

5-Aug-24

Objective: Complete **iGCSE questions** on working with functions.

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(iii) Find $f^{-1}(x)$.

$f^{-1}(x) = \dots\dots\dots$ [2]

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(b) $g(x)$ is a function with an inverse function $g^{-1}(x)$.

Write down the value of $g(g^{-1}(3))$.

$\dots\dots\dots$ [

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13 (a) (i)	2.25 oe	2	M1 for $1 = 2(5 - 2x)$ or $5 - 2x = \frac{1}{2}$ c
(ii)	$-5 + 4x$ final answer	2	B1 for $5 - 2(5 - 2x)$
(iii)	$\frac{5-x}{2}$ oe final answer 2 3	2 1	M1 for $2x = 5 - y$ or $x = 5 - 2y$ or $\frac{y}{2}$

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10 $f(x) = x^2 - x - 30$ $g(x) = x^2 - 36$ $h(x) = 2x + 7$

(a) Find $h(f(7))$.

.....

(b) Find $h^{-1}(x)$.

$h^{-1}(x) =$

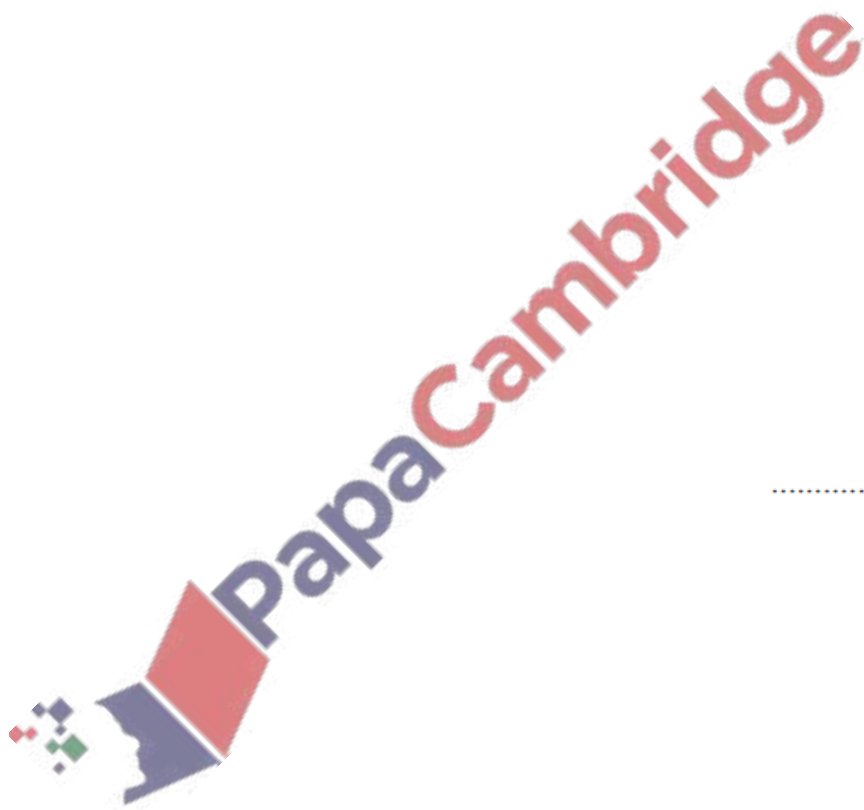
(c) Find $g(h(x))$ in its simplest factorised form.

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(d) Simplify $\frac{f(x)}{g(x)}$.



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Question	Answer	Mark	Part Marks
10 (a)	31	2	B1 for $[f(7) =] 12$ or M1 for $2(x^2 - x - 30) + 7$
(b)	$\frac{x-7}{2}$ oe	2	M1 for $y - 7 = 2x$ or $x = 2y + 7$ or $\frac{y}{2} = x + \frac{7}{2}$
(c)	$(2x+13)(2x+1)$ final answer	3	B2 for $(2x+7+6)(2x+7-6)$ or $4x^2 + 28x + 13$ or M1 for $(2x+7)^2 - 36$
(d)	$\frac{x+5}{x+6}$ final answer nfw	4	B2 for $(x-6)(x+5)$ or SC1 for $(x+a)(x+b)$ where $ab = -30$ or $a + b = 13$ and B1 for $(x+6)(x-6)$

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12 $f(x) = 4x + 2$ $g(x) = 5 - 2x$ $h(x) = x^2 - 3$

(a) Find $g(-3)$.

.....

(b) Find $f(h(2))$.

.....

(c) Find x when $f(x) = -10$.

$x =$

(d) Write down the range of $h(x)$.

..... [1]

(e) Find $f^{-1}(x)$.

$f^{-1}(x) =$ [2] Slid

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(f) $k(x) = 10 - 4x$

Describe fully the **single** transformation that maps the graph of $y = g(x)$ onto the graph of $y = k(x)$.

.....
 [3]

(g) The graph of $y = h(x)$ is translated by the vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

Find the equation of the graph of the image.
 Write your answer in the form $y = ax^2 + bx + c$.

$y =$ [3] S

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12(a)	11	1	
12(b)	6	2	B1 for $h(2) = 1$ soi or $4(x^2 - 3) + 2$ or better
12(c)	-3	2	M1 for $4x = -10 - 2$
12(d)	$h(x) \geq -3$	1	Allow $y \geq -3$
12(e)	$\frac{x-2}{4}$ oe final answer	2	M1 for $y - 2 = 4x$ or $x = 4y + 2$ or $\frac{y}{4} = x - 2$
12(f)	Stretch x-axis invariant [Scale factor] 2 OR Reflection $y = -2.70 + 6.75$ OR Rotation (2.5, 0) 167 (167.47) or 12.5 (12.53) clockwise	3 M1 A2 M1 A1 A1	B1 for each
12(g)	$[y =] x^2 - 4x + 1$	3	M2 for $y = (x - 2)^2 - 3$ or M1 for $x - 2$ seen in a quadratic If 0 scored, SC1 for $y = x^2 + 4x + 1$

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$$f(x) = x^2 + 1$$

$$g(x) = 3 + 2x$$

$$h(x) = \frac{1}{x+1}, x \neq -1$$

(a) Find $f(-3)$.

.....

(b) Find the value of $g(h(1))$.

.....

(c) Simplify $f(g(x)) + f(x)$.

.....

(d) Find $h^{-1}(x)$.

$$h^{-1}(x) = \dots\dots\dots [3]$$

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(e) Solve.

(i) $g(x) = 1$

$x = \dots\dots\dots$ [2]

(ii) $g^{-1}(x) = 1$

$x = \dots\dots\dots$ [1]

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8(a)	10	1	
8(b)	4	2	M1 for $[h(1) =] \frac{1}{2}$ or for $[gh(x) =] 3 + 2\left(\frac{1}{x+1}\right)$
8(c)	$5x^2 + 12x + 11$	3	M1 for $(3 + 2x)^2 + 1 + x^2 + 1$ B1 for $9 + 6x + 6x + 4x^2$ or better
8(d)	$\frac{1}{x} - 1$ or $\frac{1-x}{x}$ oe final answer	3	M1 correct first step M1 correct second step
8(e)(i)	- 1	2	M1 for $3 + 2x = 1$
8(e)(ii)	5	1	

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11 $f(x) = 10^x$ $g(x) = 2x - 1$

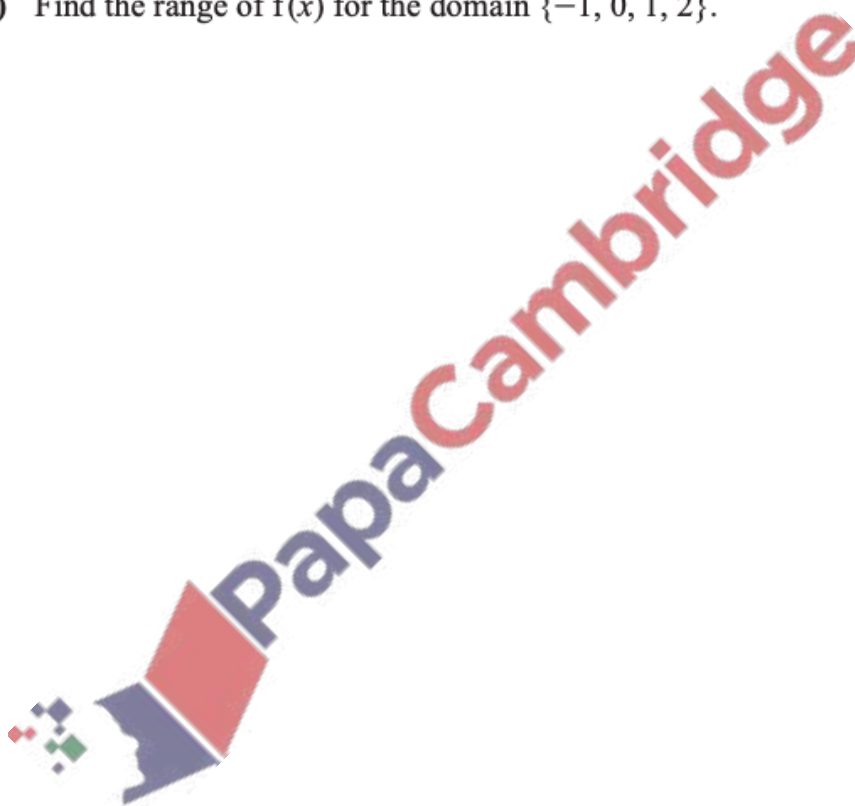
(a) Find the value of $g(3)$.

.....

(b) Find the range of $f(x)$ for the domain $\{-1, 0, 1, 2\}$.

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(c) Find x when $g(x) = 12$.

$x =$

(d) The graph of $y = g(x)$ is translated by the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ onto the graph of $h(x)$.

Find $h(x)$.

Give your answer in its simplest form.

$h(x) =$

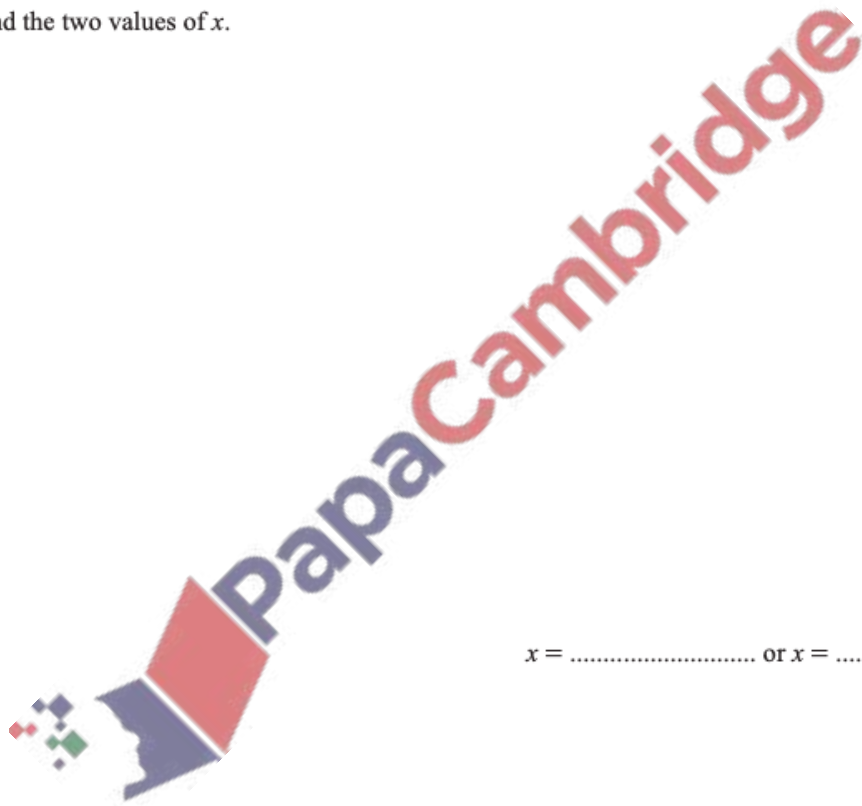
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(e) Find $f^{-1}(x)$.

$$f^{-1}(x) = \dots\dots\dots [2]$$

(f) $\tan(g(x)) = 1$ and $0^\circ \leq x \leq 180^\circ$.

Find the two values of x .



$$x = \dots\dots\dots \text{ or } x = \dots\dots\dots [3]$$

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11(a)	5	1	
11(b)	0.1oe, 1, 10, 100	2	B1 for 3 correct or all correct seen
11(c)	6.5 oe	2	M1 for $2x - 1 = 12$
11(d)	$2x - 2$ or $2(x - 1)$	3	B2 for correct unsimplified answer OR M1 for substituting $x - 2$ for x M1 for adding 3 to a function in $y = 2x + c$ OR M1 for $y = 2x + c$ ($c \neq -1$) leading to $y = 2x - 2$ with gradient 2 M1 for substituting coords of value $x = 1$ into $y = 2x + c$
11(e)	$\log x$	2	M1 for $\log y = x$ or $x = 10^y$
11(f)	23, 113	3	B2 for 23 or B1 for $[g(x) =]$ 45 soi

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11 $f(x) = 2x - 7$ $g(x) = \sqrt{x}$ $h(x) = \frac{1}{x}, x \neq 0$

(a) (i) Find $f(3)$.

.....

(ii) Solve $f(x) = 1$.

$x =$

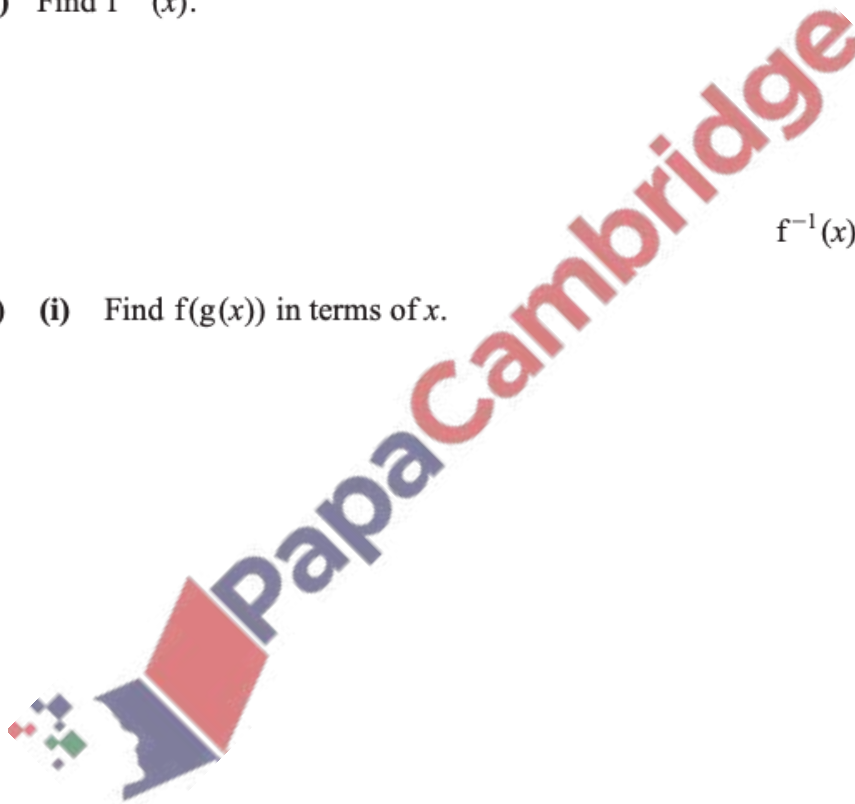
(b) Find $f^{-1}(x)$.

$f^{-1}(x) =$

(c) (i) Find $f(g(x))$ in terms of x .

.....

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(ii) Solve $f(g(x)) = 5$.

$x = \dots\dots\dots$

(d) (i) Find $h(g(f(x)))$ in terms of x .

$\dots\dots\dots$

(ii) Find an inequality in terms of x for which $h(g(f(x)))$ exists.

$\dots\dots\dots$

11(a)(i)	-1	1	
11(a)(ii)	4	2	M1 for $2x - 7 = 1$ or better
11(b)	$\frac{x+7}{2}$ oe	2	M1 for $y + 7 = 2x$ or $\frac{y}{2} = x - \frac{7}{2}$ or $x = 2y$ Allow $f(x)$ for y

Question	Answer	Marks	Partial Marks
11(c)(i)	$2\sqrt{x} - 7$ final answer	1	Allow $x^{\frac{1}{2}}$ for $\sqrt{x}$
11(c)(ii)	36	3	M2 for $\sqrt{x} = \frac{5+7}{2}$ or better e.g. $\sqrt{x} = 6$ or M1 for $2\sqrt{x} - 7 = 5$ or <i>their (c)(i) = 5</i>
11(d)(i)	$\frac{1}{\sqrt{2x-7}}$ oe final answer	2	M1 for $\sqrt{2x-7}$ If 0 scored SC1 for answer $\frac{1}{\text{their(c)(i)}}$
11(d)(ii)	$x > 3.5$	2	M1 for $2x - 7 > 0$

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12 $f(x) = 2x + 1$

$g(x) = 4 - 3x$

$h(x) = 2^x - 1$

(a) Find $h(-2)$.

.....

(b) Find $g^{-1}(x)$.

$g^{-1}(x) = \dots\dots\dots$

(c) Find $g(f(3))$.

.....

(d) Find and simplify $g(g(x))$.

.....

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(e) Find $h^{-1}(7)$.

(f) Write as a single fraction in its simplest form.

$$\frac{1}{f(x)} + \frac{1}{g(x)}$$

12(a)	$-\frac{3}{4}$ or -0.75	1	
12(b)	$\frac{4-x}{3}$ oe final answer	2	M1 for $x = 4 - 3y$ or $y + 3x = 4$ or $y - 4 = -3x$ or $\frac{y}{3} = \frac{4}{3} - x$
12(c)	-17	2	B1 for $[f(3)] = 7$ or M1 for $4 - 3(2x + 1)$ soi

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12(d)	$9x - 8$ final answer	2	M1 for $4 - 3(4 - 3x)$
12(e)	3	2	M1 for $2^x - 1 = 7$ or $\log_2(x + 1)$ oe
12(f)	$\frac{5-x}{(2x+1)(4-3x)}$ oe final answer	3	M1 for $4 - 3x + 2x + 1$ M1 for common denominator (2x + 1)

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(vi) Write as a single fraction in its simplest form.

$$\frac{3}{f(x)} + \frac{2}{g(x)}$$

(b) The function $h(x)$ has an inverse function $j(x)$.

Write down, in its simplest form, $j(h(x))$.



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10(a)(i)	11	1	
10(a)(ii)	-23	2	M1 for $5 - 2(3 \times 4 + 2)$ soi or $5 - 2(3x + 2)$
10(a)(iii)	$\frac{1}{8}$ oe	3	M1 for $5 - 2x = 2(3x + 2)$ oe M1FT for $5 - 4 = 6x + 2x$ or better
10(a)(iv)	$\frac{5-x}{2}$ oe final answer	2	M1 for $2x + y = 5$ or better or $x = 5 - 2y$ or $\frac{y}{2} = \frac{5}{2} - x$
10(a)(v)	$17 - 6x$ oe final answer	2	M1 for $3(5 - 2x) + 2$
10(a)(vi)	$\frac{5x+16}{(5-2x)(3x+2)}$ or $\frac{5x+16}{10+11x-6x^2}$ final answer	3	M1 for common denominator $(5-2x)(3x+2)$ M1 for $3(3x+2) + 2(5-2x)$ oe
10(b)	x	1	

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(f) (i) Find $j(243)$.

.....

(ii) Find x when $j(x) = 1.5$.

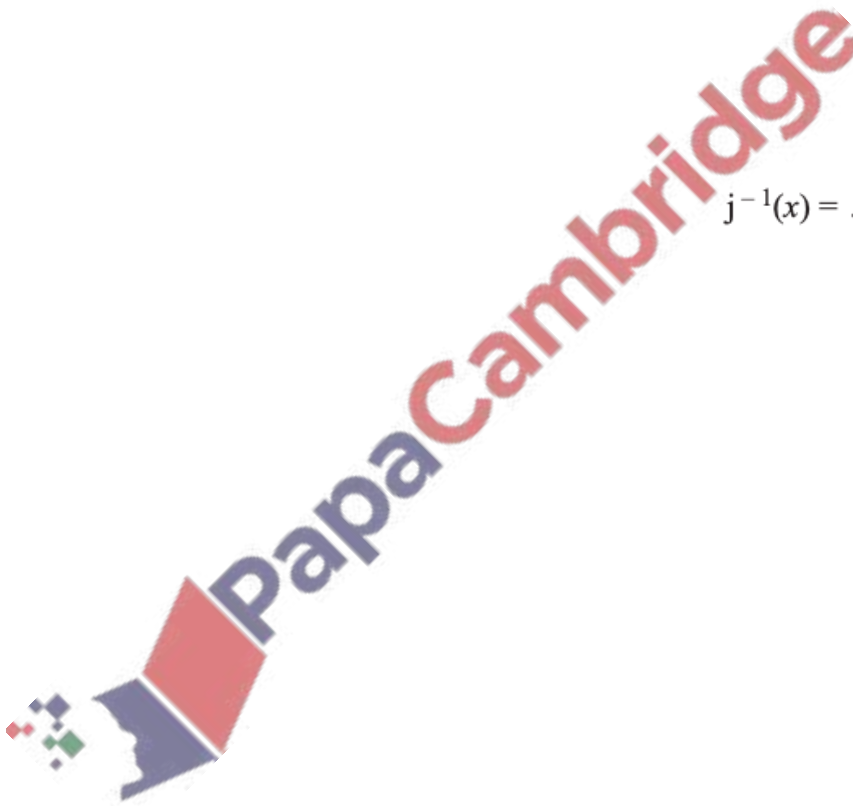
$x =$

(iii) Find $j^{-1}(x)$.

$j^{-1}(x) =$

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12 $f(x) = 5x - 2$ $g(x) = \frac{6}{4x+1}, x \neq -\frac{1}{4}$ $h(x) = 5x^2 + 3x - 2$

(a) Find $f(g(1))$.

Answer(a)

(b) Find and simplify these expressions.

(i) $g(f(x))$

Answer(b)(i)

(ii) $f^{-1}(x)$

Answer(b)(ii)

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12	(a)	4 nfw	2	B1 for $\frac{6}{4+1}$ oe seen or M1 for $5\left(\frac{6}{4x+1}\right)$
	(b) (i)	$\frac{6}{20x-7}$ final answer	2	M1 for $\frac{6}{4(5x-2)+1}$
	(ii)	$\frac{x+2}{5}$ oe final answer	2	M1 for $y+2=5x$ or $x=5y-2$ or $\frac{y}{5}=x$ or better
	(c) (i)	$\frac{1}{x+1}$ final answer	3	M2 for $\frac{5x-2}{(5x-2)(x+1)}$ oe or M1 for $\frac{5x-2}{(5x+a)(x+b)}$ oe where $ab =$ or $a+5b=3$ or SC1 for $(5x-2)(x+1)$ seen

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(ii)	$\frac{26x-13}{(4x+1)(5x-2)}$ oe final answer	3	M1 for common denominator $(4x+1)(5x-2)$ M1 for $6(5x-2) - (4x+1)$ oe
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(ii) $g^{-1}(x),$

(iii) $\frac{2}{f(x)} - \frac{3}{g(x)}.$

Answer(b)(ii)

Answer(b)(iii)

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12	(a) (i)	-2	1	
	(ii)	-7	1FT	
	(b) (i)	$6 - 6x$ oe	2	B1 for $4 - 2(3x - 1)$
	(ii)	$\frac{4-x}{2}$ or $2 - \frac{x}{2}$ oe	2	B1 for $x = 4 - 2y$ or $2x + y = 4$
	(iii)	$\frac{11-13x}{(3x-1)(4-2x)}$	3	M2 for $\frac{2(4-2x)-3(3x-1)}{(3x-1)(4-2x)}$ or B1 for $2(4-2x)-3(3x-1)$ or SC2 for $\frac{5-13x}{(3x-1)(4-2x)}$ or M1 for common denominator ($3x$

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11 $f(x) = 3x + 1$ $g(x) = \log x$

(a) Find the value of $g(f(33))$.

.....

(b) Find the value of x when $g(x) = f(-1)$.



$x =$

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(c) Find

(i) $f^{-1}(x)$,

$f^{-1}(x) = \dots\dots\dots$

(ii) $g^{-1}(x)$.

$g^{-1}(x) = \dots\dots\dots$

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11	(a)	2	2	B1 for $[f(33) =] 100$ or M1 for $\log(3x + 1)$
	(b)	$\frac{1}{100}$ or $[0].01$	2	M1 for $g(x) = 3(-1) + 1$ oe
	(c) (i)	$\frac{x-1}{3}$ oe	2	M1 for $x = 3y + 1$ or $y - 1 = 3x$
	(ii)	10^x	2	M1 for $x = \log y$ or $10^y = x$

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12 $f(x) = 5 - 3x$ $g(x) = 2x + 3$

(a) Solve $f(x) = 11$.

$x = \dots\dots\dots$

(b) Find $f^{-1}(x)$.

$f^{-1}(x) = \dots\dots\dots$

(c) Solve $f(x) \times g(x) = 0$.

$\dots\dots\dots$

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12(a)	-2	2	M1 for $-3x = 11 - 5$ oe
12(b)	$\frac{5-x}{3}$ oe final answer	2	M1 for $3x = 5 - y$ or $x = 5 - 3y$ or $y - 5 = -3x$ or $\frac{y}{5} = \frac{5}{3} - x$ or better
12(c)	$\frac{5}{3}$ oe $-\frac{3}{2}$ oe final answers	2	B1 for each
12(d)(i)	x	1	
12(d)(ii)	$11x - 7$	3	M2 for $5 - 3(5 - 3x) + 2x + 3$ or M1 for $5 - 3(5 - 3x)$
12(d)(iii)	$\frac{26-8x}{(5-3x)(2x+3)}$ final answer	3	M1 for $2(2x+3) + 4(5-3x)$ oe M1 for common denominator $(5-3x)(2x+3)$

(d) Simplify.

(i) $g^{-1}(g(x))$

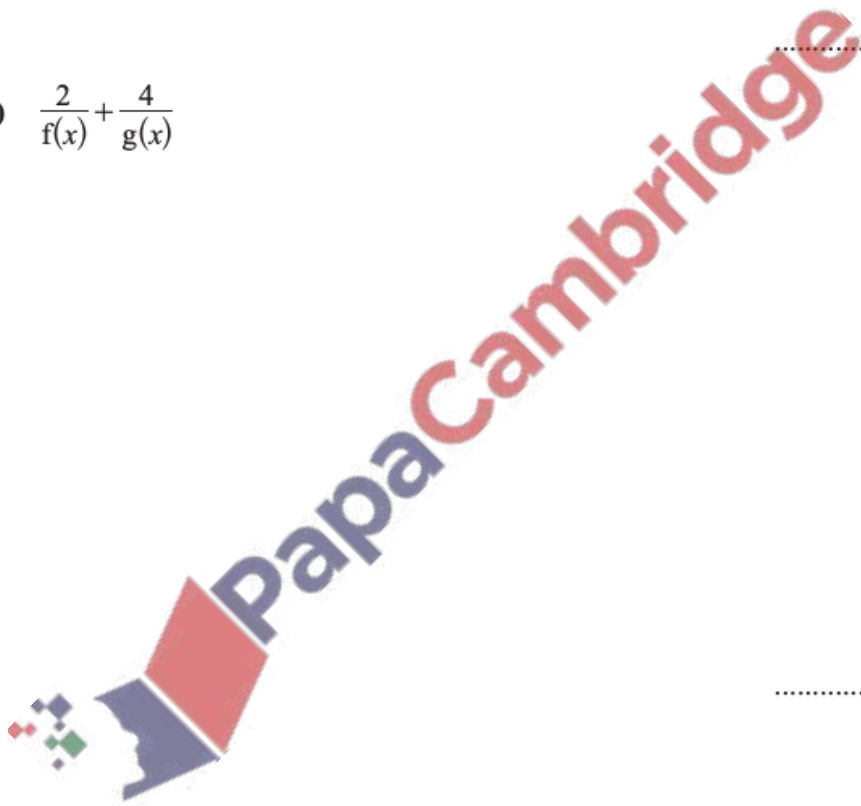
..... [1]

(ii) $f(f(x)) + g(x)$

..... [3]

(iii) $\frac{2}{f(x)} + \frac{4}{g(x)}$

..... [3]



12 $f(x) = 3x - 1$ $g(x) = 4 - 2x$

(a) Find

(i) $g(3)$,

Answer(a)(i)

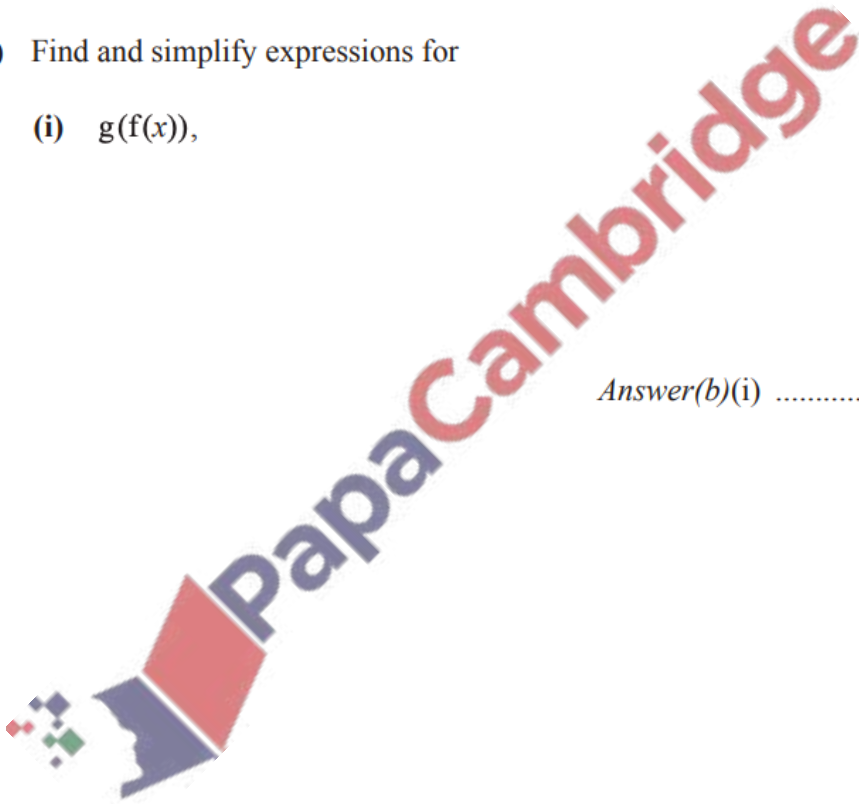
(ii) $f(g(3))$.

Answer(a)(ii)

(b) Find and simplify expressions for

(i) $g(f(x))$,

Answer(b)(i)



(c) Simplify.

(i) $\frac{f(x)}{h(x)}$

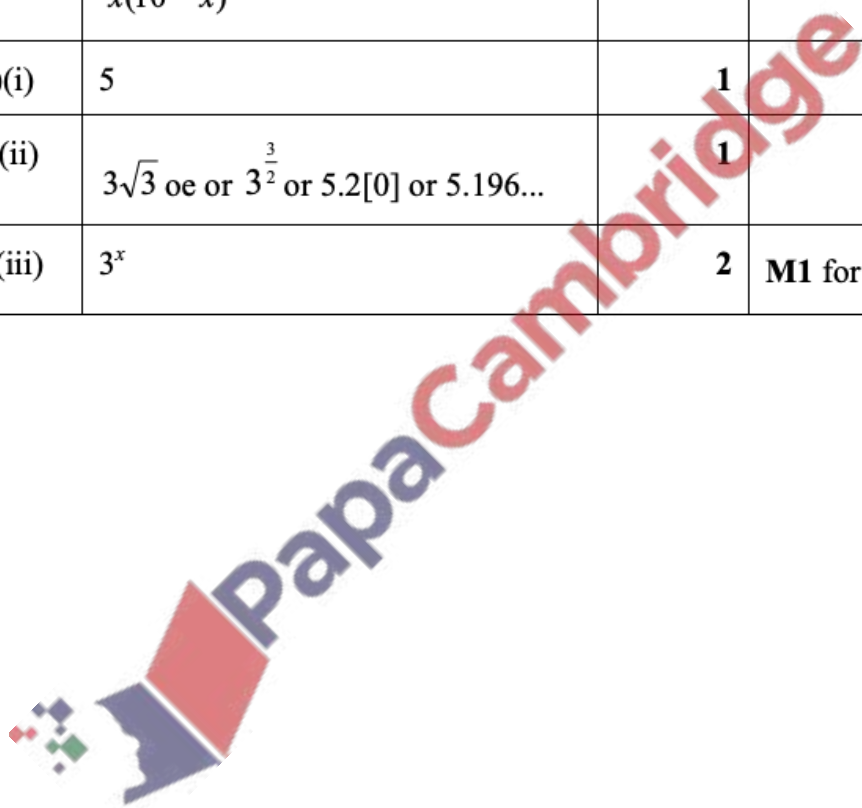
Answer(c)(i)

(ii) $g(x) - \frac{1}{f(x)}$

Answer(c)(ii)



12(a)	10	1	
12(b)	9.5 oe	2	M1 for $10 - \frac{1}{x}$ soi e.g. $10 - \frac{1}{2}$
12(c)	$x^2 - 20x + 101$	3	M1 for $(10 - x)^2 + 1$ B1 for $100 - 10x - 10x + x^2$ oe
12(d)	f(x) and h(x)	2	B1 for each
12(e)	$\frac{10 - 2x}{x(10 - x)}$ oe	3	M1 for common denominator $x(10 - x)$ B1 for $(10 - x) - x$ oe seen
12(f)(i)	5	1	
12(f)(ii)	$3\sqrt{3}$ oe or $3^{\frac{3}{2}}$ or 5.2[0] or 5.196...	1	
12(f)(iii)	3^x	2	M1 for $x = \log_3 y$ or $x = 3^y$



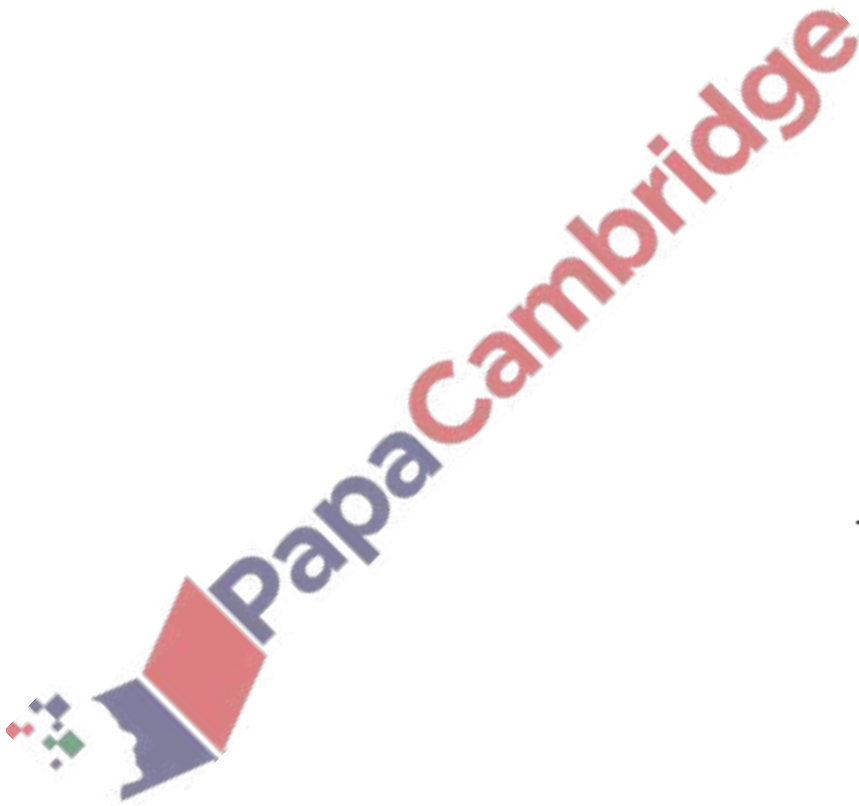
(d) For some functions, $p^{-1}(x) = p(x)$.

Write down which two functions, $f(x)$, $g(x)$, $h(x)$ or $j(x)$, have this property.

..... and

(e) Write $h(x) - \frac{1}{f(x)}$ as a single fraction in its simplest form.

.....



12 $f(x) = 10 - x$ $g(x) = x^2 + 1$ $h(x) = \frac{1}{x}$ $j(x) = \log_3 x$

(a) Find $g(3)$.

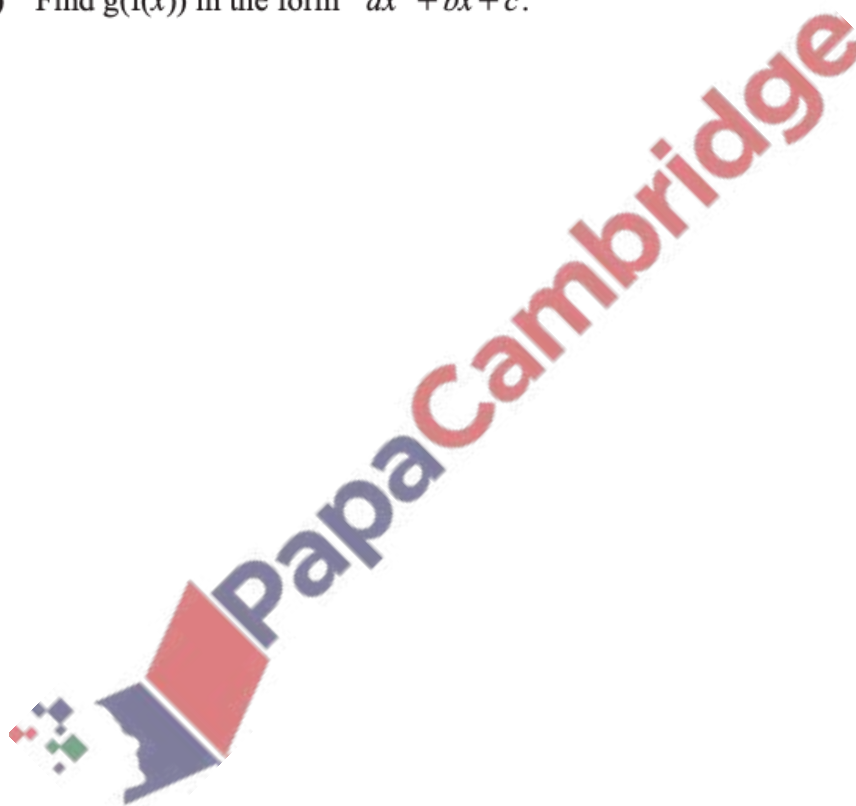
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(b) Find $f(h(2))$.

.....

(c) Find $g(f(x))$ in the form $ax^2 + bx + c$.

.....

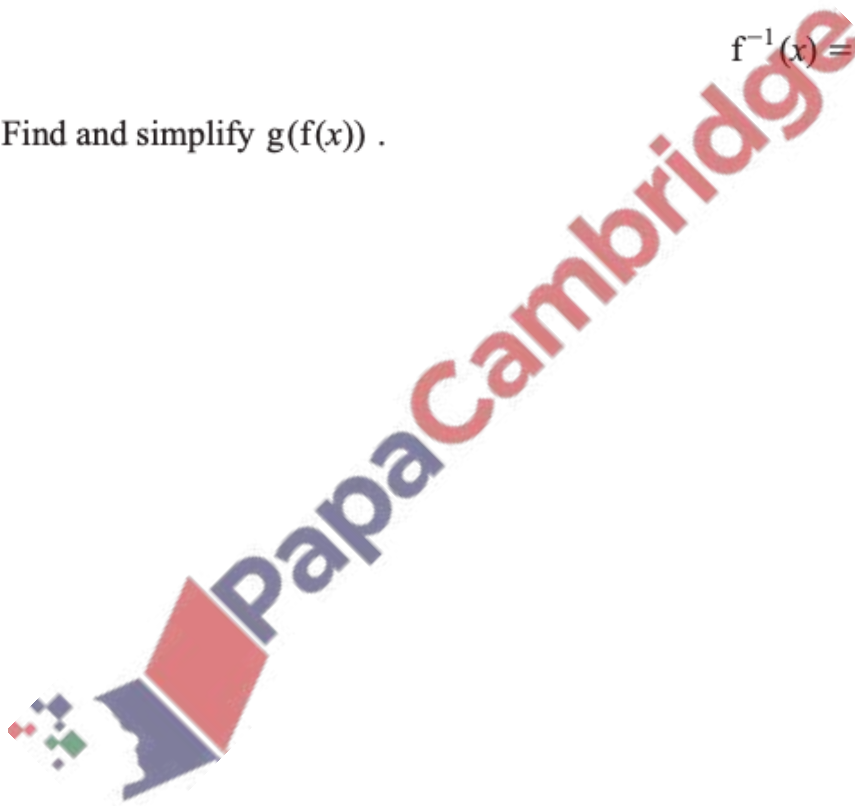


(iv) Find $f^{-1}(x)$.

$f^{-1}(x) = \dots\dots\dots$

(v) Find and simplify $g(f(x))$.

$\dots\dots\dots$



10 (a) $f(x) = 5 - 2x$ $g(x) = 3x + 2$

(i) Find $f(-3)$.

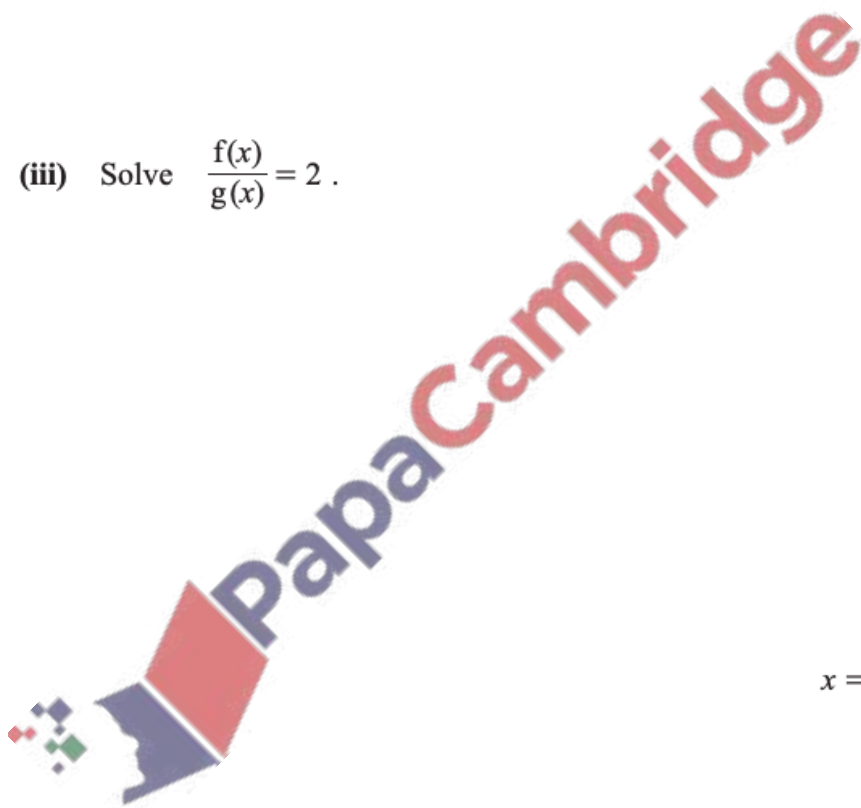
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(ii) Find $f(g(4))$.

(iii) Solve $\frac{f(x)}{g(x)} = 2$.

.....

$x =$



13 (a) $f(x) = 5 - 2x$

(i) Solve $\frac{1}{f(x)} = 2$.

$x =$ [2]

(ii) Find and simplify $f(f(x))$.

..... [2]

