FURTHER MATHEMATICS

Paper 9795/01
Further Pure Mathematics

Key messages

There are several major points that candidates need to bear in mind before they sit this paper, many of which are detailed in the later comments on individual questions. Firstly, however, candidates need to be aware that this is a long and tough paper and that it is not always possible for them to make complete attempts at all questions. With this in mind, it becomes clear that the element of time is a crucial factor while working through one of these papers, and that this element represents a valuable, but finite, resource which they must spend wisely.

Candidates need to be mindful of certain underlying principles in order to give their best performance:

- The number of marks allocated to each question-part: on this year's paper, for instance, **Question 5(b)** elicited a large number of responses which ran to a page-and-a-half of detailed algebraic working and it really should have been clear, from the small number (2) of marks assigned to it, that this was not what was intended. Indeed, a moment's clear thought upfront should have revealed that this could easily be answered with a response taking up a single line of working.
- The structure of the question: often candidates seemed unwilling to follow some carefully crafted signposting that was there to help them figure out how to proceed. This especially applied in **Questions 7(a)**, **8(b)**, **9**, **10**, **11** and **12**.
- The order in which questions are presented may not best suit each individual.
- Clarity of explanation: there were several places on this paper where written comments, explanations or justifications were required (**Questions 1**, **4**, **7** and **9** in particular). Many candidates seem to be strongly averse to writing any words down while others seem to work on the basis that if they throw enough of the right sorts of words and phrases down on the page then, somewhere in the mix of it, something will have 'hit the mark'. Comments need to be brief and to the point. In many cases, the tactic of saying everything that might be relevant is as clear a way as any of indicating that the respondent does not really know what should be being said.
- Clarity of presentation: there was some very poor presentation in a significant proportion of the scripts, with some handwriting being barely legible and numbers written so badly that individual digits were difficult to discern. There were also several cases where solutions to questions were fragmented over multiple answer booklets with little indication of continuation or labelling. If candidates' work is not decipherable then Examiners will not be able to award it much credit.
- Answering the question: for instance, **Question 1** is an Induction question; around a dozen candidates scored 0 by quoting the standard summation results for Σr and Σr^2 instead. In **Question 6(a)**, candidates were required to draw two loci on 'a single Argand diagram', so it was unsuitable for candidates to draw two completely separate pictures. In both **Question 7(a)** and **Question 10(a)**, there appeared the instruction 'deduce…', which a lot of candidates simply ignored. While there are occasions when an alternative approach *can* be found acceptable, it is always going to be a gamble to use a method which the question's wording seems to have precluded.
- Sketching functions: a significant number of candidates are still drawing sketches on graph paper. This inclines them to be constrained by scales and so struggle to represent the key features in a clear manner. Furthermore, some are induced to waste much time in attempts to plot multiple points accurately. All that is required is a clear representation of the key features. Graph paper should not be used.

General comments

The standard of work on this paper has always been high but seemed to rise even higher this year Most candidates managed to make full attempts at all 12 questions, including many who produced three or four attempts at parts of **Questions 11** and **12**. A significant proportion of candidates achieved very high marks, an admirable achievement on this paper. None scored very low marks.

One of the main differences in this year's paper was that a far greater proportion of candidates were completing their attempts on all of the questions. This despite the presence of two long and demanding questions at the end, each of which required considerable concentration, skill and technical proficiency. Only **Questions 5** and **10** proved to be perhaps slightly too easy for candidates. Otherwise, each question seemed to give the well-trained candidates the appropriate opportunities to prove their worth whilst offering all candidates a suitable level of challenge.

Comments on specific questions

Question 1

This was a straightforward starter to the paper and was found to be so by almost everyone. However, as mentioned above, a small number of candidates chose not to attempt a proof by induction and scored nothing on this question. For those that did follow the instruction, the only mark that was generally lost was the one for the explanation, at the end, of the logic behind the inductive reasoning; closely followed by the mark for either not getting S_{k+1} in a visibly correct form (to match the required form of the result) or by not explaining it properly in some other way (such as pointing out in advance what it should look like).

Question 2

This question used a result – that seemed to be little-known amongst candidates – regarding a method for calculating the area of a triangle in the Cartesian plane just from the coordinates of its vertices. It was one of the highest-scoring questions on the paper and, by and large, candidates just did as they were told on it. A few made heavy-going of the algebra in **(a)** while, in **(b)**, almost all candidates expanded the determinant by either the first row or the first column, when the much simpler *Sarrus' Rule* could have been employed. A small number of candidates overlooked the need to show that their answers to **(a)** and **(b)** were demonstrably equal.

Question 3

Although the demands of this question were fairly routine, a lot of heavy weather was made by candidates, many of whom produced large numbers of lines of almost entirely unnecessary working – especially in **part (b)**, where was only necessary to substitute into the newly-added third plane any one point on the line of intersection found in **(a)**.

Question 4

This was a relatively straightforward groups question but the explanations/reasoning required in all three parts, especially in **(b)** and **(c)**, usually meant that marks were dropped somewhere along the line. Completing the table in **(a)** proved to be of little difficulty and checking through the group axioms should have been found equally straightforward, though very few candidates earned all 5 of the marks in **part (a)**.

The principal reason for this shortfall was the slight misunderstanding of what is required by way of explanation or justification. Most common of these marginally defective responses was the one for justifying the closure property of the given binary operation acting on the given set of elements. Many candidates said something like, 'Closed from the table'; but, without expanding on this statement, it scored no marks: it was required that they explain *how* the table illustrates closure. Others said something along the lines of, 'All elements of *S* appear in the table' – which is true, but which still needs to be that little bit clearer by observing that, '**Only** the elements of *S* appear in the table'.

The same sort of thing applies to the statements regarding the identity and, in particular, the inverses. To say that the identity appears in each row and column so that every element has an inverse is insufficient; it must, of course, be the same 'row identity' and 'column identity' for every (other, non-identity) element in order for this group axiom to hold. For groups of small order, it is simplest just to state the inverse for each (non-identity) element.

Exactly the same issues regarding the making of precise, *relevant* statements to support the appropriate conclusions arose in **parts (b)** and **part (c)** – many candidates were making true statements, but they were frequently not sufficiently relevant or complete to earn the marks.

Question 5

This question was the highest-scoring on the paper, being relatively routine in nature and without any twists to test any out-of-the-ordinary aspect of the topic. Apart from occasional slips with the numerical evaluation of the coefficients, almost everyone knew exactly what to do here and did so very effectively. Just a very small number of candidates stopped when they arrived at the general solution.

Question 6

A significant minority of candidates declined to draw the two loci of **part (a)** on the same diagram. Apart from losing a mark, the biggest problem that this presented was that these candidates often then were unable to notice – in **part (c)** – that the region required was cut off by a quadrant of the circle, which made the working for this area rather trickier for them than it should have been.

Here again, in **(b)**, there was frequently a lot of algebra working to be found, often stretching to many, many lines. More thoughtful candidates just wrote the two Cartesian equations straight down.

Question 7

In **part (a)**, a large number of candidates did not follow the instruction to ‘deduce’, invariably using calculus instead. On this occasion some leniency was applied, but this may not always be the case. Candidates must use the directed approach where one is specified.

In **part (b)**, the details for the sketch were often worked out independently of whatever had gone before in **part (a)**, and marks scored here were usually very high.

Question 8

This question began with a sketch. As with all sketches, *some* helpful detail is needed and a lot of candidates drew something approximately correct but then omitted to put anything on their diagram that gave an indication of scale. Although technically inappropriate, most candidates drew Cartesian axes. Many others resorted to plotting points, which was understandable here.

It is worth noting that a sketch at the beginning of a question might require less detail than one asked for at the end of a question, *after* other properties or features have been established. On this occasion, the ‘cusp’ at the pole (origin) was not required, since nothing (apart from the plotting of points) would have led to the visualisation of this feature of the curve. The important points were the closed curve, symmetry in the ‘x-axis’, and some indication of scale in suitable places.

There was some minor confusion in **part (b)** as to what was required, and many candidates overlooked the request to turn terms of the form (e.g.) $\cos(\theta + \frac{1}{2}\pi)$ into a suitable $\cos\theta$ or $\sin\theta$ expression. As a result, a significant proportion of candidates made little or no effort to answer **(b) (ii)**. On the other hand, many who had struggled to deal with the angles in **(b) (i)** still completed **(b) (ii)** successfully, as it actually only relied on the rs and not the θs.

Question 9

The explanations/reasoning required in **parts (a)** and **(b)** provided many candidates with an unwelcome hurdle. **Part (a)**, in particular, was a frequent source of lost marks, as the majority of candidates failed to appreciate that an ‘if and only if’ proof was wanted; indeed, in many cases, there was a lack of grasp as to how to connect their expansion of $(\alpha\beta - 1)(\beta\gamma - 1)(\gamma\alpha - 1)$ to the issue of ‘one (unspecified) root being the reciprocal of another (distinct but also unspecified) root’.

For **part (b)**, there were two main approaches and they appeared in roughly equal measure. There were some candidates who did not quite know how to go about this and then there were those who were somewhat careless with the negative sign(s) involved, but this was generally completed very capably.

Many candidates spotted that the given cubic equation in **(c)** had a fairly obvious integer root, which allowed them to solve the equation without recourse to the steering provided in the earlier parts of the question.

Question 10

This question used a relatively simple application of the method of differences and proved to be the second-highest scoring question on the paper, after **Question 5**. In general, there were very few ways in which candidates lost marks: in **(a)**, there were those who did not make any attempt to use the opening result to deduce the second. In **(b)**, some candidates saw what was required in the cancelling of terms and so did not put any suitable amount of working down to support their conclusion. Some slipped up in identifying the two remaining terms at the end, getting either or both of the endpoints wrong.

Question 11

This question was reasonably long and tough, but more than a third of candidates scored full marks on it, which was very impressive. The reduction formula of **part (a)** was almost as straightforward as it could have been, but the technical efficiency required to make a complete success of **parts (b)** and **(c)** were significantly non-trivial. For instance, the widespread awareness that almost all candidates showed in both looking for, and then obtaining, $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2$ in the form of a perfect square (in order to obtain a manageable integrand) is definitely a high-level skill and yet so many candidates still managed to accomplish this.

In **part (c)**, the structure of the question pointed towards expressing S in the form of the reduction formula of **part (a)**, but the wide array of skills and competences on display meant that a high proportion of successful attempts managed to employ alternative methods with great efficiency.

Question 12

There are several ways in which this question was very demanding and it proved to be the only one with a mean score under 50 per cent of the available marks. Some candidates were clearly pushed for time, while others spent considerable amounts of time making little real progress on **parts (a)** and **(b)**, often going round in circles juggling a range of potentially useful identities that never really seemed to get drawn together in a cohesive way.

There are so many ways to do **part (a)**, but the most straightforward one must be to use the result of $\sinh^{-1}$ in log. form from the Booklet of Formulae (MF20) and then substitute it into $\tanh\frac{1}{2}x = \frac{e^x - 1}{e^x + 1}$, which is a standard enough result to be quoted or which can be quickly deduced from $\tanh = \frac{\sinh}{\cosh}$.

The biggest obstacle to successful progress in **(b)** – which, of course, had a knock-on effect in **(c)** – was the oversight of the factor of $\frac{1}{2}$ when using the chain rule. This was closely allied to the previously mentioned tendency to play around with, in some cases, almost every possible hyperbolic-function identity available in order to obtain the given answer.

Part (c) was genuinely tough and even among those who had made very successful progress thus far, it was frequently the case that solutions effectively finished at the point where the correct intermediate result $\int \frac{2\sinh^2 x}{\cosh x}\,dx$ appeared. For many candidates, however, even this point proved to be a bridge too far, and it was often their 'single maths' skills associated with the employment of an integration-by-substitution method that were found wanting, generally through failing to replace the θ s throughout with x s. Those who connected $\int \frac{2\sinh^2 x}{\cosh x}\,dx$ to the derivative found in **part (b)** tended to be the ones who made it through to the end. There were not very many completely correct final answers.

FURTHER MATHEMATICS

Paper 9795/02
Further Applications of Mathematics

Key messages

- Give full details in 'show that' questions and provide logical structure to reasoning.
- Write clearly, concisely and precisely when asked to 'explain'.
- Use significant figures sensibly in working in order to prevent rounding error in the final answer.
- Show all intermediate steps and give them in numerical, rather than general, form.
- Read the questions carefully and provide what is requested in the manner requested.
- Draw large, clear and complete diagrams using a ruler and pencil.
- Define any newly introduced symbols and show them on a diagram, if relevant.
- Try to answer questions in a single place (leave space if it is intended to attempt further work later).
- If a question has to be continued in a different place then indicate clearly that this is the case and where the continuation can be found.

General comments

As in previous years this paper contained some difficult question parts which most candidates found very hard and it was still clear that many candidates were not well enough versed in some basic definitions or methodologies. Most candidates remain good at applying standard techniques but have problems applying techniques and principles to unfamiliar situations.

There was some poor and illogical presentation, especially when it came to answering 'explain' or 'show that' questions. Since answers should generally be presented to 3 sf (unless otherwise instructed or exact values are requested) candidates should know that they need to work with more significant figures (or exact values), especially in cases such as finding the difference between two similar numbers when significant figures are 'lost'. Keeping exact values is usually a good idea although it can be cumbersome. Keeping complicated general expressions is not a good idea – it tends to go wrong and it is hard to award any credit unless formulae are actually applied to the given scenario.

In particular, candidates must realise that method marks cannot generally be awarded unless a method is shown or, at least, unambiguously implied. Candidates should be told that it is hard to give partial credit if intermediate values are not shown; it is not sufficient to quote a formula in symbols and then write down an answer; candidates must show that the equation has been used. Writing should be clear; often it seems to be small and scrawled; if it cannot be read then it can gain no credit. Numbers should be distinguishable. In general, the most direct route to the answer is the best; convoluted methods carry enormous extra risk of error.

Use of sensible notation also reduces the risk of error and makes it far easier for candidates to see where they are going. Candidates should also be encouraged to include indicators to help the flow of reasoning; for example, the use of logical connectors, the naming and quoting of standard results and making it clear where and how previous results are being utilised can easily improve and clarify an otherwise apparently structureless mass of work.

Overall candidates seemed to find the Statistics section easier than the Mechanics section, perhaps because Statistics tends to be more methodological and less dependent on an understanding of the underlying principles. In Mechanics especially, candidates should be encouraged to understand the principles rather than simply memorising formulae and methodologies. Of course, since the Mechanics section comes after the Statistics section it might also be the case that candidates are running short of time by the time that they reach it.

Candidates should appreciate that if the answer is given in a particular question then the onus is on them to demonstrate that they have reached this answer properly and not simply 'fudged it'. Their arguments should also be complete; it should not be for the person reading the solution to have to infer how a candidate has got from one step to the next.

Finally, candidates should also be encouraged to strike out any work which they do not wish to be assessed and to do this as neatly and clearly as possible; leaving such redundant work risks the unnecessary loss of marks.

<u>Comments on specific questions</u>

Section A: Statistics

Question 1

This question was done well in general but illustrates some important points. In **part (a)** candidates should be encouraged to present answers properly; in this case, simply writing '35, 35' gained full credit but in a future case this may not be so. In **part (b)** candidates were requested to 'find the smallest value of *r*'. It was therefore necessary to indicate somehow (in words or symbols) that the value provided was indeed the smallest value. Also, while it was clear that almost all candidates knew the correct approach to take, some candidates forgot to use a continuity correction or applied it incorrectly. A sketch can be really helpful to prevent such confusion. A small number of candidates derived an incorrect probability statement; again, a sketch can be very helpful to prevent this. A surprisingly common error was the use of a *z*-value of 2.54 instead of 2.054.

Question 2

Again, this question was generally done well but hardly any candidates got full marks; loss of marks was usually due to incorrect or imprecise or uncontextualised explanations of the assumptions required for a Poisson distribution to apply. Statements such as 'The probability of weeds occurring is independent' are not using the word 'independent' in its mathematical sense (in which it is *events* which are independent of each other). Candidates should also recognise that it is required that the *average* rate at which weeds occur which needs to be constant. Candidates should also be able to distinguish between an assumption and a condition (for example that the weeds grow singly) or a consequence (for example, that the mean and variance of a Poisson distribution are the same). Most candidates found the actual Poisson calculation in **(b)** straightforward. In **(c)** most candidates knew the approach to take although a number did not show the relevant probabilities in spite of the fact that these were specifically required by the question. Some candidates were confused between the area, *A*, and the Poisson parameter (λ). A small number of candidates were evidently confused about how to form the appropriate inequality; again, a sketch can really help. A further, rather subtle point is that *A* itself is actually just a number since the area has been defined as being $A\ \text{m}^2$. Candidates will not, in general, be penalised for providing *A* with the correct units (so the answer $0.72\ \text{m}^2 < A < 0.74\ \text{m}^2$ obtained full marks). However, candidates who provided *A* with incorrect units (e.g. m) did not receive the final accuracy mark.

Question 3

Again, this question was mostly done well although it was rare that a candidate managed to answer all three non-calculational parts convincingly. Candidates should learn that 'independent' and 'random' are different concepts; for example, samples can be random without being independent if they are drawn from parent populations which are not themselves independent. Candidates are also expected to use correct and recognised mathematical notation when presenting their answers. A confidence interval is a numerical interval and so should be presented as such; a statement such as $12.73 < \mu < 14.07$ is, in any case, incorrect since the confidence interval may well not contain μ; such statements, unless clearly explained, did not receive the final A mark.

It was interesting to note that a lot of candidates were not able to express clearly why the Central Limit Theorem is necessary, the most common misconception being that a large sample size is required in order for us to assume that the parent population is normally distributed.

In **(b)**, it should be noted that 2 d.p. were specifically requested; (12,7, 14.1) did not obtain the final accuracy mark.

In **(d)**, a large number of candidates attempted to pool the boys' and girls' variances. This was surprising given that the correct formula for the distribution of the differences is given in the formula book and also because the sample variances are clearly different and so the assumption that the population variances are the same is both unnecessary and also unfounded.

In **parts (b)** and **(d)** it was surprising to see how many candidates considered samples of size 40 and 60 as being 'small'. Such candidates were not penalised, provided that they used a correct method and a sensible *t*-value. However, in this specification, as is common practice, any sample of size 30 or more can be considered large.

Question 4

In **part (a)** candidates were requested to 'Show that' the given function 'satisfies the requirements for a probability density function. It was therefore incumbent on candidates to make it clear that they understand what these requirements are. Simply finding the integral was insufficient for full marks without some indication that it is a requirement for the integral to be equal to 1. It is also necessary that the function is everywhere non-negative but most candidates did not seem to appreciate this.

Some candidates mistakenly thought that a PDF had to be continuous or that it had to equal 0 at certain points or that it had to be less than 1 everywhere; such claims were ignored unless they contradicted any actual requirement. However, a lot of time was wasted in proving such properties.

It was also clear that some candidates do not quite understand the nature of a continuous probability distribution; statements such as 'the probabilities must add to 1' were common. In this case such comments were treated leniently but in future cases they may not be.

Again, the use of sketches would have helped enormously with **parts (a)**, **(b)** and **(c)**; it was surprising how many candidates evaluated an integral to solve **(b)** when the instruction was 'Write down' and only 1 mark was available. Equally it was surprising how many candidates thought that the expected value was $\frac{\pi}{4}$, which is actually the *y*-value at the relevant point. Many different routes to solution were deployed for **part (c)** but only a few candidates found the most straightforward of these, although most in the end derived the solution.

In **part (d)** it was very common to be able to write down the correct integral but then either not be able to evaluate it or to use a highly convoluted method. A great deal of time must have been wasted using, for example, trig identities or integration by parts when in fact spotting the integral as a straightforward case of the reverse chain rule gave the solution in a matter of seconds.

Question 5

Most candidates were able to have a reasonable attempt at this question although it was evident that many did not fully understand the difference between the number of rounds played, which follows a geometric distribution, and the number of throws of the die, *X*, which does not. Explanation of this difference was crucial in **part (c)** for any candidates using the standard result for the PGF of a geometric distribution, although it was much easier simply to use the definition and the given and derived probabilities.

Question 6

It was evident that candidates are still not very confident about the topic of estimators. Most candidates still appear to be confused by the very concept; answers to **(a)** along the lines of $\mathrm{E}(\theta) = \theta$ or $\mathrm{E}(\theta) = T$ were common, as were answers to **(d)** of $\mathrm{E}\left(\frac{n(X-\overline{X})^2}{n-1}\right)$; candidates should know that an estimator is a random variable while a quantity such as E(*X*) is simply a number.

The question was highly scaffolded and so it was not surprising that many candidates managed to derive the given identity, although it was evident that it was not always from a complete understanding, as was clear from the missing or slightly incorrect details in the proof. For example, many candidates simply assumed, incorrectly, that the statistic $(X-\overline{X})(\overline{X}-\mu)$ was zero. Other candidates assumed that $\mathrm{E}\left[(X-\overline{X})(\overline{X}-\mu)\right] = \mathrm{E}(X-\overline{X})\mathrm{E}(\overline{X}-\mu)$ even though they were told that this quantity was 0.

Section B: Mechanics

Question 7

It was surprising how many problems this straightforward ‘energy budget’ question caused. In **part (a)** all that was required was consideration of what happens to the energy made available by the engine; some of it was used against friction while the rest increased the kinetic energy of the car. Candidates should have realised that since only 2 marks were available for this part would not require a great deal of work. In **part (b)** many candidates were confused by signs and derived an answer of 100 000 J rather than 180 000 J, although a little thought would indicate that the presence of friction must mean that the engine needs to produce more energy (some of which is wasted) than if there were no friction. Some candidates assumed that the car accelerated uniformly; if they presented this as an assumption then they were not penalised if their calculation was correct. However, deriving the answer did not require this assumption and the direct calculation was actually far simpler.

Question 8

This question was generally done well, although some candidates got into a bit of a muddle about the connection between angles θ and ϕ. Many candidates simply put θ in their ‘ϕ triangle’ (or vice-versa) on their diagram and then produced equations of motion involving just one angle; this provided a very straightforward route to the solution. A lot of candidates noted that $\sin\theta = \cos\phi$ and that $\cos\theta = \sin\phi$, which quickly produced the solution, although some introduced a rogue minus sign or two. A small number of candidates in effect changed θ to ϕ. but also ϕ to θ; this did not advance their cause greatly. Many candidates used trig identities in order eliminate the angles; while this did the trick, if done correctly, it was often quite a convoluted method. **Part (b)** presented few difficulties; the most common error for the small number of candidates who got this part wrong was to attempt to find a solution in terms of symbols rather than simply to convert to numbers as far as possible and as soon as possible.

Question 9

This question was found to be very difficult and was, by and large, poorly attempted by the majority of candidates. It was very surprising to observe how loath candidates were to use vectors; in vector notation the entire question can be done straightforwardly with no trigonometry and no issue with signs. In **part (a)**, most candidates did not seem to appreciate the significance of the instruction ‘Show that’; with this command candidates should realise that the onus is on them to explain their working; it is not sufficient, for example, to introduce a new angle without either defining it or showing it clearly on a diagram. Candidates should also have realised that even the velocity **v** should, if used, be explained as being the velocity of the snooker ball after the first collision since this has not been defined in the question. Candidates at this level should also realise that statements such as ‘$u\sin\theta = u\sin\theta$ are of limited value. Introducing a minus sign at the end of working, in contradiction to earlier working, simply so as to accord with the given answer is also insufficient for credit. Many candidates seemed to think that the angle of incidence was equal to the angle of reflection. Many other candidates introduced new angles but did not then know how to deal with them; confusion frequently ensued. Many candidates thought that showing that $w = 0.6v$ was sufficient to show that $\mathbf{w} = -0.6\mathbf{v}$. Most candidates did not seem to be able to convey, algebraically, that the direction of the velocity perpendicular to the cushion would be reversed, which was really the critical aspect of the question. A small number of candidates thought that $\mathbf{v} = 0.6\mathbf{w}$ or that momentum would be conserved in the collisions. Similar issues arose in **part (b)**, with many candidates seeming to think that the magnitudes of the impulses in the two collisions could simply be added to form the overall magnitude of the total impulse. Many candidates seemed to think that the impulse was $m(\mathbf{u} - \mathbf{v})$, which gave them the correct answer but meant they did not get credit for the direction. A surprisingly large number of candidates omitted the m in their calculations or left the answer in terms of u, which was given. A number of candidates somehow found a value for θ which is actually not possible. All in all there seemed to be considerable scope for improving the approach to solving 2 dimensional collision problems.

Question 10

Most candidates made a decent attempt at both parts of this question, although it was very clear that many of them would benefit from taking a more structured approach. In **part (a)** some candidates did not start off with the simple approach of putting components of the contact forces at *A* and *B*. For those who did the most common problem was to worry about the components of the contact forces along the wall; consideration of these can be avoided simply by taking moments about *A* or *B* (or, indeed, any point along the wall). However, some candidates took moments about other points (for example, *C* or *D*) which ensured a quite difficult route to solution. Candidates should know that it is generally advantageous to take moments about the point through which most unknown forces (or their components) act. In this case, taking moments about either *A* or *B*, and considering forces in the horizontal, led straightforwardly to the solution even though some candidates who did this were confused by the signs. **Part (b)**, although a more complicated situation, generally caused fewer problems; most candidates knew what they had to do although once again it was surprising to note how many candidates did not take moments about *A,* which was surely the obvious approach. Candidates would also be advised to label all forces on their diagrams and to follow a clear methodology to solve such problems. Finally, once again candidates were instructed to find 'the least possible value of μ' so to gain the final accuracy mark candidates either had to show in their working that they understood why their derived value would be the least possible value or, at least, to declare their value as such (which was permitted in this case although may not be in future cases).

Question 11

A lot of candidates had no difficulties whatsoever with this question and it was common for candidates to gain full marks. By far the most common error in **(a)** was simply to assume that all that was required for the particle to reach the top was sufficient energy; of course, just because a particle has sufficient energy to do something this does not mean that it can or will do so and a proper dynamical analysis was required for any credit. Since the journey was from bottom to top the algebra for this analysis was potentially very simple; some candidates, however, made the solution far more complicated than it needed to be and as a consequence many made errors with signs. **Part (b)** was straightforward enough for candidates who took the standard methodological approach to a dynamics problem, identifying the force and then resolving this to derive the acceleration. Candidates who started by recalling formulae for the acceleration components did not generally know then how to proceed. A similar pattern emerged in **part (c)**; those candidates who completed **part (b)** generally then proceeded by recalling and applying the formulae for the acceleration components. Those who struggled in **part (b)** also struggled in **part (c)**.

Question 12

Generally, most candidates were able to make a reasonable attempt at **parts (a)** and **(b)**. However, a lot of candidates struggled with **part (c)** even though with a sensible approach it is really no harder than **part (b)**. In order to solve the differential equation, most candidates simply used the given integral from the formula book. This was not a problem since the question did not ask for detailed reasoning. However, candidates should note that the $\tanh^{-1}$ version of the solution does not strictly provide a complete general solution since it only works (unless great care is taken) for $v < u$ (both, being speeds, are non-negative). Candidates who selected this version of the integral were therefore not given full credit although they had the opportunity to gain the residual marks in **part (c)** where the distinction became critical. Some candidates omitted the constant of integration and others simply added it on at the end, after exponentiation or tanh-ing, both rather surprising errors at this level. While most candidates did obtain a correct answer it was evident that most were not used to solving such simultaneous equations; a large proportion of candidates had terms like e^{kt+c}, rather than the more usual Ae^{kt}, in their final answers. This had ramifications for **part (c)** especially and made the algebra unnecessarily harder anyway. In **part (b)**, which was generally well done, some candidates left the answer in terms of *k* rather than substitute the value. Candidates generally found **part (c)** more of a challenge, especially if they left their constant in the exponent since it would then need to be complex. However, candidates who used the general form of the solution in **part (a)** usually managed to complete **part (c)(i)** and from there most understood what was required in **part (c)(ii)**; pleasingly, the awarding of full marks for this question was not unusual.