 2$

$x^2 + 9x - 7x - 63$

$x(x+9) - 7(x+9)$

$(x+9)(x-7)$

$(x+9)(x-7)$ .... [2]

(iii) Solve the equation $x^2 + 2x - 63 = 0$ to find the difference between the perimeters of the two rectangles.

$(x+9)(x-7) = 0$

$x + 9 = 0 \quad x - 7 = 0$

$x = -9 \quad x = 7$

Ignore negative value.

Large Rectangle Perimeter $= 2(L + w)$

$= 2(17 + 6)$

$= 2(23)$

$= 46$ cm

Smaller rectangle $= 2(L + w)$

$= 2(8 + 5)$

$= 26$ cm

Difference between Perimeters

$= 46\text{cm} - 26\text{ cm}$

$= 20$ cm

20 cm .... cm [2]

7 **(a)** The price of a book increases from \$2.50 to \$2.65 .

Calculate the percentage increase.

2·50 to 2·65

Increase $= 2.65 - 2.50$

$= 0.15$

% Increase $= \frac{0.15}{2.50} \times 100\% = 6\%$

6 .... % [3]

(b) Scott invests \$500 for 14 years at a rate of 1.5% per year simple interest.

Calculate the value of his investment at the end of the 14 years.

$\boxed{\text{Amount} = \text{Principal} + \text{Interest}}$

$I = P \times \frac{R \times T}{100}$

$500 \times \frac{1.5}{100} \times 14$

$= 105$

Amount Invested $= 500 + 105$

$= 605$

\$ 605 .... [3]

(c) Marie invests \$500 for 14 years at a rate of 1.5% per year compound interest.

Calculate the value of her investment at the end of the 14 years.

$A = P(1 + r/100)^n$

$= 500(1 + \frac{1.5}{100})^{14}$

$= 500(1.015)^{14}$

$= 615.8779$

$= 615.88$

\$ ....615.88........ [2]

(d) Pedro invests \$500 at a rate of r% per year compound interest.
At the end of 14 years the value of his investment is \$586.80 .

Find the value of r.

$A = P(1 + r/100)^n$

$\sqrt[14]{\frac{586.80}{500}} = \frac{\cancel{500}(1 + r/100)^{14}}{\cancel{500}}$

$\sqrt[14]{1.1736} = \sqrt[14]{(1 + r/100)^{14}}$

$1.0114996 = 1 + r/100$

$1.0114996 - 1 = r/100$

$100 \times 0.0114996 = \frac{r}{\cancel{100}} \times \cancel{100}$

$1.14996 = r$

$r = \simeq 1.15$

$r =$ ....1.15........ [3]

8 **(a)** Solve the equation $2x^2 + 3x - 4 = 0$.
Show all your working and give your answers correct to 2 decimal places.

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times -4}}{4}$

$\frac{-3 \pm \sqrt{9 - (-32)}}{4}$

$\frac{-3 \pm \sqrt{41}}{4}$

$\frac{-3 \pm 6.403}{4}$

$x = 0.850$

$\simeq 0.85$

or $\frac{-3 - 6.403}{4}$

$= -2.35075$

$\simeq -2.35$

$x =$ ...... 0.85 ...... or $x =$ ...... −2.35 ...... [4]

(b) Solve the following equations.

(i) $\sqrt{x} - 1 = 1 - 2\sqrt{x}$

$\sqrt{x} + 2\sqrt{x} = 2$

$(3\sqrt{x})^2 = (2)^2$

$\frac{9x}{9} = \frac{4}{9}$ $x = 4/9$

$x =$ ...... 4/9 ...... [2]

(ii) $5^{x-3} = 1$

$5^{x-3} = 5^0$

$x - 3 = 0$

$x = 3$

$x =$ ...... 3 ...... [1]

9 $\quad f(x) = 7x - 2 \qquad g(x) = x^2 + 1 \qquad h(x) = 3^x$

(a) Find g(h(2)).

$h(2) = 3^2$

$g(9) = 9^2 + 1$

$= 81 + 1$

$= 82$

82 .... [2]

(b) Find $f^{-1}(x)$.

$y = 7x - 2$

$x = 7y - 2$

$\frac{x+2}{7} = \frac{7y}{7}$

$y = \frac{x+2}{7}$

$f^{-1}(x) = \frac{x+2}{7}$

$f^{-1}(x) =$ $\frac{x+2}{7}$ .... [2]

(c) $g(g(x)) = ax^4 + bx^2 + c$

Find the values of a, b, and c.

$g(x) = (x^2+1)(x^2+1)$

$= x^2(x^2+1) + 1(x^2+1)$

$= x^4 + x^2 + x^2 + 1$

$= x^4 + 2x^2 + 1 + 1$

$= x^4 + 2x^2 + 2$

$ax^4 + bx^2 + c$

$a = 1 \quad b = 2, \; c = 2+$

$a =$ 1 ....

$b =$ 2 ....

$c =$ 2 .... [3]

(d) Find x when $h(f(x)) = 81$.

$f(x) = 7x - 2$

$h(7x-2) = 3^{7x-2} = 81$

$= 3^{7x-2} = 3^4$

$= 7x - 2 = 4$

$\frac{7x}{7} = \frac{6}{7}$

$x = 6/7$

$x =$ 6/7 .... [3]

10 The volume of each of the following solids is $1000\,\text{cm}^3$.

Calculate the value of x for each solid.

(a) A cube with side length x cm.

$V = l^3$

$\sqrt[3]{1000}$

$= \underline{\underline{10}}$

$x =$ 10 cm .... [1]

(b) A sphere with radius x cm.

$3 \times 1000\,\text{cm}^3 = \frac{4}{3}\pi r^3 \times 3$

$\frac{3000\,\text{cm}^3}{4\pi} = \frac{4\pi r^3}{4\pi}$

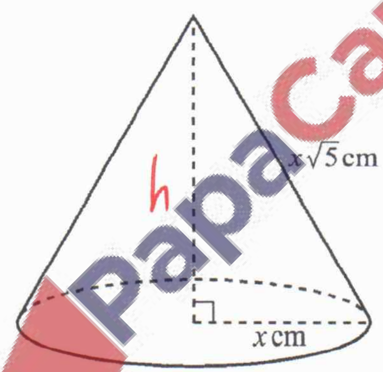
$r^3 = \sqrt[3]{238.7324}$

$r = 6.2035$

$\approx \underline{\underline{6.2\text{ cm}}}$

$x =$ 6.2 .... [3]

(c)



NOT TO SCALE

A cone with radius x cm and sloping edge $x\sqrt{5}$ cm.

$h^2 = (x\sqrt{5})^2 - x^2$

$h^2 = (x\sqrt{5})(x\sqrt{5}) - x^2$

$h^2 = 5x^2 - x^2$

$h^2 = \sqrt{4x^2}$

$h = \underline{2r}$

$3 \times 1000 = \frac{1}{3}\pi \times r \times r \times 2r \times 3$

$\frac{3000}{2\pi} = \frac{2\pi r^3}{2\pi}$

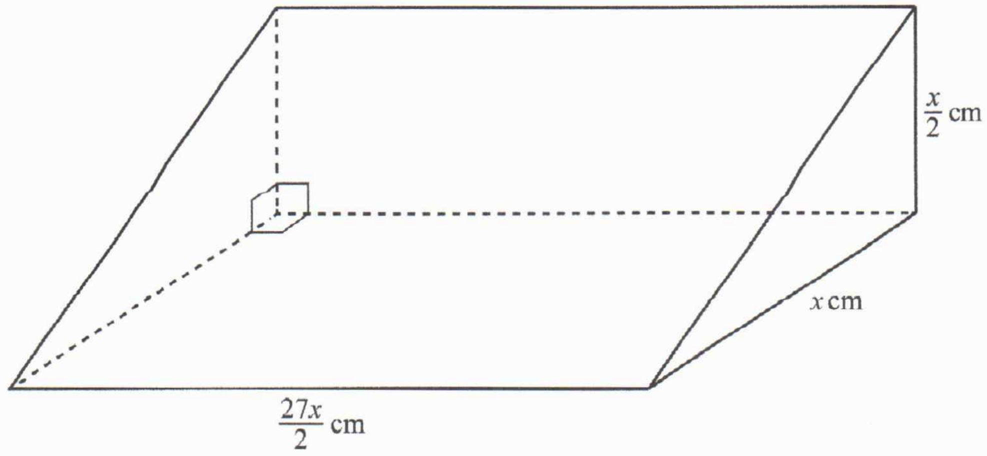
$r^3 = \sqrt[3]{\frac{3000}{2\pi}}$

$r = 7.8159$

$r = \underline{\underline{7.82}}$

$x =$ 7.82 cm .... [4]

(d)



NOT TO SCALE

A prism with a right-angled triangle as its cross-section.

$= \frac{1}{2}(x)\left(\frac{x}{2}\right)$

$= \frac{x^2}{4}$

$8 \times 1000 = \frac{x^2}{4} \times \frac{27x}{2}$

$\frac{8000}{27} = x^3$

$x = \sqrt[3]{\frac{8000}{27}}$

$x = 6.667$

$= \underline{6.67 \text{ cm}}$

$x =$ ......6.67...... [4]

Question 11 is printed on the next page.

11 Brad traveled from his home in New York to Chamonix.

- He left his home at 1630 and traveled by taxi to the airport in New York.
  This journey took 55 minutes and had an average speed of 18 km/h.

- He then traveled by plane to Geneva, departing from New York at 2215.
  The flight path can be taken as an arc of a circle of radius 6400 km with a sector angle of 55.5°.
  The local time in Geneva is 6 hours ahead of the local time in New York.
  Brad arrived in Geneva at 1125 the next day.

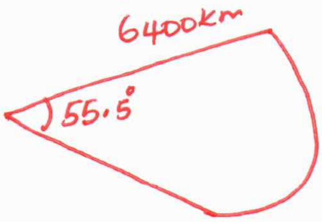
- To complete his journey, Brad traveled by bus from Geneva to Chamonix.
  This journey started at 1300 and took 1 hour 36 minutes.
  The average speed was 65 km/h.
  The local time in Chamonix is the same as the local time in Geneva.

Find the overall average speed of Brad's journey from his home in New York to Chamonix.
Show all your working and give your answer in km/h.

Distance = S × T

S = $= 18 \times \frac{55}{60}$

$= 16.5$ km



$l = \frac{\theta}{360} \times 2\pi r$

$= \frac{55.5}{360} \times 2\pi \times 6400$

$= 6199.$

$\simeq 6200$ km

$d = 65 \times \frac{96}{60} = 104$ km

Total distance $= 16.5 + 6200 + 104$ km

$= 6320.5$

$1630 \xrightarrow{55} 1725 \xrightarrow{4.50} 22.15 \xrightarrow{1:45} 2400$

$2400 \xrightarrow{5:25} 11.25 \xrightarrow{1:35} 1300$

Total time: 16hrs 06mins

Average speed $= \frac{\text{Total Distance}}{\text{Total time taken}}$

$= \frac{6320.5}{16.1}$

$= 392.5776398$

$\simeq 392.58$ km/h

393 .... km/h [11]