Cambridge Assessment International Education

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CANDIDATE NAME

CENTRE NUMBER | | | | | |

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2813398976

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/61**

Paper 6 Investigation and Modelling (Extended) **October/November 2022**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 5) and part **B** (Questions 6 to 9).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

[Turn over

Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 TO 5)

ISOSCELES TRAPEZIUMS (30 marks)

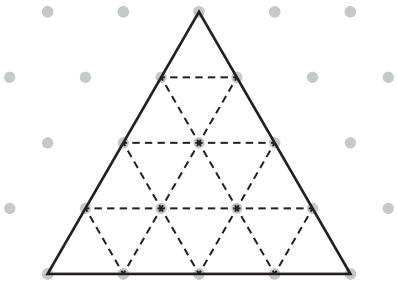
You are advised to spend no more than 50 minutes on this part.

This investigation looks at equilateral triangles in isosceles trapeziums drawn on 1 cm isometric grids. There is a spare isometric grid on page 9.

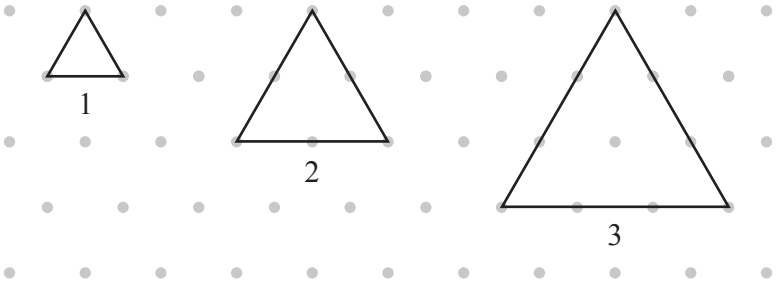
A *unit triangle* is an equilateral triangle of side length 1.

In this task, all shapes can be divided into unit triangles.

1 There are 16 unit triangles in the equilateral triangle with side length 4.



These are the first three equilateral triangles in a sequence.



(a) Complete the table.

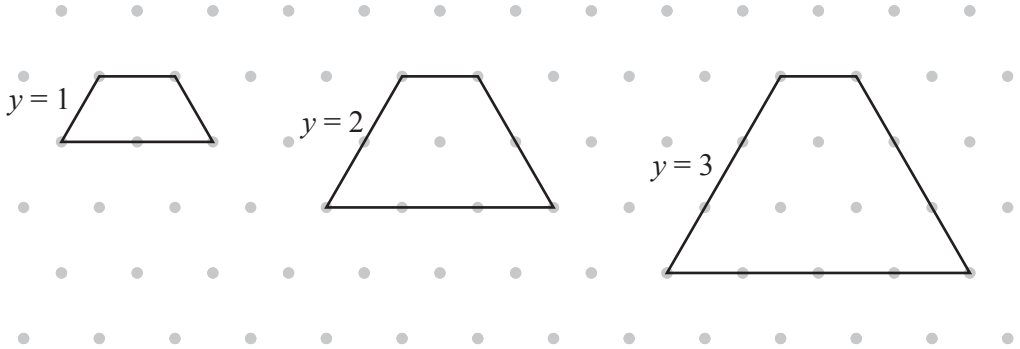
Side length (x)	1	2	3	4	5
Number of unit triangles	1			16	

[2]

(b) Write down an expression, in terms of x, for the number of unit triangles.

..... [1]

2 These are the first three isosceles trapeziums in a sequence.
The length of each sloping side is y.



(a) Complete the table.

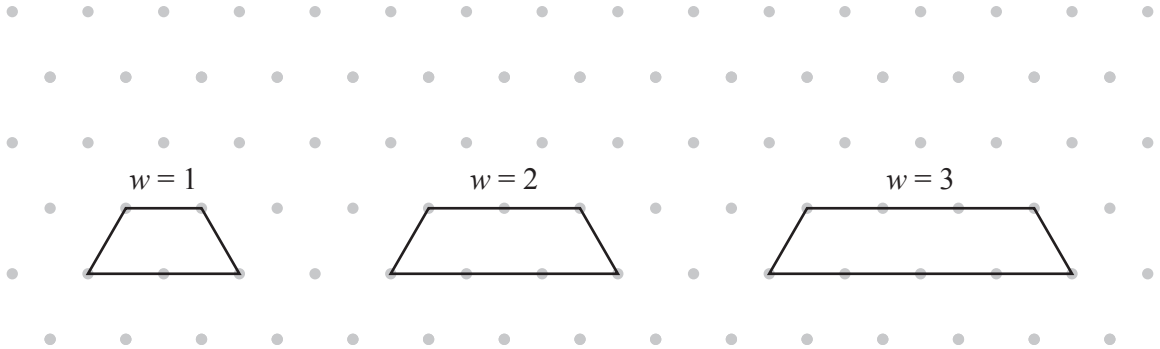
Length of sloping side (y)	1	2	3	4
Number of unit triangles	3			24

[1]

(b) Find an expression, in terms of y, for the number of unit triangles.
Give your answer in its simplest form.

..... [3]

3 These are the first three trapeziums in another sequence.
The length of each sloping side is 1.
The length of the shorter parallel side is w.



(a) Complete the table.

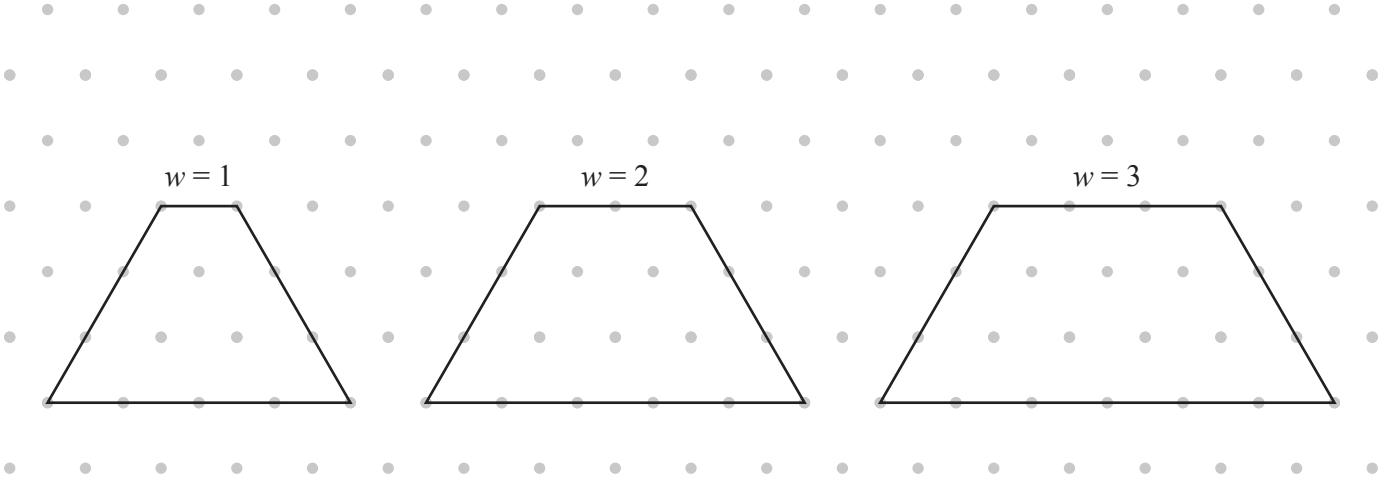
Length of shorter parallel side (w)	Number of unit triangles
1	2^2 – = 3
2	– 2^2 =
3	– = 7
4	– =

[2]

(b) Find an expression, in terms of w, for the number of unit triangles.
Give your answer in its simplest form.

..... [2]

4 These are the first three trapeziums in another sequence.
The length of each sloping side is 3.
The length of the shorter parallel side is w.



(a) Complete the table.

Length of shorter parallel side (w)	Number of unit triangles
1	4^2 – = 15
2	– 2^2 =
3	– = 27
4	– =

[2]

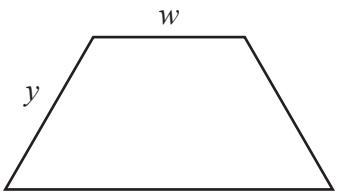
(b) Find an expression, in terms of w, for the number of unit triangles.
Give your answer in its simplest form.

..... [2]

(c) Find the value of w when there are 93 unit triangles.

..... [2]

5



In each trapezium:
- the length of the shorter parallel side is w
- the length of each sloping side is y.

(a) Show that an expression for the number of unit triangles is $y(y+2w)$.

[3]

(b) In a trapezium the length of the shorter parallel side is equal to the length of the sloping side.

Is it possible for the number of unit triangles to be 300?

[4]

(c) Find the lengths of the sides for all the trapeziums with exactly 160 unit triangles.

[6]

SPARE GRID

B MODELLING (QUESTIONS 6 TO 9)

PIZZA BUSINESS (30 marks)

You are advised to spend no more than 50 minutes on this part.

This task is about the costs in a pizza business.

Pablo is starting a pizza business.
He collects information from pizza businesses to set up mathematical models.
He assumes that he sells all the pizzas he makes.

6 Model 1

Pablo sets the following conditions:

- He sells one type of pizza with a selling price of \$10.
- He sells n pizzas per week.
- It costs him \$2 to make one pizza.
- His running costs are \$400 per week.
- He employs 2 people each costing \$300 per week.

This is how he works out his profit.

profit = income from selling pizzas − his costs

(a) Show that a model for the weekly profit, \$$P$, in terms of n, is

$$P = 8n - 1000.$$

[1]

(b) Pablo wants a profit of at least \$500 per week.

Use the model to find the minimum number of pizzas he must sell per week.

..... [3]

(c) Pablo expects sales to vary.
He sets the following conditions.

The pizza business:

- opens 4 days per week
- opens 6 hours per day
- makes 16 pizzas per hour for half of the time
- makes 8 pizzas per hour for the rest of the time.

Can Pablo make a weekly profit of $500?
Use the conditions above and your answer to **part (b)** to show how you decide.

[3]

7 Pablo makes changes to the conditions.

These are the conditions now:

- He sells one type of pizza with a selling price of $\$x$.
- He sells n pizzas per week.
- It costs him \$2 to make one pizza.
- His running costs of \$400 increase by 50%.
- He employs 3 people each costing \$300 per week.

(a) Use these conditions to show that the model for the weekly profit, $\$P$, is

$$P = (x-2)n - 1500.$$

[3]

(b) The pizza business is now open for 6 hours per day and 6 days per week.
It continues to make 16 pizzas per hour for half of the time and 8 pizzas per hour for the rest of the time.

Work out the number of pizzas the business can make per week.

..... [2]

(c) Pablo wants a weekly profit of at least $500.

Use the model in **part (a)** and your answer to **part (b)** to work out the minimum selling price of a pizza.

..... [3]

8 Model 2

Pablo makes a model for the relationship between the price of a pizza and the number of pizzas he sells.

He uses the following information:

- He sells 480 pizzas per week when the selling price is \$5 per pizza.
- He sells 60 pizzas per week when the selling price is \$20 per pizza.

Pablo assumes this relationship between the selling price of a pizza, \$$x$, and the number of pizzas he sells per week, n.

$$n = ax + b$$

Write down and solve a pair of simultaneous equations to show that $a = -28$ and $b = 620$.

[4]

9 Model 3

Pablo wants a model for the weekly profit, \$$P$, in terms of the selling price, \$$x$, of the pizzas.

(a) Use **Question 7(a)** and **Question 8** to show that a model for the weekly profit, in terms of x, is

$$P = -28x^2 + 676x - 2740.$$

[3]

(b) Sketch the graph of the model on the axes.

[3]

(c) Find the maximum weekly profit and the selling price that gives this.

Maximum weekly profit .....

Selling price ..... [2]

(d) Write an inequality for the values of x that give a weekly profit, $\$P$, of at least \$500.

..... [3]

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