CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Pre-U Certificate

MARK SCHEME for the May/June 2015 series

9795 FURTHER MATHEMATICS

9795/02 Paper 2 (Further Application of Mathematics), maximum raw mark 120

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2015 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

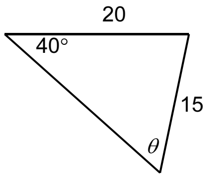
CAMBRIDGE
International Examinations

1	(i)	$2\mu - 50 = 0 \Rightarrow \mu = \mathbf{25}$ $44 + 25\sigma^2 = 144 \Rightarrow \sigma^2 = \mathbf{4}$	M1A1 M1A1 [4]	 $44 + 5\sigma^2$ or $22 + 5\sigma^2$ $\to$ 20 or 24.4: M1A0
	(ii)	$X - Y \sim \text{N}(15, 15)$ $z = \frac{10-15}{\sqrt{15}} = -1.291$ $\text{P}(X - Y > 10) = \mathbf{0.902}$	M1 M1 A1 A1 [4]	$\text{N}(\mu - 10,\ 11 + \sigma^2)$ Standardise, including $\sqrt{\sigma^2}$ Allow +1.29(1) [N(15, 31) $\to$ 0.898 $\to$ 0.815: M1M1A0A0]
2	(i)	$\bar{x} = \frac{114}{20} = 5.7$, $s^2 = \frac{2.382}{19} = 0.1254$ $t_{19} = 2.5395$ 98% c.l.: $5.7 \pm \left(2.5395 \times \frac{\sqrt{0.1254}}{\sqrt{20}}\right)$ 98% C.I. is **(5.499, 5.901)**	B1B1 B1 M1 A1A1 [6]	 Normal: B2B0M1A0
	(ii)	5.5 is (just) within the confidence interval. Some evidence to suggest that the average pH is 5.5 in villages where rhododendrons grow well.	B1FT B1FT [2]	Relate to CI Conclusion FT on their confidence interval
3	(i)	$\text{G}'(t) = \frac{1}{81} \times 4\left(t + \frac{2}{t}\right)^3\left(1 - \frac{2}{t^2}\right)$ $\text{E}(X) = \text{G}'(1) = \frac{4}{81} \times 27 \times (-1) = -\frac{4}{3}$ $\text{G}''(t) = \frac{4}{27}\left(t + \frac{2}{t}\right)^2\left(1 - \frac{2}{t^2}\right)^2 + \frac{4}{81}\left(t + \frac{2}{t}\right)^3 \times \frac{4}{t^3}$ $\text{G}''(1) = \frac{4}{27} \times 9 \times 1 + \frac{4}{81} \times 27 \times 4 = \frac{20}{3}$ $\text{Var}(X) = \frac{20}{3} + \left(-\frac{4}{3}\right) - \frac{16}{9} = \mathbf{\frac{32}{9}}$	M1 A1 M1 M1A1 [5]	For information: $G(t) = \frac{1}{81}\left(t^4 + 8t^2 + 24 + \frac{32}{t^2} + \frac{16}{t^4}\right)$ $G'(t) = \frac{1}{81}\left(4t^3 + 16t - \frac{64}{t^3} - \frac{64}{t^5}\right)$ $G''(t) = \frac{1}{81}\left(12t^2 + 16 + \frac{192}{t^4} + \frac{320}{t^6}\right)$
	(ii)	x: –4, –2, 0, 2, 4 $y = \frac{1}{2}(x+4)$: 0, 1, 2, 3, 4 $\text{P}(X = x) = \text{P}(Y = y)$: $\frac{16}{81}$, $\frac{32}{81}$, $\frac{24}{81}$, $\frac{8}{81}$, $\frac{1}{81}$ Recognising as terms of the expansion of $\left(\frac{2}{3} + \frac{1}{3}\right)^4$ State $\boldsymbol{n} = \mathbf{4}$ and $\boldsymbol{p} = \mathbf{\frac{1}{3}}$	B1 B1 B1B1 [4]	x and probabilities y Or: $G_{X+4}(t) = t^4 G_X(t)$ $G_{\frac{1}{2}(X+4)}(t) = G_{X+4}\left(\sqrt{t}\right)$ Independent of method BUT max 3 if binomial not shown

4 **(i)** **(a)**	$\mathrm{M}(t)=\sum\frac{\lambda^r}{r!}\mathrm{e}^{-\lambda}.\mathrm{e}^{tr}=\mathrm{e}^{-\lambda}\sum\frac{(\lambda\mathrm{e}^t)^r}{r!}$	M1	Allow via expansion
	$=\mathrm{e}^{-\lambda}.\mathrm{e}^{\lambda\mathrm{e}^t}=\mathrm{e}^{\lambda(\mathrm{e}^t-1)}$	M1A1 [3]	Needs to recognise series for M1 PGF quoted: M0M0A0
(b)	$\mathrm{M}_Z(t)=\mathrm{M}_X(t).\mathrm{M}_Y(t)=\mathrm{e}^{\mu(\mathrm{e}^t-1)}.\mathrm{e}^{\nu(\mathrm{e}^t-1)}$	M1	Multiply two MGFs
	$=\mathrm{e}^{(\mu+\nu)(\mathrm{e}^t-1)}\Rightarrow Z\sim\mathrm{Po}(\mu+\nu)$	A1 [2]	Needs one intermediate step
(ii) **(a)**	From tables $2+k=3.1\Rightarrow k=1.101=$ **1.10** (3sf)	M1A1 [2]	Or $\mathrm{e}^{-(2+k)}=0.045$ $\Rightarrow 2+k=\ln 22.22$ $\Rightarrow k=1.101=1.10$ (3sf)
(b)	$\mathrm{P}(3)=\mathrm{e}^{-1.1}\times\frac{1.1^3}{3!}=$ **0.0740**	M1 A1 [2]	Or P(≤3) – P(≤2) = 0.9743 – 0.9004 Answer in range [0.0738, 0.0740] $\lambda=k$ needed for M1
(c)	Using mean of 3.1: P(≤ 5) – P(≤ 1) = 0.9057 – 0.1847 = 0.7210 = **0.721** (3 sf)	M1 A1 [2]	Or from series ±1 term
5	$\frac{60.5-\mu}{\sigma}=2.083$, $\frac{39.5-\mu}{\sigma}=1.417$	B1M1 A1A1	z correct to 3 sf,
	$\boldsymbol{\mu}$ **= 48.0,** $\boldsymbol{\sigma}$ **= 6**	M1A1	Allow 3 sf, can be implied
	$\mu=np,\ \sigma=\sqrt{npq}$	B1B1	
	$1-p=36\div48=\frac{3}{4}$	M1	
	$\Rightarrow$ $\boldsymbol{p}$ **= 0.25,** $\boldsymbol{n}$ **= 192**	A1 A1 [11]	$p\in[0.248, 0.250]$ 192 or 193, must be integer
6 **(i)**	$\mathrm{f}_u(u)=\frac{1}{40}$ $U=\mathrm{g}^{-1}(V)=\frac{40V}{V-40}$	B1 M1	PDF of u U in terms of V
	$\mathrm{f}_v(v)=\mathrm{f}_u(\mathrm{g}^{-1}(v))\times\lvert\mathrm{g}^{-1\prime}(v)\rvert$	M1A1	Formula; mod sign needed for A1
	$=\frac{1}{40}\times\left\lvert\frac{-1600}{(v-40)^2}\right\rvert=\frac{40}{(v-40)^2}$ (AG)	A1 [5]	Mod sign needed for A1
	Or: $\mathrm{F}_u(u)=\frac{u-80}{40}$; $U=\frac{40V}{V-40}$	B1 M1	CDF of U U in terms of V
	$\mathrm{P}(V<v)=\mathrm{P}\left(U>\frac{40V}{V-40}\right)=1-\mathrm{P}\left(U<\frac{40V}{V-40}\right)$	M1	Turn $\mathrm{F}_U(u)$ into $\mathrm{F}_V(v)$, allow no 1–
	$=2-\frac{40}{v-40}$ so $\mathrm{f}_v(v)=\frac{40}{(v-40)^2}$ (AG)	A1A1 [5]	Correct $\mathrm{F}_V(v)$; correctly obtain AG

(ii) (a)	$F(v) = 2 - \dfrac{40}{(v-40)}$ $\quad 60 \le v \le 80$ $2 - \dfrac{40}{v-40} = \dfrac{1}{2} \Rightarrow v = \mathbf{\dfrac{200}{3}}$ (OE)	M1 A1 [2]	(Or by integration of pdf from 60 to median = 0.5; M1 needs limits or c) Allow from 1 – F(v)
(b)	$E(V) = \int_{60}^{80} \dfrac{40v}{(v-40)^2} dv = \int_{60}^{80} \dfrac{40}{v-40} + \dfrac{1600}{(v-40)^2} dv$ $= \left[40\ln\lvert v-40\rvert - \dfrac{1600}{v-40}\right]_{60}^{80}$ $=$ **40 ln 2 + 40** (= 67.7)	M1 M1A1 M1A1 A1 [6]	Or by $x = v - 40$: $40\left[\ln x - \dfrac{40}{x}\right]_{20}^{40}$
7 (i)	$x^2 + 9a^2 = (4a - x)^2 \Rightarrow \ldots \Rightarrow x = \dfrac{7}{8}a$ (AG)	M1A1 [2]	
(ii)	$T\cos\theta = mg$ $T + T\sin\theta = \dfrac{mv^2}{r}$ $\cos\theta = \dfrac{24}{25}$, $\sin\theta = \dfrac{7}{25}$ or $\tan\theta = \dfrac{7}{24}$ Solve to obtain $v = \sqrt{\dfrac{7ag}{6}}$	B1 M1*A1 B1 dep*M1 A1 [6]	M1 needs two forces One correct, may be implied Solve
8 (i)	Hooke's law: $T = \dfrac{8x}{0.4} = 20x$ Newton II: $\dfrac{1}{5}\ddot{x} = -20x$ $\Rightarrow \ddot{x} = -100x$ which is SHM Period $= \dfrac{2\pi}{10}$ $= \dfrac{1}{5}\pi$ seconds	B1 M1 A1 A1 [4]	Can be specific x Needs general x, – sign A.e. exact f.
(ii)	$-0.1 = 0.2\cos 10t$ $\Rightarrow 10t = \dfrac{2\pi}{3}$ $\Rightarrow t = \dfrac{1}{15}\pi$ seconds	M1A1 M1A1 [4]	Or: Obtain $\dfrac{1}{60}\pi$ Add quarter period to get $\dfrac{1}{15}\pi$

9 (i)	Gain in KE = $\frac{1}{2}\times 800\times(25^2-10^2) = 210\,000\,\text{J}$	M1	$\frac{1}{2}mv^2-\frac{1}{2}mu^2-mg\times 400\sin 2^\circ$
	Loss in PE = $800\times 10\times 400\sin 2^\circ = 111\,678\,\text{J}$	A1	Sign wrong: M1A0
	Work done = 210 000 – 111 768 = **98 322** J (= 98.3 kJ)	A1 [3]	
(ii)	$a=\frac{25^2-10^2}{800}=0.65625$ or $\frac{21}{32}$	M1A1	Allow in part (i) only if used in part (ii)
	$v^2 = 10^2 + 400 \times 0.65625 \quad \Rightarrow v = 19.0394$	A1	
	$F + 800 \times 10 \times \sin 2^\circ = 800 \times 0.65625 \quad \Rightarrow F = 245.8$	M1A1	3 terms needed for M1
	$P = Fv = 4680 \quad \Rightarrow$ Power is **4.68 kW**.	A1 [6]	
10 (i)	$-(mg+mkv^2)=m\frac{\text{d}v}{\text{d}t}$	B1	Allow + only if ↓ explicit
	$-\int_0^T \text{d}t=\int_u^0 \frac{1}{g+kv^2}\text{d}v$	M1	Separate and insert integral signs
	$=\frac{1}{k}\int_u^0 \frac{1}{\frac{g}{k}+v^2}\text{d}v$		
	$T=\frac{1}{k}\left[\sqrt{\frac{k}{g}}\tan^{-1}\sqrt{\frac{k}{g}}v\right]_0^u$	M1 A1	Or indefinite integral and find c Correct indefinite integral
	$=\frac{1}{\sqrt{gk}}\tan^{-1}\sqrt{\frac{k}{g}}u$	A1 [5]	
(ii)	$-(mg+mkv^2)=mv\frac{\text{d}v}{\text{d}x}$	B1	Allow + only if ↓ explicit
	$-\int_0^H \text{d}x=\frac{1}{2k}\int_u^0 \frac{2kv}{g+kv^2}\text{d}v$	M1	
	$H=\frac{1}{2k}\left[\ln\left\|g+kv^2\right\|\right]_0^u$	M1	Or indefinite integral and find c
		A1	Correct indefinite integral
	$=\frac{1}{2k}\ln\left(\frac{g+ku^2}{g}\right)$	A1 [5]	

11 **(i)**	Right angled triangle with angle $\tan^{-1}\frac{4}{3} - 50° = 3.13°$	B1	$(20t - 5\sin 50)^2 + (5\cos 50 - 15t)^2$ $\Rightarrow t = 0.1997$
	Shortest distance = 5 sin 3.13° = **0.273 km**	M1A1 [3]	
(ii) (a)	A correct velocity triangle. 	B1	
	$\frac{\sin\theta}{20} = \frac{\sin 40°}{15} \Rightarrow \theta = 58.986\ldots°$	M1A1	
	Bearing is **008.99°**	A1 [4]	(Accept 9° or 8.99°)
(b)	$\frac{v}{\sin 81.013\ldots°} = \frac{15}{\sin 40°} \Rightarrow v = 23.049$	M1A1	
	Time $= \frac{5}{23.049} \times 60 =$ **13.0** minutes.	M1A1 [4]	(Accept 13 minutes.)
12 **(i)**	Taking axes along and perpendicular to plane:		
	$\dot{x} = u\cos\theta - gt\sin\alpha$; $\dot{x} = 0 \Rightarrow t = \frac{u\cos\theta}{g\sin\alpha}$	M1 A1	Equation for $\dot{x}$, equated to 0 Find (eliminate) t
	$y = ut\sin\theta - \frac{1}{2}gt^2\cos\alpha$; $y = 0 \Rightarrow t = \frac{2u\sin\theta}{g\cos\alpha}$	M1 A1	Equation for y, equated to 0 Find (eliminate) t
	$\frac{u\cos\theta}{g\sin\alpha} = \frac{2u\sin\theta}{g\cos\alpha} \Rightarrow 2\tan\alpha\tan\theta = 1$ (AG)	M1A1 [6]	Equate and simplify; get AG
(ii) (a)	$x = u\cos\theta.\frac{u\cos\theta}{g\sin\alpha} - \frac{1}{2}g\sin\alpha.\frac{u^2\cos^2\theta}{g^2\sin^2\alpha}$	M1	Equation for x, substitute t
	$\Rightarrow x\sin\alpha = \frac{u^2}{2g}\cos^2\theta$	A1	Correct expression for $x\sin\alpha$
	But $\cos^2\theta = \frac{1}{1+\tan^2\theta} = \frac{1}{1+\frac{1}{4\tan^2\alpha}} = \frac{4\tan^2\alpha}{1+4\tan^2\alpha}$	M1A1	$\cos\theta$ in terms of $\tan\alpha$
	$\Rightarrow x\sin\alpha = \frac{2u^2\tan^2\alpha}{g(1+4\tan^2\alpha)}$ (AG)	A1 [5]	Obtain given answer
(b)	Time of flight $= \frac{u\cos\theta}{g\sin\alpha} = \frac{u}{g\sin\alpha}\sqrt{\frac{4\tan^2\alpha}{1+4\tan^2\alpha}}$	M1	Find t in terms of $\tan\alpha$
	$= \frac{2u}{g\cos\alpha\sqrt{1+4\tan^2\alpha}} = \frac{2u\sec\alpha}{g\sqrt{1+4\tan^2\alpha}}$ (AG)	M1A1 [3]	Simplify to given answer
	(N.B. Other orders of doing this may be seen.)		

Alternative method for 12:

12	**(i)**	Taking axes horizontally and vertically, where $\beta = \alpha + \theta$		
		$y = x\tan\beta - \frac{gx^2}{2u^2}\left(1+\tan^2\beta\right)$ and $y = \tan\alpha$ (on landing)	M1 A1	
		$\Rightarrow \tan\alpha = \tan\beta - \frac{gx}{2u^2}\left(1+\tan^2\beta\right)$ (I)	M1A1	
		$\left(\frac{dy}{dx} = \right)$ $\tan\beta - \frac{gx}{u^2}\left(1+\tan^2\beta\right) = \frac{-1}{\tan\alpha}$ (II)		
		(I) & (II) $\Rightarrow \tan\beta + \frac{1}{\tan\alpha} = 2\left(\tan\beta - \tan\alpha\right)$	M1	$\left(= \frac{gx}{u^2}\left(1+\tan^2\beta\right)\right)$
		$\Rightarrow \tan\beta = 2\tan\alpha + \frac{1}{\tan\alpha}$ (III)		
		$\Rightarrow \frac{\tan\alpha + \tan\theta}{1-\tan\alpha\tan\theta} = \frac{2\tan^2\alpha+1}{\tan\alpha} \Rightarrow \ldots$ $\Rightarrow 2\tan\alpha\tan\theta = 1$ (AG)	A1	
			[6]	
	(ii) (a)	(I) & (III) $\Rightarrow \frac{gx^2}{2u^2}\left(1+\tan^2\beta\right) = \frac{2\tan^2\alpha+1}{\tan\alpha} - \tan\alpha$ $= \frac{\tan^2\alpha+1}{\tan\alpha}$	M1A1	
		$\Rightarrow x\tan\alpha = \frac{2u^2}{g}\left\{\frac{1+\tan^2\alpha}{1+\tan^2\beta}\right\} = \frac{2u^2}{g}\left\{\frac{1+t^2}{1+\left(\frac{2t^2+1}{t}\right)^2}\right\}$, where $t = \tan\alpha$. (Required height is $x\tan\alpha$.)	M1A1	
		$\Rightarrow \ldots \Rightarrow x\tan\alpha = \frac{2u^2\tan^2\alpha}{g\left(1+4\tan^2\alpha\right)}$ (AG)	A1	
			[5]	
	(b)	$T = \frac{x}{u\cos\beta} = \frac{2u^2t^2}{g\left(1+4t^2\right)}.\frac{1}{t}.\frac{1}{u\cos\beta}$	M1	
		$\Rightarrow T^2 = \frac{4u^2t^2\left(1+\tan^2\beta\right)}{g^2\left(1+4t^2\right)^2} = \frac{4u^2t^2\left(1+\left[2t+\frac{1}{t}\right]^2\right)}{g^2\left(1+4t^2\right)^2} = \ldots$	M1	
		$= \frac{4u^2}{g^2}\left(\frac{1+t^2}{1+4t^2}\right) \Rightarrow T = \frac{2u\sec\alpha}{g\sqrt{1+4\tan^2\alpha}}$ (AG)	A1	
			[3]	