



GCSE Mathematics

For first teaching from September 2010

Teacher Guide

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Part 1: Summary Information

1 Key features

- This is a unitised specification. This means that students have the opportunity to sit at least one unit in the first year of teaching.
- The specification may be taught as either a linear or modular course.
- All Units (T1, T2, T3 and T4) and both of the Completion Tests (T5 and T6) will be available in January and June.
- Units may be taken in the same examination series or in different examination series.
- Each tier offers a choice of units that are suited to a wide range of abilities and will enable students to demonstrate achievement.
- Every candidate has access to Grade C.

2 Assessment objective weightings

The table below sets out the assessment objective weightings for the overall GCSE Mathematics qualification. All units address all three assessment objectives, and, where appropriate, questions may require knowledge and use of problem-solving strategies. Each written paper includes structured questions, questions set in context and some questions requiring the unprompted solution of multi-step problems.

Assessment Objectives		Weighting
AO1	Recall and use their knowledge of the prescribed content (AO1).	45–55%
AO2	Select and apply mathematical methods in a range of contexts (AO2).	25–35%
AO3	Interpret and analyse problems and generate strategies to solve them (AO3).	15–25%

3 Functional elements

In this specification, the term *functional elements* refers to the skills and abilities candidates need in order to take an active and responsible role in their communities, everyday lives, workplaces and educational settings. Functional mathematics requires candidates to use mathematics effectively in a wide range of contexts.

Functional elements are embedded in this specification, which provides opportunities for candidates to develop and apply these mathematical skills to real-life contexts.

3.1 Functional elements weighting

Assessments in GCSE Mathematics allocate a weighting of 20–30 percent on Higher Tier and 30–40 percent on Foundation Tier for the functional elements of mathematics. Each unit's assessment contains questions set in a functional context.

4 Quality of Written Communication

For the first time, quality of written communication (QWC) will be assessed in GCSE Mathematics through the candidate's ability to communicate clearly using both mathematical symbols and language and the written word. This may be through, for example, an explanation of the geometrical properties of a given shape or, for example, through concise mathematical argument in a multi-step problem.

Questions assessing the quality of written communication are indicated in the papers.

On each of the examination papers we have indicated which question(s) will assess QWC:

- on the front cover **in bold**; and
- at the start of the question **in bold**.

5 Specification at a Glance

The tables below summarise the assessment structure of this GCSE course.

5.1 Foundation Tier

Content	Assessment	Weighting	Availability
Unit T1:	External written examination <u>with</u> calculator 1 hour 30 minutes	45%	January and Summer (beginning summer 2011)
Unit T2:	External written examination <u>with</u> calculator 1 hour 30 minutes	45%	January and Summer (beginning summer 2011)
Unit T5: Foundation Completion Test (Compulsory)	2 external written examinations: <ul style="list-style-type: none"> • Paper 1 <u>without</u> calculator 1 hour • Paper 2 <u>with</u> calculator 1 hour 	55%	January and Summer (beginning summer 2012)

5.2 Higher Tier

Content	Assessment	Weighting	Availability
Unit T3:	External written examination <u>with</u> calculator 2 hours	45%	January and Summer (beginning summer 2011)
Unit T4:	External written examination <u>with</u> calculator 2 hours	45%	January and Summer (beginning summer 2011)
Unit T6: Higher Completion Test (Compulsory)	2 external written examinations: <ul style="list-style-type: none"> • Paper 1 <u>without</u> calculator 1 hour 15 minutes • Paper 2 <u>with</u> calculator 1 hour 15 minutes 	55%	January and Summer (beginning summer 2012)

6 Reporting, Grading and Uniform Mark Scale

6.1 Assessment opportunities

For a full award, candidates must complete two units: a Unit Test and a Completion Test:

- Foundation Tier candidates would normally enter for T1 or T2 and T5.
- Higher Tier candidates would normally enter for T3 or T4 and T6.

Candidates can choose to resit individual assessment units once. The 40% terminal rule states that units making up at least 40% of the qualification must be taken in the examination series in which the qualification is certificated. The final result will include the results for the unit that satisfies the 40% terminal requirement even if there is a better score for an earlier attempt at this unit.

Assessment Components	Percentage Assessment Weighting
Unit Test T1 or Unit Test T2	45%
Completion Test T5	55%

Assessment Components	Percentage Assessment Weighting
Unit Test T3 or Unit Test T4	45%
Completion Test T6	55%

Candidates entered for Higher Tier must also have studied all of the content of the Foundation Tier.

Candidates may sit units from different tiers and the results from each unit will be combined to give a total uniform mark.

The final grade will be determined by the total uniform mark for the better results in both the Unit Test and the Completion Test.

6.2 Reporting and grading

We report the results of individual assessment units on a uniform mark scale that reflects the assessment weighting of each unit. We determine the grades awarded by aggregating the uniform marks that candidates obtain on individual assessment units.

We award GCSE qualifications on an eight grade scale from A*–G, with A* being the highest. For candidates who fail to obtain a grade G, we report their results as unclassified (U).

There are two tiers of entry:

- We award grades C–G to candidates entered for the Foundation Tier.
- We award grades A*–D to candidates entered for the Higher Tier. We provide an allowed grade E as a ‘safety net’ for Higher Tier candidates just failing to secure a grade D.

To ensure access to a grade A at the Higher Tier, we strongly recommended that candidates take Unit T4 and Unit T6.

We award grades that match the grade descriptions published by the regulatory authorities.

Unit Tests T1, T2, T3 and T4 are timetabled on the same date and time. Therefore, candidates may enter only one of these examinations in each session.

Completion Tests T5 and T6 are timetabled concurrently, on a different day to T1, T2, T3 and T4. For both the Foundation and Higher Tier Completion Tests, Paper 1 (without calculator) and Paper 2 (with calculator) are scheduled one immediately after the other.

6.3 Uniform Marks Grid

All results will be reported on a Uniform Mark Scale (UMS)

	Unit T1, T2, T3, T4	Completion Test T5	Completion Test T6	AWARD
Max UMS	180	220	220	400
Grade				
A*	162 – 180		198 – 220	360 – 400
A	144 – 161		176 – 197	320 – 359
B	126 – 143		154 – 175	280 – 319
C	108 – 125	132 – 153	132 – 153	240 – 279
D	90 – 107	110 – 131	110 – 131	200 – 239
E	72 – 89	88 – 109	88 – 109	160 – 199
F	54 – 71	66 – 87	(66 – 87)	120 – 159
G	36 – 53	44 – 65	(44 – 65)	80 – 119
U	0 – 35	0 – 43	0 – 43	0 – 79

Notes: This qualification will be graded A*–G.

Candidates who fail to reach the minimum requirement for a grade G will be recorded as U, unclassified.

A candidate cannot achieve a uniform mark on a paper higher than the range of grades for that paper; for example, on Unit T2 a candidate cannot obtain more than 125 UMS.

Elaboration of Subject Content

Unit T1: Foundation Tier

This unit is targeted at grades E, F and G. Assessments based on this unit contain questions that are set in a functional context.

Content	Learning Outcomes	Elaboration
Number and Algebra	<p>Students should be able to:</p> <ul style="list-style-type: none"> add, subtract, multiply and divide initially with whole numbers and progressing to numbers up to three decimal places; 	<p>Write numbers in words and figures.</p> <p>Understand and use the language of number, [for example, whole, decimal, terminating, fraction, percentage, prime, square, cube, root, factor, multiple, common factor; positive and negative, integer, natural, sum, difference, product, numerator, denominator, common denominator, equivalent].</p> <p>Consolidation knowledge of number facts, including multiplication to 10×10; use this knowledge to find facts that they cannot recall; Calculate mentally that there are 63 days in 9 full weeks. Calculate 70×100 leading to $70 \times 500 = 35000$; $800 \div 10$ leading to $800 \div 20 = 40$</p> <p>Work out in a variety of ways how many buses are needed to carry 234 people to a football match if each bus holds 57 people. How many seats are not filled? Including rounding whole numbers to the nearest 10 and 100. Work out mentally how much heavier an object weighing 75kg is than one weighing 48kg. Approximate to specified degrees of accuracy including a given number of decimal places and significant figures. Calculate with negative numbers.</p>

Content	Learning Outcomes	Elaboration
Number and Algebra (cont.)	<p>Students should be able to:</p> <ul style="list-style-type: none"> order rational numbers; use the concepts and vocabulary of <i>factor (divisor), multiple and common factor</i>; use the terms <i>square, positive and negative square root, cube and cube root</i>; use index notation for squares, cubes and powers of 10; understand equivalent fractions, simplifying a fraction by cancelling all common factors; add and subtract fractions; use decimal notation and recognise that each terminating decimal is a fraction; understand that <i>percentage</i> means ‘number of parts per 100’ and use this to compare proportions, and simple calculations; use calculators effectively and efficiently; 	<p>Understand that 0.24 is greater than 0.235 Write numbers in order, such as 0.8, 0.89, 0.9 or $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, or -6, -2, 0, 4</p> <p>Find all primes between 0 and 100. Understand that if $x^2 = 16$, $x = \pm 4$</p> <p>Understand that $\sqrt{16} = 4$ Work out 10^2, 6^3</p> <p>Including equivalences such as $50\% = 0.5 = \frac{1}{2}$ $60\% = 0.6 = \frac{3}{5}$</p> <p>Simple fractions only i.e. not mixed numbers.</p> <p>Know that 0.235 is 2 tenths and 3 hundredths and 5 thousandths or 235 thousandths.</p> <p>Know that 5000 is 5 thousands or 50 hundreds or 500 tens or 5000 ones.</p> <p>Know that 7 books out of a total of 100 books represents 7% Find 20% of £3.00</p> <p>Understand the use of brackets to distinguish between</p> $9 + \frac{46}{80} \quad \text{and} \quad \frac{9 + 46}{80}$ <p>Find $\frac{2}{3}$ of £51</p>

Content	Learning Outcomes	Elaboration
Number and Algebra (cont.)	<p>Students should be able to:</p> <ul style="list-style-type: none"> distinguish the different roles played by letter symbols in algebra, using the correct notation; manipulate algebraic expressions by collecting like terms and by multiplying a single term over a bracket; solve simple equations; generate terms of a sequence using term-to-term definitions of the sequence; 	<p>Understand and use conventional notations of algebra, for example,</p> <p>$1a$ is written as a $b + b + b$ is written as $3b$ $3c + 4c$ is written as $7c$ $7x - 2x + y$ is written as $5x + y$ $a \times b \times 2$ is written as $2ab$ $y \times y$ is written as y^2 $2x^2 + 3x^2$ is written as $5x^2$ $a \div b$ is written as a/b or $\frac{a}{b}$</p> <p>Write expressions from a given problem.</p> <p>Know that $2(a+b)$ is the same as $2a + 2b$</p> <p>Solve $5x - 3 = 7$</p> <p>Explore, explain and generalise number patterns, and predict and check subsequent numbers; Understand the patterns in addition and multiplication tables, including symmetry of results and relationships between multiplication by 2, 4 and 8 etc.</p> <p>Produce a sequence in which the third and each subsequent number is the sum of the previous two numbers.</p> <p>Generalise, mainly in words, patterns which arise in various situations. [for example, through spatial arrangement]; Construct match stick squares using the appropriate number of match sticks to make 1,2,3,4... squares. Recognise square and triangular numbers and the relationship between them.</p>

Content	Learning Outcomes	Elaboration
Number and Algebra (cont.)	Students should be able to: <ul style="list-style-type: none"> • use the conventions for coordinates in the plane, and plot points in all four quadrants, including using geometric information; and • calculate with money and solve simple problems in the context of finance (for example, profit and loss, discount, current accounts, cheques, statements, lodgements and withdrawals). 	Name shapes formed by plotting coordinates. When using a calculator to find a sum of money in £, a display of 26.3 should be written as £26.30
Geometry and Measures	<ul style="list-style-type: none"> • recall and use properties of: <ul style="list-style-type: none"> – angles at a point; angles at a point on a straight line (including right angles); – perpendicular lines; and – opposite angles at a vertex; • understand and use the angle properties of triangles and quadrilaterals; • recall the properties and definitions of special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; 	Use appropriate language for lines, angles and shapes, including vertical, horizontal, perpendicular, parallel, acute, obtuse, right and reflex. Understand the notion of angle in the context of turning; give and understand instructions for moving through $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{4}$ turns and right angles. Understand clockwise and anticlockwise. Explore shape through drawing and practical work using a wide range of materials; recognise and describe a range of regular and irregular 2-D and 3-D shapes, including squares, rectangles, triangles, hexagons, pentagons, circles, cubes, cuboids, cylinders, pyramids . Classify and define types of triangles including scalene, right-angled, equilateral and isosceles. Draw triangles and other 2-D shapes using a ruler and protractor. Use the properties of triangles and quadrilaterals.

Content	Learning Outcomes	Elaboration
Geometry and Measures (cont.)	<p>Students should be able to:</p> <ul style="list-style-type: none"> • understand congruence; • distinguish between centre, radius, chord, diameter, circumference, tangent, arc, sector and segment; • use 2D representations of 3D shapes; • understand and use metric units of measurement; • use and interpret maps and scale drawings; • interpret scales on a range of measuring instruments and recognise the inaccuracy of measurements; • convert measurements from one unit to another; • solve problems involving length, weight, time, capacity and temperature; • make sensible estimates of a range of measures; 	<p>Group together congruent shapes from a range of triangles and quadrilaterals.</p> <p>Use a compass to draw a circle of given size.</p> <p>Draw nets of solids and use associated language including face, edge and vertex. Draw plans and elevations of simple 3D shapes.</p> <p>Understand and use scale in the context of maps and drawings, for example, calculate the actual distance as the crow flies between two places on a map. Know the eight points of the compass when answering questions on maps.</p> <p>Use two units, such as millilitres and litres to measure the capacity of the same jug. Work out that 2.4 kg is equivalent to 2400g</p> <p>Read digital and analogue displays; use a calendar; understand and use the twelve and twenty-four hour clock; use timetables involving the twenty-four hour clock. Use positive and negative temperatures</p> <p>Estimate the length of a car, the capacity of a teacup, the “weight” of a school bag. Estimate the time taken to complete a task.</p>

Content	Learning Outcomes	Elaboration
Geometry and Measures (cont.)	Students should be able to: <ul style="list-style-type: none"> • measure and draw lines and angles; • draw triangles and other 2D shapes using a ruler and protractor; • calculate perimeters and areas of triangles and rectangles; and • calculate volumes of cubes and cuboids. 	
Statistics and Probability	<ul style="list-style-type: none"> • understand and use the statistical problem-solving process; • understand and use the handling data cycle; • design data-collection sheets, distinguishing between different types of data; • extract data from printed tables and lists; • design and use two-way tables for discrete and grouped data; 	<p>Formulate questions that can be considered using statistical methods and undertake enquiries based on data analysis.</p> <p>Devise a simple habitat recorder for an ecological survey. Investigate the number of children crossing the road to school in order to establish the need for a crossing patrol and the necessary hours of duty. Conduct a survey of cars passing with one, two, three . . . occupants. Determine the best location for a pedestrian crossing. Use a given decision tree diagram to sort a collection of items.</p> <p>Use a table of distances between towns to plan a journey.</p>

Content	Learning Outcomes	Elaboration
Statistics and Probability (cont.)	Students should be able to: <ul style="list-style-type: none"> • produce charts and diagrams for various data types, including pie charts and frequency tables; • calculate median, mean, range and mode, and understand their uses; • interpret a wide range of graphs and diagrams and draw conclusions; and • look at data to find patterns and exceptions. 	<p>Use frequency diagrams and stem and leaf diagrams; pictogram using whole symbols to represent one or more than one item; part symbols; pie charts for categorical data; frequency tables and charts for ungrouped or grouped discrete data; line graphs; understand that intermediate values may or may not have meaning;</p> <p>Calculate or estimate and use the mean and range of sets of discrete data; distinguish between and be able to find the mean, median and mode of discrete data; consider the suitability of the mean, mode or median in different circumstances.</p> <p>Compare sets of data using the mean, median, mode and range.</p> <p>Interpret diagrams as stated above.</p>

Unit T2: Foundation Tier

This unit is targeted at grades C, D and E. Assessments based on this unit contain questions that are set in a functional context.

Students should have knowledge of the content Unit T1 before undertaking study or assessment of this unit.

Content	Learning Outcomes	Elaboration
Number and Algebra	<p>Students should be able to:</p> <ul style="list-style-type: none"> add, subtract, multiply and divide any number; use the concepts and vocabulary of <i>factor (divisor), multiple, common factor, highest common factor, least common multiple, prime number and prime factor decomposition</i>; use index laws for multiplication and division of integer powers; add and subtract fractions, including mixed numbers; recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals; use percentages, including calculating simple interest; interpret fractions, decimals and percentages as operations; 	<p>Including negative numbers and simple fractions.</p> <p>Express 147 as $3 \times 7 \times 7$ or 3×7^2 Find the Highest Common Factor (HCF) and Lowest Common Multiple (LCM) of two whole numbers.</p> <p>Excluding negative powers. Evaluate $3^2 \times 3^3 = 3^5$ and $\frac{4^5}{4^2} = 4^3$</p> <p>$3\frac{1}{3} + 1\frac{3}{4}$</p> <p>$1/3 = 0.3333333\dots = 0.\dot{3}$</p> <p>£1500 is invested at a rate of 3.6% p.a. Find the total amount after 3 years simple interest. Express one number as a percentage of another.</p> <p>Work out the cost of a laptop which is offered at 15% discount in a sale.</p> <p>Find $\frac{2}{3}$ of £51 ; 15% of 170m; 37% of £234</p> <p>Write 12 out of 40 as a percentage.</p>

Content	Learning Outcomes	Elaboration
Number and Algebra (cont.)	<p>Students should be able to:</p> <ul style="list-style-type: none"> • use ratio notation, including reduction to its simplest form and its various links to fraction notation; • manipulate algebraic expressions by: <ul style="list-style-type: none"> – collecting like terms; – multiplying a single term over a bracket; and – by taking out common factors; • set up and solve simple equations; • use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them; • generate terms of a sequence using term-to-term and position-to-term definitions of the sequence; • use linear expressions to describe the n^{th} term of an arithmetic sequence; • recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding their gradient; and 	<p>Simplifying ratios. $12:18 = 2:3$</p> <p>Know that $2(a + b)$ is the same as $2a + 2b$ and the converse. $x^2 - 3x = x(x - 3)$</p> <p>To include evaluating simple algebraic expressions for example,</p> <p>Find the value of $4x + 2y$ when $x = 3$ and $y = -2$</p> <p>To include fractions, for example,</p> $\frac{2x}{5} - 3 = 7$ <p>Find the side of a square whose area is 78cm^2 in the following way: $9^2 = 81$, $8^2 = 64$, so the side is more than 8cm, but less than 9cm. As 8.5^2 is 72.25, the side is greater than 8.5cm etc. Solve $x^2 + x = 10$ or $x^3 + x = 20$ by such a method.</p> <p>Understand that 1, 2, 4 . . . may be part of the sequence 1, 2, 4, 8, 16 . . . or the sequence 1, 2, 4, 7, 11 . . . etc. Use the difference method to explore sequences.</p> <p>Express in symbols the rule for the following sequence: 1, 3, 5 . . . $[2n - 1]$. . . Determine possible rules for generating a sequence.</p> <p>For example, draw the graph of $y = 3x - 2$</p>

Content	Learning Outcomes	Elaboration
Number and Algebra (cont.)	Students should be able to: <ul style="list-style-type: none"> calculate with money and solve problems in the context of finance (for example, currency exchange rates, discount, profit and loss, current accounts, cheques, statements, VAT, lodgements and withdrawals). 	
Geometry and Measures	<ul style="list-style-type: none"> understand and use the angle properties of parallel and intersecting lines; calculate and use the sums of the interior and exterior angles of polygons; understand congruence and similarity; use Pythagoras' theorem in 2D; understand and use bearings; find circumferences and areas of circles; calculate volumes of right prisms; find the mid-point and length of a line given in 2D co-ordinates; and 	<p>Use appropriate language and notation including vertically opposite, adjacent, alternate and corresponding angles; explain why the angle sum of any quadrilateral is 360°</p> <p>Use exterior and interior angle properties of regular polygons.</p> <p>Calculate the side of a right angled triangle when the other two sides are known.</p> <p>Locate position; use the eight points of the compass and three figure bearings to specify direction. Location of a boat.</p> <p>Calculate the perimeter and area of a semi-circle with radius 7cm.</p> <p>To include cylinders and triangular prisms etc.</p> <p>Application of Pythagoras' theorem.</p>

Content	Learning Outcomes	Elaboration
Geometry and Measures (cont.)	Students should be able to: <ul style="list-style-type: none"> • solve problems requiring calculations, including length, time, weight, capacity and temperature. 	Develop an understanding of the continuous nature of measure and approximate nature of measurement. Know the difference between 4.60 and 4.6 as measurements; realise that a length of l written as 9.7cm correct to one decimal place means that $9.65 \leq l < 9.75$ Use timetables in 12 and 24 hour clock. Use positive and negative temperatures in temperature change.
Statistics and Probability	<ul style="list-style-type: none"> • identify possible sources of bias; • design an experiment or survey; • find median, mean, range, mode and modal class; • interpret and display information in a variety of ways, including scatter graphs, flow charts and frequency polygons; • recognise correlation and draw and/or use lines of best fit by eye, understanding what these lines represent; • compare distributions and make inferences; and • Use relevant statistical functions on a calculator 	Including from frequency tables. Using paper and ICT. Including stem and leaf diagrams . Distinguish between positive, negative and no correlation. Compare the means and range for different distributions and draw conclusions.

Unit T3: Higher Tier

This unit targets grades B, C and D. Assessments based on this unit contain questions that are set in a functional context. Students should have knowledge of the content Unit T1 and Unit T2 before undertaking study or assessment of this unit.

Content	Learning Outcomes	Elaboration
Number and Algebra	<p>Students should be able to:</p> <ul style="list-style-type: none"> • use percentage and repeated proportional change; • calculate upper and lower bounds; • manipulate algebraic expressions by; <ul style="list-style-type: none"> – collecting like terms; – multiplying a single term over a bracket; – taking out common factors; – multiplying two linear expressions; – setting up and solving simple equations, including simultaneous linear equations in two unknowns; – factorising and solving quadratic equations; – factorising quadratic expressions including the difference of two squares; and – simplifying rational expressions; 	<p>Find the original value given a percentage change.</p> <p>Understand and calculate the upper and lower bounds of the values of expressions involving numbers expressed to a given degree of accuracy.</p> <p>Given the sides of a rectangle correct to the nearest unit, calculate the range of values within which the area lies.</p> <p>Expand and simplify $(x + 4)(x - 2)$</p> <p>Know that $(a \pm b)^2 = a^2 \pm 2ab + b^2$</p> $2x^2 - 4x = 2x(x - 2)$ <p>Solve equations such as</p> $\frac{4x + 3}{10} + \frac{6x - 5}{5} = \frac{13}{2}$ <p>Use algebra to solve a problem such as ‘If I double a number, then add 1 and the result is 49, what is the number?’</p> <p>Solve simultaneous equations algebraically.</p> $x^2 - 8x + 15 = (x - 3)(x - 5)$ $x^2 - 16 = (x - 4)(x + 4)$

Content	Learning Outcomes	Elaboration
Number and Algebra (cont.)	Students should be able to:	Simplify: (i) $\frac{6x^2y}{8xy^3} = \frac{3x}{4y^2}$ (ii) $\frac{2x^2}{y} \times \frac{3y^2}{6x}$ Adding and subtracting fractions such as $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$ Multiplying and dividing fractions such as $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
	<ul style="list-style-type: none"> understand that the form $y = mx + c$ represents a straight line and that m is the gradient of the line and c is the value of the y – intercept; and calculate with money, to include compound interest, insurance, taxation, wages, salaries and interest rates. 	Derive a linear relationship from a straight line graph; solve simultaneous linear equations using a graphical method. Draw the graph of $3x - 4y = 7$ Calculation of compound interest restricted to a maximum of three iterations. Determine weekly pay for a $37\frac{1}{2}$ hour week at £4.265 per hour.
Geometry and measures	<ul style="list-style-type: none"> use Pythagoras' theorem in 2D. calculate perimeters and areas of shapes made from triangles, rectangles and other shapes; and understand and use the trigonometric ratios of sine, cosine and tangent to solve 2D and 3D problems. 	Calculate a side of a right-angled triangle when the other two sides are known. Including simple arcs, simple sectors and composite shapes. Calculate the perimeter of a sector of a circle with radius 10cm and angle 60° Including angles of elevation and depression.

Content	Learning Outcomes	Elaboration																	
Statistics and Probability	<p>Students should be able to:</p> <ul style="list-style-type: none"> calculate median, mean, range, quartiles and inter-quartile range, mode and modal class; construct cumulative frequency tables; construct and use the cumulative frequency curve; interpret and display information in a variety of ways, including box plots and stem and leaf diagrams; and use relevant statistical functions on a calculator. 	<p>Prepare tables; calculate an estimate for the mean for discrete or continuous data.</p> <p>(i) Measurement of heights:</p> <p>Use 10 cm intervals from 120-150 cm. Class intervals defined as:</p> <table border="1" data-bbox="895 577 1324 784"> <thead> <tr> <th>Interval</th> <th>Mid Point</th> </tr> </thead> <tbody> <tr> <td>$120 < h \leq 130$cm</td> <td>125 cm</td> </tr> <tr> <td>$130 < h \leq 140$cm</td> <td>135cm</td> </tr> <tr> <td>$140 < h \leq 150$cm</td> <td>145cm</td> </tr> </tbody> </table> <p>h = height (centimetres)</p> <p>(ii) Examination marks:</p> <p>Range 0-100, intervals of 10 marks</p> <table border="1" data-bbox="895 1008 1380 1153"> <tbody> <tr> <td>0-9</td> <td>midpoint</td> <td>4.5</td> </tr> <tr> <td>10-19</td> <td>midpoint</td> <td>14.5</td> </tr> <tr> <td>20-29</td> <td>midpoint</td> <td>24.5 etc.</td> </tr> </tbody> </table> <p>Find the median, the upper quartile, the lower quartile and the interquartile range; describe the dispersion of data.</p>	Interval	Mid Point	$120 < h \leq 130$ cm	125 cm	$130 < h \leq 140$ cm	135cm	$140 < h \leq 150$ cm	145cm	0-9	midpoint	4.5	10-19	midpoint	14.5	20-29	midpoint	24.5 etc.
Interval	Mid Point																		
$120 < h \leq 130$ cm	125 cm																		
$130 < h \leq 140$ cm	135cm																		
$140 < h \leq 150$ cm	145cm																		
0-9	midpoint	4.5																	
10-19	midpoint	14.5																	
20-29	midpoint	24.5 etc.																	

Unit T4: Higher Tier

This unit targets grades A*, A and B. Assessments based on this unit contain questions that are set in a functional context.

Students should have knowledge of the content Unit T1, Unit T2 and Unit T3 before undertaking study or assessment of this unit.

Content	Learning Outcomes	Elaboration
Number and Algebra	<p>Students should be able to:</p> <ul style="list-style-type: none"> use index laws for multiplication and division of integer, fractional and negative powers; understand and use direct and indirect proportion; use calculators effectively and efficiently, including trigonometrical functions; set up and solve simple equations, including simultaneous linear equations in two unknowns; 	<p>Calculate using integer indices, positive, zero or negative; Evaluate $3^2 \times 2^3$</p> <p>Know that $10^{-1} = \frac{1}{10}$ etc.</p> <p>Evaluate $27^{2/3}$, $8^{-4/3}$ Know that $2^a \times 2^b = 2^{a+b}$</p> <p>Inverse and square laws. The current, I amps, in a circuit is inversely proportional to the resistance R ohms. The current is 2 amps when the resistance is 250 ohms. Find I when $R = 200$ The power P varies as the square of the current I. When $I = 2$, $P = 1000$ Find P when $I = 5$</p> <p>Extend to include fractional equations</p> $\frac{2}{x+2} + \frac{3}{2x-1} = 1$ <p>and one linear with one non-linear.</p> $2x + y = 1$ $x^2 + y = 1$ <p>Degree of $x \leq 2$</p>

Content	Learning Outcomes	Elaboration
Number and Algebra (cont.)	<p>Students should be able to:</p> <ul style="list-style-type: none"> • solve quadratic equations using factors and the formula; • understand the gradients of parallel and perpendicular lines. 	<p>Use these techniques with a range of more complex expressions; To include quadratic equations where the coefficient of $x^2 \neq 1$. $2x^2 - 18 = 0$ $6x^2 + 5x - 4 = 0$</p> <p>Finding the equation of a line through two points.</p> <p>$y = -5x$ and $y = -5x + 3$ represent parallel lines with gradient -5 $y = -5x$ and $5y = x + 4$ are perpendicular.</p>
Geometry and Measures	<ul style="list-style-type: none"> • use Pythagoras' theorem in 3D; • understand and use the sine and cosine rules to solve problems in 2D and 3D; • calculate the area of a triangle using $\frac{1}{2} ab \sin C$; and • understand and construct geometrical proofs using circle theorems. 	<p>Finding the length of a space diagonal.</p> <p>Finding the angle between a line and a plane.</p> <p>To include:</p> <p>Angle in a semicircle, angle at the centre and at the circumference, angles in the same segment, cyclic quadrilaterals, angle between tangent and radius, tangent kite, alternate segment theorem.</p>
Statistics and Probability	<ul style="list-style-type: none"> • understand and use: <ul style="list-style-type: none"> – sampling schemes; – histograms for grouped continuous data; and – frequency density; and • use relevant statistical functions on a calculator or spreadsheet. 	<p>Including random and stratified sampling.</p>

Unit T5: Foundation Completion Test

This unit targets grades C, D, E, F and G. Assessments based on this unit contain questions that are set in a functional context.

Students should have knowledge of the content Unit T1 before undertaking study or assessment of this unit.

Content	Learning Outcomes	Elaboration
Number and Algebra	<p>Students should be able to:</p> <ul style="list-style-type: none"> approximate to specified or appropriate degrees of accuracy including a given power of 10, number of decimal places and significant figures; understand and use number operators (+, −, ×, ÷) and the relationships between them, including inverse and reciprocal operations and hierarchy of operations (for example, BODMAS); 	<p>Estimate that: $1472 - 383$ is about 1100</p> <p>$278 \div 39$ is about 7</p> <p>$\frac{0.25 \times 83.4}{5.7}$ is about 3 or 4</p> <p>Estimate $\sqrt{97}$</p> <p>Multiply and divide mentally single-digit multiples of any power of ten, and realise that, when multiplying or dividing by a number less than one, multiplication has a decreasing effect, and division an increasing effect. Work out mentally 80×0.2 and $600 \div 0.2$</p> <p>Understand ‘reciprocal’ as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is one and that zero has no reciprocal, because division by zero is not defined. Apply order of precedence, first without brackets and then with brackets; solve problems requiring application of order of precedence.</p> <p>Distinguish correctly between:</p> <p>$3 + 2 \times 5$ and $(3 + 2) \times 5$</p> <p>$\frac{7.2}{9.8 + 12.7}$ and $\frac{7.2}{9.8} + 12.7$</p> <p>Understand the use of a counter-example. Explain why $2n - 1$ is always odd.</p>

Content	Learning Outcomes	Elaboration
Number and Algebra (cont.)	<p>Students should be able to:</p> <ul style="list-style-type: none"> • divide a quantity in a given ratio; • understand the meaning of the words <i>equation</i>, <i>formula</i> and <i>expression</i> and distinguish between them; • derive a formula, substitute numbers into a formula and change the subject of a formula; • solve linear inequalities in one variable, and represent the solution set on a number line; • construct linear functions from real-life problems and plot their corresponding graphs; • discuss, plot and interpret graphs (which may be non-linear) modelling real situations; 	<p>Understand and use ratios in a variety of situations. Use the approximation of 5 miles to 8 km to find the equivalent of 12 miles. State the lengths 8 cm and 12 cm in a drawing are in the ratio 2:3 Adapt a recipe for six people to one for eight people. Divide £10 between two people in the ratio 3:5</p> <p>Distinguish the different roles played by letter symbols in algebra, knowing that letter symbols represent definite unknown numbers in equations [e.g. $5x + 1 = 16$], defined quantities or variables in formulae [e.g. $V = IR$], general unspecified and independent numbers in expressions [e.g. $3x + 2x = 5x$ for all values of x] and in functions they define new expressions or quantities by referring to known quantities [e.g. $y = 2x$].</p> <p>Understand, construct and evaluate formulae related to mathematics or other subjects or real-life situations. Evaluate expressions. Work out $s = ut + \frac{1}{2}at^2$ where u or a may have negative values. Make t the subject of the formula. $v = u + at$.</p> <p>List the values of the integer n such that $-10 < 2n \leq 20$ or solve the inequality $2n - 3 \geq 7$ illustrating the solution on a number line. Solve $x \leq 3x - 5$ where x is a real number.</p> <p>For example, conversion graphs.</p> <p>For example, distance-time graphs including intersecting travel graphs.</p>

Content	Learning Outcomes	Elaboration
Number and Algebra (cont.)	<p>Students should be able to:</p> <ul style="list-style-type: none"> generate points and plot graphs of simple quadratic functions, and use these to find approximate solutions; use index laws in algebra for multiplication and division of integer powers; and calculate with money to include hire purchase, VAT, taxation, wages and salaries. 	<p>To include drawing graphs of: $y = ax^2 + bx + c$</p> <p>Simplify expressions involving positive indices only, such as: $x^6 \div x^4$, $x^2 \times x^3$ and $(x^2)^3$</p> $\frac{6x^2y}{8xy^3} \quad \frac{2x^2}{y} \times \frac{3x^2}{6x}$ <p>Solve problems in the context of finance, e.g. currency exchange rates, loans, deposit accounts, credit cards, general bank accounts including overdrafts, interest rates and mortgages, simple interest, mail order sales, insurance and unemployment benefit.</p>
Geometry and Measures	<ul style="list-style-type: none"> recognise reflection and rotation symmetry of 2D shapes; describe and transform 2D shapes using single or combined rotations, reflections, translations, or enlargements by a positive scale factor distinguish properties that are preserved under particular 2D transformations; 	<p>Find the orders, centres and axes of symmetry. Reflect shapes in a mirror line. Turn shapes using tracing paper. Reflection to include rectilinear figures only. Know and use symmetry properties of triangles, quadrilaterals and other polygons.</p> <p>Rotations will be limited to $\pm 90^\circ$ and 180° about a point. Reflections will be limited to reflections in lines parallel to the co-ordinate axes.</p> <p>Vector notation will be used for translations. Use transformations to create and analyse spatial patterns. Recognise that enlargements preserve angle but not length.</p>

Content	Learning Outcomes	Elaboration
Geometry and Measures (cont.)	<p>Students should be able to:</p> <ul style="list-style-type: none"> • understand the effect of enlargement for perimeter, area and volume of shapes and solids; • understand and use compound measures; • know and use imperial measures still in common use and their approximate metric equivalents; • use a ruler and a pair of compasses to do constructions; • construct loci; • calculate perimeters and areas of shapes made from triangles and rectangles; and • solve problems (which may be set in context) that require calculations including a range of measures, for example, length, area, volume, weight, time and temperature. 	<ul style="list-style-type: none"> • Work out average speed (distance/time) or density (mass/volume). <p>Includes foot, yard, mile, pound and pint. Recall that 1 kg is about 2.2 lb, 8 km is approximately 5 miles, 1 litre is about 1.75 pints.</p> <p>Including an equilateral triangle with a given side, the mid-point and perpendicular bisector of a line segment, the perpendicular from a point to a line, the perpendicular from a point on a line and the bisector of an angle.</p> <p>Region bounded by a circle and an intersecting line.</p> <p>Calculate the area of a trapezium, parallelogram, rhombus and kite.</p> <p>Calculate the surface areas and volume of cubes, cuboids and simple right prisms.</p>

Content	Learning Outcomes	Elaboration
Statistics and Probability	<p>Students should be able to:</p> <ul style="list-style-type: none"> understand and use the vocabulary of probability and the probability scale; understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), and from relative frequency; list all outcomes for single events, and for two successive events, in a systematic way and derive related probabilities; identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1; compare experimental data and theoretical probabilities; and understand that if they repeat an experiment, they may, and usually will, get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics. 	<p>Place events in order of ‘likelihood’ and use appropriate words to identify chance. Understand and use 0 and 1 as the limits of the probability scale.</p> <p>Know that for equally likely outcomes, the probability of an event is the number of desirable outcomes divided by the number of possible outcomes. Know that if there are six identical beads numbered, 1, 1, 2, 2, 3 and 4, the probability of selecting a bead labelled 1 is $\frac{2}{6}$</p> <p>Recognise situations where probabilities can be based on equally likely outcomes and others where estimates must be based on sufficient experimental evidence and make these estimates; understand and use relative frequency as an estimate of probability.</p> <p>List all the outcomes when tossing two coins, HH, TT, TH, HT.</p> <p>Make a table of all the outcomes when throwing two dice and show the total sums arising.</p> <p>Recognise that if the probability of a machine failing is 0.05 then the probability of it not failing is 0.95</p> <p>Understand possible outcomes of random trials or experiments; understand that there is a degree of uncertainty about the occurrence of some events, and others are certain or impossible. Know that you do not always get 5 heads in 10 tosses of a ‘fair’ coin and very occasionally there will be none.</p>

Unit T6: Higher Completion Test

Students should have knowledge of all Foundation Tier content as well as the content of Unit T3 before undertaking study or assessment of this unit.

Content	Learning Outcomes	Elaboration
Number and Algebra	<p>Students should be able to:</p> <ul style="list-style-type: none"> use surds and π in exact calculations; Change a recurring decimal to a fraction; interpret, order and calculate with numbers written in standard index form; distinguish in meaning between the words <i>equation</i>, <i>formula</i>, <i>identity</i> and <i>expression</i>; derive a formula, substitute numbers into a formula and change the subject of a formula, including cases where the subject appears in more than one term or where a power of the subject appears; solve linear inequalities in one or two variables, and represent the solution set on a number line or suitable diagram; 	<p>Distinguish between rational and irrational numbers; know that $\sqrt{2}$ and π are irrational.</p> <p>Simplification of surds including rationalise a denominator;</p> $\sqrt{12} = 2\sqrt{3}; \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3};$ <p>Write $(5 - \sqrt{5})^2$ in the form $a + b\sqrt{5}$.</p> <p>Know the significance of recurring and non-recurring decimals.</p> <p>Calculate with numbers in standard index form using both positive and negative powers of ten.</p> <p>Calculate $\frac{3.2 \times 10^4}{1.6 \times 10^{-3}}$;</p> <p>Use standard index form on a calculator.</p> <p>Know the meaning of and use the word 'identity'.</p> <p>Understand the identity symbol.</p> <p>Use the formula $F = \frac{GMm}{r^2}$ to calculate one variable given the others.</p> <p>Transform formulae such as $v = u + at, \quad A = \pi r^2,$ $p = \frac{100(s - c)}{c}$</p> <p>List the values of the integer n such that $-10 < 2n \leq 20$ or solve the inequality $2n - 3 \geq 7$ illustrating the solution on a number line. Solve $x \leq 3x - 5$ where x is a real number.</p> <p>Use straight line graphs to locate regions representing linear inequalities; For example, $x < 10, y \geq 6, y < 2x + 3$.</p>

Content	Learning Outcomes	Elaboration
Number and Algebra (cont.)	<p>Students should be able to:</p> <ul style="list-style-type: none"> • find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions; • draw, sketch and recognise graphs of: <ul style="list-style-type: none"> – simple cubic functions; – the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$; – the function $y = k^x$ for integer values of x and simple positive values of k; and – the trigonometric functions $y = \sin x$, $y = \cos x$ and $y = \tan x$ • construct the graphs of simple loci; • construct linear, quadratic and other functions from real-life problems and plot their corresponding graphs; use growth and decay rates and display these graphically; and 	<p>For example, Solve $y = 5x - 6$ and $y = x^2$, by drawing the graph of each function. Use the graph of $y = x^2 + 5x$ to solve $x^2 + 5x = 7$</p> <p>Make tables of such functions, sketch and interpret their graphs using graphical calculators and computers to understand their behaviour.</p> <p>To include drawing graphs of: $y = ax + b$ $y = ax^2 + bx + c$ $y = \frac{a}{x}$ where $a \neq 0$ and $x \neq 0$ $y = a^x$ where $a = 2, 3, 4$ Use the graphs of $y = x^2 + 5x$ and $y = x^3$ to solve $x^3 = x^2 + 5x$</p> <p>Recognise the characteristic shapes of these functions. Within the range 0° to $+360^\circ$</p> <p>Including the region bounded by a circle and an intersecting line.</p> <p>For example, distance-time graphs including intersecting travel graphs.</p> <p>Know about rates of economic growth and decline and the half-life of radioactive elements.</p>

Content	Learning Outcomes	Elaboration
Number and Algebra (cont.)	Students should be able to: <ul style="list-style-type: none"> use index laws in algebra for multiplication and division of integer, fractional and negative powers. 	Simple expressions such as $6x^6 \div 3x^4, 2x^2 \times 3x^3, (3x^2)^3;$ $\frac{6x^2y}{8xy^3} \quad \frac{2x^4}{y} \times \frac{-3y^2}{6x^2}$ Use : $x^0 = 1, y^{-3} = \frac{1}{y^3}, \frac{x^2}{x^3} = \frac{1}{x} = x^{-1}, x^{\frac{1}{2}} \times x^{\frac{3}{2}} = x^2$
Geometry and Measures	<ul style="list-style-type: none"> describe and transform 2D shapes using single or combined rotations, reflections, translations, or enlargements by a positive scale factor; use positive fractional and negative scale factors; distinguish properties that are preserved under particular transformations; understand and use the effect of enlargement for perimeter area and volume of shapes and solids; solve mensuration problems that involve more complex shapes (including arc length and area of sector) and solids (including cones, spheres and frustums); and understand and use dimensions of formula for perimeter, area and volume. 	Column vector notation for translation should be known. To include reflection in the line $y = \pm x$ Find the inverse of transformations. Recognise that translations, rotations and reflections preserve length and angle; recognise that enlargements preserve angle but not length. Use mathematical similarity and prove triangles similar. Know that angles remain unchanged and corresponding sides are in the same ratio. Use the relationship between the surface areas of similar 3-D shapes and between volumes of similar 3-D shapes, including the frustum of a cone. Finding surface area and volume of compound solids constructed from cubes, cuboids, cones, spheres, hemispheres, cylinders and prisms. Recognise that πd is a linear measurement and that πr^2 is an area measurement. Identify from a range of formulae those which denote (say) volume: $4\pi r^2, \frac{4}{3} \pi r^3, \frac{1}{3} \pi r^2 h, r(\pi + 2)$

Content	Learning Outcomes	Elaboration
Statistics and Probability	<p>Students should be able to:</p> <ul style="list-style-type: none"> know when to add or multiply two probabilities: if A and B are mutually exclusive, then the probability of A or B occurring is $P(A) + P(B)$, whereas if A and B are independent events, the probability of A and B occurring is $P(A) \times P(B)$; and use tree diagrams to represent outcomes of compound events, recognising when events are independent. 	<p>Selection with or without replacement. Understand that when dealing with two independent events, the probability of them both happening is less than the probability of either of them happening (unless the probability is 0 or 1). Know that the probability of getting two consecutive sunny days over the weekend is less than the probability of getting a sunny Saturday or Sunday. Draw a tree diagram or use a tabulation to define all of the possible outcomes e.g. tossing a coin 3 times</p> <p>To include dependent and independent events.</p> <p>Use of a tree diagram to calculate conditional probability.</p> <p>Given that there are 2 sets of traffic lights on the way to school and the probability of getting straight through the lights without having to stop are 0.6 and 0.4 respectively, find the probability of a cyclist having to stop at one set of lights, using a tree diagram or otherwise.</p>