

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the May/June 2015 series

4037 ADDITIONAL MATHEMATICS

4037/12

Paper 1, maximum raw mark 80

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Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

1	$k^2 - 4(2k + 5) < 0$	M1	use of $b^2 - 4ac$, (not as part of quadratic formula unless isolated at a later stage) with correct values for a , b and c
	$k^2 - 8k - 20 < 0$		
	$(k - 10)(k + 2) < 0$	M1	Do not need to see $<$ at this point attempt to obtain critical values
	critical values of 10 and -2	A1	correct critical values
	$-2 < k < 10$	A1	correct range
	Alternative 1:		
	$\frac{dy}{dx} = 2(2k + 5)x + k$	M1	attempt to differentiate, equate to zero and substitute x value back in to obtain a y value
	When $\frac{dy}{dx} = 0$, $x = \frac{-k}{2(2k + 5)}$, $y = \frac{8k + 20 - k^2}{4(2k + 5)}$	M1	consider $y = 0$ in order to obtain critical values
	When $y = 0$, obtain critical values of 10 and -2	A1	correct critical values
	$-2 < k < 10$	A1	correct range
	Alternative 2:		
	$y = (2k + 5) \left(\left(x + \frac{k}{2(2k + 5)} \right)^2 - \frac{k^2}{4(2k + 5)} \right) + 1$	M1	attempt to complete the square and consider ' $1 - \frac{k^2}{4(2k + 5)}$ '
	Looking at $1 - \frac{k^2}{4(2k + 5)} = 0$ leads to	M1	attempt to solve above = to 0, to obtain critical values
	critical values of 10 and -2	A1	correct critical values
	$-2 < k < 10$	A1	correct range

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2	$\frac{\tan \theta + \cot \theta}{\operatorname{cosec} \theta} = \frac{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}}{\frac{1}{\sin \theta}}$ $= \frac{\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}}{\frac{1}{\sin \theta}}$ $= \frac{1}{\cos \theta}$ $= \sec \theta$ Alternative: $\frac{\tan \theta + \cot \theta}{\operatorname{cosec} \theta} = \frac{\frac{\tan^2 \theta + 1}{\tan \theta}}{\operatorname{cosec} \theta}$ $= \frac{\sec^2 \theta}{\tan \theta \frac{1}{\sin \theta}}$ $= \frac{\sec^2 \theta}{\sec \theta}$ $= \sec \theta$	M1 M1 M1 A1 M1 M1 A1	for $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\cot \theta = \frac{\cos \theta}{\sin \theta}$ and $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$; allow when used dealing correctly with fractions in the numerator; allow when seen use of the appropriate identity; allow when seen must be convinced it is from completely correct work (beware missing brackets) for either $\tan \theta = \frac{1}{\cot \theta}$ or $\cot \theta = \frac{1}{\tan \theta}$ and $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$; allow when used dealing correctly with fractions in numerator; allow when seen use of the appropriate identity; allow when seen must be convinced it is from completely correct work
3	$\mathbf{A}^{-1} = \frac{1}{2} \begin{pmatrix} 3 & -2 \\ -5 & 4 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 3 & -2 \\ -5 & 4 \end{pmatrix} \begin{pmatrix} 8 \\ 9 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 6 \\ -4 \end{pmatrix}$ $x = 3, y = -2$	B1 B1 M1 A1, A1	$\frac{1}{2}$ multiplied by a matrix for matrix attempt to use the inverse matrix, must be pre-multiplication

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4	(i)	Area = $\left(\frac{1}{2} \times 12^2 \times 1.7\right) + \left(\frac{1}{2} \times 12^2 \sin(2\pi - 1.7 - 2.4)\right)$ = awrt 181	B1,B1 M1 A1	B1 for sector area, allow unsimplified B1 for correct angle BOC, allow unsimplified correct attempt at area of triangle, allow unsimplified using <i>their</i> angle BOC (Their angle BOC must not be 1.7 or 2.4)
	(ii)	$BC^2 = 12^2 + 12^2 - (2 \times 12 \times 12 \cos 2.1832)$ or $BC = 2 \times 12 \times \sin\left(\frac{2\pi - 4.1}{2}\right)$ $BC = 21.296$ Perimeter = $(12 \times 1.7) + 12 + 12 + 21.296$ $= 65.7$	M1 A1 B1 M1 A1	correct attempt at BC, may be seen in (i), allow if used in (ii). Allow use of <i>their</i> angle BOC. for arc length, allow unsimplified for a correct 'plan' (an arc + 2 radii and BC)
5	(a) (i)	20160	B1	
	(ii)	$3 \times {}^6P_4 \times 2$ $= 2160$	B1,B1	B1 for 6P_4 (must be seen in a product) B1 for all correct, with no further working
	(iii)	$5 \times 2 \times {}^6P_4$ $= 3600$ Alternative 1: ${}^6C_4 \times 5! \times 2$ $= 3600$ Alternative 2: $({}^7P_5 - {}^6P_5) \times 2$ $= 3600$ Alternative 3: $2!({}^6P_4 + ({}^6P_1 \times {}^5P_3) + ({}^6P_2 \times {}^4P_2) + ({}^6P_3 \times {}^3P_1) + {}^6P_4)$ $= 3600$	B1,B1 B1 B2 B1 B2 B1 B2 B1	B1 for 6P_4 (must be seen in a product) B1 for 5 (must be in a product) B1 for all correct, with no further working for ${}^6C_4 \times 5!$ for ${}^6C_4 \times 5! \times 2$ for $({}^7P_5 - {}^6P_5)$ for $({}^7P_5 - {}^6P_5) \times 2$ 4 terms correct or omission of 2! in each term all correct

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(b) (i)	${}^{14}C_4 \times {}^{10}C_4$ or ${}^{14}C_8 \times {}^8C_4$ (or numerical or factorial equivalent) $= 210210$	B1,B1	B1 for either ${}^{14}C_4$ or ${}^{14}C_8$ as part of a product B1 for correct answer, with no further working
	(ii) ${}^8C_4 \times {}^6C_4$ $= 1050$	B1,B1	B1 for either 8C_4 or 6C_4 as part of a product B1 for correct answer with no further working
6 (i)	10ln4 or 13.9 or better	B1	
	(ii) $\left(\frac{dx}{dt}\right) \frac{20t}{t^2 + 4} - 4$ When $\frac{dx}{dt} = 0$, $\frac{20t}{t^2 + 4} = 4$ leading to $t^2 - 5t + 4 = 0$ $t = 1, t = 4$	M1	attempt to differentiate and equate to zero
		B1	$\frac{20t}{t^2 + 4}$ or equivalent seen
		DM1	attempt to solve <i>their</i> $\frac{dx}{dt} = 0$, must be a 2 or 3 term quadratic equation with real roots
		A1	for both

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(iii)	If $(v =) \frac{20t}{t^2 + 4} - 4$ $(a =) \frac{20(t^2 + 4) - 20t(2t)}{(t^2 + 4)^2}$ $20(4 - t^2)$ or $80 - 20t^2$ or $4 - t^2$ or equivalent expression involving $-t^2$ When acceleration is 0, $t = 2$ only Alternative 1 for first 3 marks: If $(v =) \frac{20t - 4t^2 - 16}{t^2 + 4}$ $(a =) \frac{(t^2 + 4)(20 - 8t) - (20t - 4t^2 - 16)(2t)}{(t^2 + 4)^2}$ Alternative 2 for M1 mark: If $(v =) 20t(t^2 + 4)^{-1} - 4$ $(a =) 20t(-2t(t^2 + 4)^{-2}) + 20(t^2 + 4)^{-1}$ Alternative 3 for the first 3 marks If $(v =) (20t - 4t^2 - 16)(t^2 + 4)^{-1}$ $(a =) (20t - 4t^2 - 16)(-2t(t^2 + 4)^{-2}) + (20 - 8t)(t^2 + 4)^{-1}$ Numerator $= -2t(20t - 4t^2 - 16) + (20 - 8t)(t^2 + 4)$	M1	attempt to differentiate <i>their</i> $\frac{dx}{dt}$
		A1	$20(t^2 + 4)$
		A1	$20t(2t)$
		A1	$20(4 - t^2)$ or $80 - 20t^2$ or $4 - t^2$
		B1	$t = 2$, dependent on obtaining first and second A marks
		M1	attempt to differentiate <i>their</i> $\frac{dx}{dt}$
		A1	for $(t^2 + 4)(20 - 8t)$
		A1	for $(20t - 4t^2 - 16)(2t)$
		M1	attempt to differentiate <i>their</i> $\frac{dx}{dt}$
		M1	attempt to differentiate <i>their</i> $\frac{dx}{dt}$
		A1	for $2t(20t - 4t^2 - 15)$
		A1	for $(20 - 8t)(t^2 + 4)$
7 (i)	$\overrightarrow{DA} = 3\mathbf{a} - \mathbf{b}$	B1	mark final answer, allow unsimplified
(ii)	$\overrightarrow{DB} = 7\mathbf{a} - \mathbf{b}$	B1	mark final answer, allow unsimplified
(iii)	$\overrightarrow{AX} = \lambda(4\mathbf{a} + \mathbf{b})$	B1	mark final answer, allow unsimplified
(iv)	$\overrightarrow{DX} = 3\mathbf{a} - \mathbf{b} + \lambda(4\mathbf{a} + \mathbf{b})$	M1	<i>their (i) + their (iii)</i> or equivalent valid method or $3\mathbf{a} - \mathbf{b} + \text{their (iii)}$
		A1	Allow unsimplified

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(v)	$3\mathbf{a} - \mathbf{b} + \lambda(4\mathbf{a} + \mathbf{b}) = \mu(7\mathbf{a} - \mathbf{b})$ Equate like vectors: $3 + 4\lambda = 7\mu$ $-1 + \lambda = -\mu$ leads to $\lambda = \frac{4}{11}, \mu = \frac{7}{11}$	M1 DM1 A1,A1	equating <i>their</i> (iv) and $\mu \times$ <i>their</i> (ii) for an attempt to equate like vectors and attempt to solve 2 linear equations for λ and μ A1 for each
8 (i)	$5e^{2x} - \frac{1}{2}e^{-2k} \quad (+c)$	B1, B1	B1 for each term, allow unsimplified
(ii)	$\left(5e^{2k} - \frac{1}{2}e^{-2k}\right) - \left(5e^{-2k} - \frac{1}{2}e^{2k}\right)$	M1 A1	use of limits provided integration has taken place. Signs must be correct if brackets are not included. allow any correct form
(iii)	$\left(5e^{2k} - \frac{1}{2}e^{-2k}\right) \left(5e^{-2k} - \frac{1}{2}e^{2k}\right) = -60$ or $\frac{11}{2}e^{2k} - \frac{11}{2}e^{-2k} = -60$ or equivalent leading to $11e^{2k} - 11e^{-2k} + 120 = 0$	B1 DB1	correct expression from (ii) either simplified or unsimplified equated to -60 , must be first line seen. must be convinced as AG
(iv)	$11y^2 + 120y - 11 = 0$ $(11y - 1)(y + 11) = 0$ leading to $k = \frac{1}{2}\ln\frac{1}{11}, \ln\frac{1}{\sqrt{11}}, -\ln\sqrt{11}, -\frac{1}{2}\ln 11$	M1 DM1 A1	attempt to obtain a quadratic equation in y or e^{2k} and solve to get y or e^{2k} (only need positive solution) attempt to deal with e to get $k =$. any of given answers only.

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9	$\frac{dy}{dx} = 4 - 6 \sin 2x$ When $x = \frac{\pi}{4}$, $y = \pi$ $\frac{dy}{dx} = -2 \text{ so gradient of normal} = \frac{1}{2}$ Normal equation $y - \pi = \frac{1}{2} \left(x - \frac{\pi}{4} \right)$ When $x = 0$, $y = \frac{7\pi}{8}$ When $y = 0$, $x = -\frac{7\pi}{4}$ $\text{Area} = \frac{1}{2} \times \frac{7\pi}{4} \times \frac{7\pi}{8} = \frac{49\pi^2}{64}$	M1,A1 B1 DM1 DM1 A1 A1 B1ft	M1 for attempt to differentiate A1 for all correct for y for substitution of $x = \frac{\pi}{4}$ into <i>their</i> $\frac{dy}{dx}$ and use of ' $m_1 m_2 = -1$ ', dependent on first M1 correct attempt to obtain the equation of the normal, dependent on previous DM mark must be terms of π must be terms of π Follow through on <i>their</i> x and y intercepts; must be exact values
10 (a)	$\cos^2 3x = \frac{1}{2}, \quad \cos 3x = (\pm) \frac{1}{\sqrt{2}}$ $3x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$ $x = 15^\circ, 45^\circ, 75^\circ, 105^\circ$	M1 A1,A1	complete correct method, dealing with sec and 3, correctly A1 for each correct pair
(b)	$3(\cot^2 y + 1) + 5 \cot y - 5 = 0$ Leading to $3 \cot^2 y + 5 \cot y - 2 = 0 \text{ or}$ $2 \tan^2 y - 5 \tan y - 3 = 0$ $(3 \cot y - 1)(\cot y + 2) = 0 \text{ or}$ $(\tan y - 3)(2 \tan y + 1) = 0$ $\tan y = 3, \quad \tan y = -\frac{1}{2}$ $y = 71.6^\circ, 251.6^\circ \quad 153.4^\circ, 333.4^\circ$	M1 M1 M1 A1,A1	use of a correct identity to get an equation in terms of one trig ratio only for $\cot y = \frac{1}{\tan y}$ to obtain either a quadratic equation in $\tan y$ or solutions in terms of $\tan y$; allow where appropriate for solution of a quadratic equation in terms of either $\tan y$ or $\cot y$ A1 for each correct 'pair'
(c)	$\sin \left(z + \frac{\pi}{3} \right) = \frac{1}{2}$ $z + \frac{\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$ $z = \frac{\pi}{2}, \frac{11\pi}{6}$ (allow 1.57, 5.76)	M1 A1 M1 A1	completely correct method of solution one correct solution in range correct attempt to obtain a second solution within the range second correct solution (and no other)