New Specification

Rewarding Learning

Centre Number

Candidate Number

ADVANCED
General Certificate of Education
2019

Mathematics

Assessment Unit A2 1
assessing
Pure Mathematics

MV18

[AMT11]
TUESDAY 28 MAY, MORNING

Time

2 hours 30 minutes, plus your additional time allowance.

Instructions to Candidates

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all twelve** questions in the spaces provided.
Do not write on blank pages or tracing paper.
Complete in black ink only.
Questions which require drawing or sketching should be completed using an H.B. pencil.
Show clearly the full development of your answers.
Answers without working may not gain full credit.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

Information for Candidates

The total mark for this paper is 150

Figures in brackets printed at the end of each question indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

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(Questions start overleaf)

1 A curve is given by the equation

$$x^3 + 3y^2 = 11$$

By using implicit differentiation find $\frac{dy}{dx}$ in terms of x and y.
[4 marks]

2 A curve is defined parametrically by

$x = at^2$ and $y = 3at$

where a is a constant and t is the parameter.

Find the Cartesian equation of this curve. [4 marks]

3 A mirror ABCDE is designed in the form of a sector of a circle, centred at B, together with two congruent right-angled triangles, BAE and BCD, as shown in **Fig. 1** below.

AC = 80 cm AE = CD = 60 cm

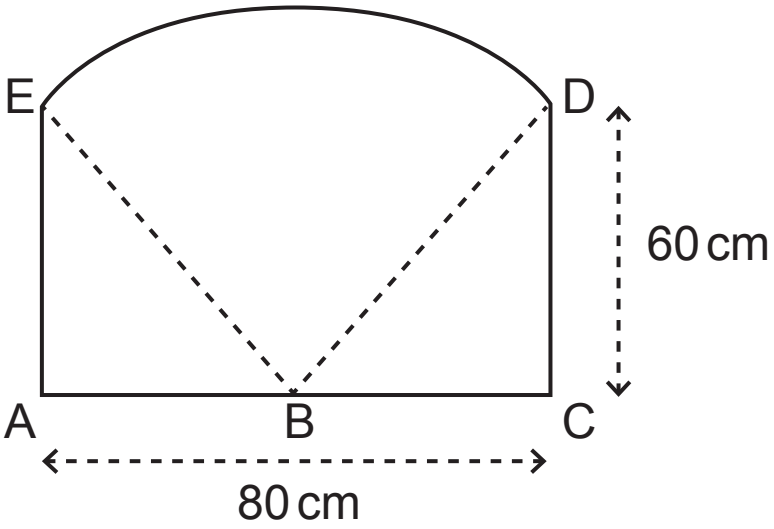


Fig. 1

(i) Find the angle EBD in radians. [5 marks]

(ii) Find the area of the mirror. [7 marks]

4 **(i)** Prove that

$\text{cosec } 2\theta - \cot 2\theta \equiv \tan \theta$ [7 marks]

(ii) Hence find the exact value of $\tan \frac{\pi}{8}$ [2 marks]

5 (a) A function f is defined by

$$\text{f: } x \to x^2 - 8, \qquad x \in \mathbb{R}, \qquad x \geqslant 0$$

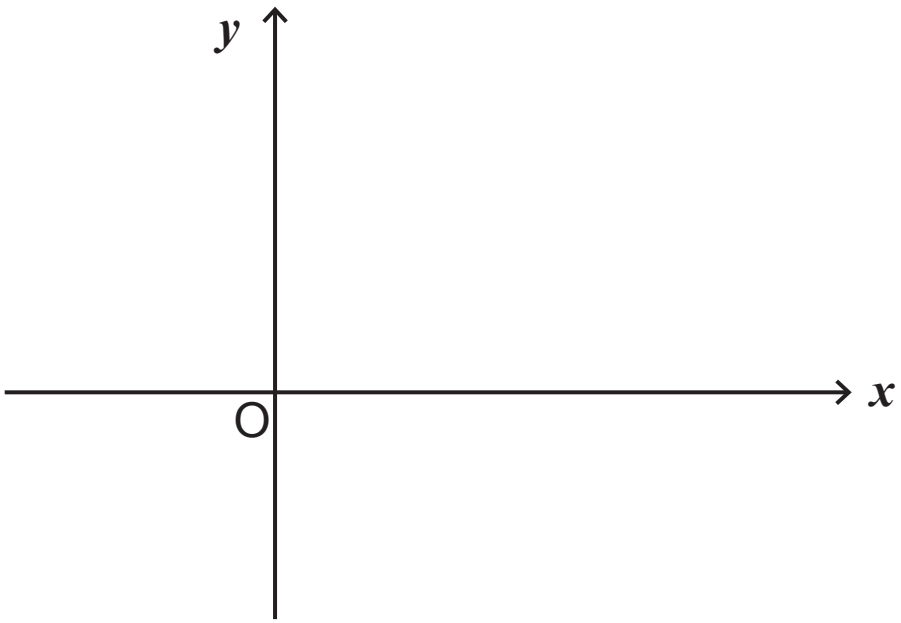
(i) State the range of the function f(x). [1 mark]

(ii) Find the inverse function $\text{f}^{-1}(x)$, clearly stating its domain. [4 marks]

A function g is defined by

g: $x \rightarrow |x - 3|, \quad x \in \mathbb{R}$

(iii) On the axes below sketch the graph of $y = g(x)$.
[2 marks]



(iv) Find the composite function gf(x). [2 marks]

(b) The graph of the function $y = h(x)$ is sketched in **Fig. 2** below.

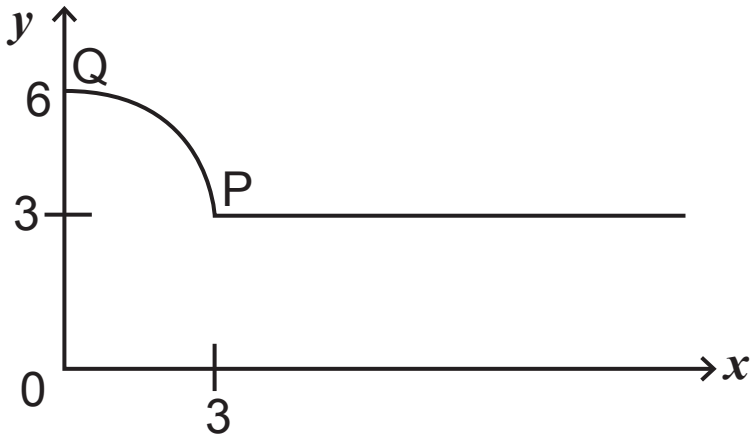
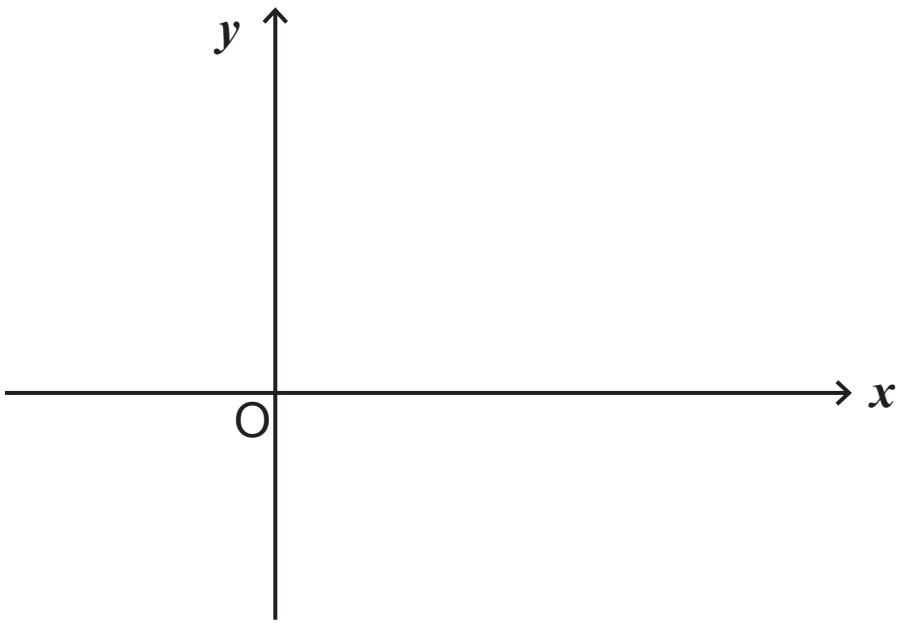


Fig. 2

(i) On the axes below sketch the graph of

$$y = \frac{1}{3}h(3x)$$

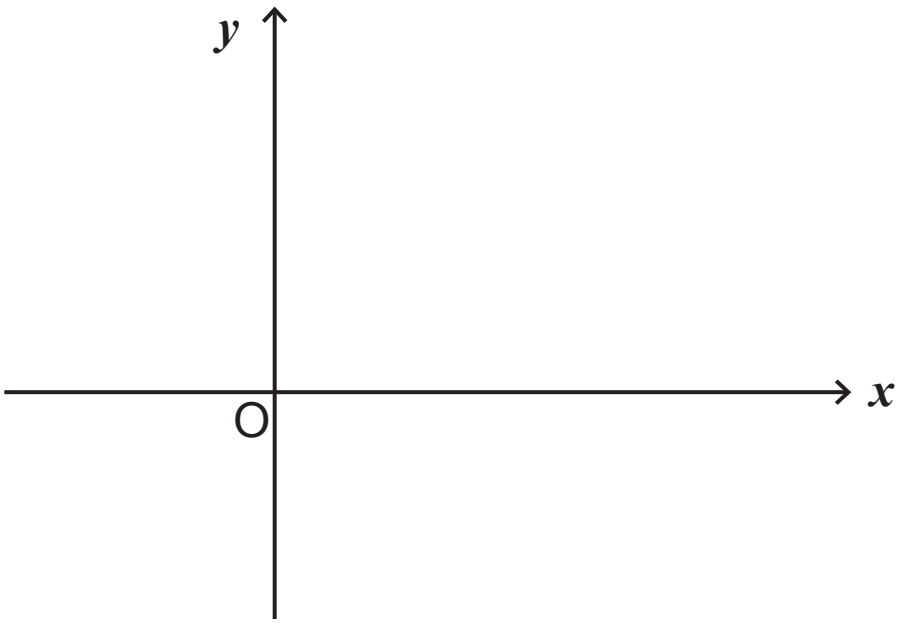
and clearly label the images of the points P and Q. [2 marks]



(ii) On the axes below sketch the graph of

$y = 6 - h(x)$

and clearly label the images of the points P and Q.
[2 marks]



6 The expression $8 \sin x + 15 \cos x$ can be written in the form $R \sin(x + \alpha)$, where R is an integer and $0° \leqslant \alpha \leqslant 90°$

(i) Find the values of R and α. [6 marks]

(ii) Hence, or otherwise, determine the maximum value of

$$\frac{18}{8 \sin x + 15 \cos x + 23}$$

and find a corresponding value of x. [4 marks]

7 **(i)** Use the Trapezium Rule with 3 ordinates to find an approximate value for

$$\int_2^3 \frac{x^2}{(x+3)(x-1)}\,\mathrm{d}x$$ [5 marks]

(ii) Use partial fractions to calculate the value of

$$\int_2^3 \frac{x^2}{(x+3)(x-1)}\,dx$$ [12 marks]

(iii) Explain how the use of the Trapezium Rule in **(i)** could be modified to obtain a better approximation to the integral

$$\int_2^3 \frac{x^2}{(x+3)(x-1)}\,\mathrm{d}x$$ [1 mark]

8 The population, P, in a housing development grows at a rate proportional to the population at any time t (years).

This can be modelled by the differential equation

$$\frac{dP}{dt} = kP$$

where k is a constant.

The initial population is P_0

(i) Show that

$$P = P_0 e^{kt}$$ [6 marks]

(ii) Given that the initial population doubles in 5 years, find the exact value of k. [3 marks]

(iii) Find the number of years until the initial population is trebled.

Give the answer to the nearest year. [3 marks]

(iv) State a limitation of this model. [1 mark]

9 A curve has the equation

$y = (x - 5) \ln x$

(i) Show that

$$\frac{dy}{dx} = 1 - \frac{5}{x} + \ln x \qquad \text{[4 marks]}$$

(ii) Show that the curve has a turning point between $x = 2$ and $x = 3$ [3 marks]

(iii) By taking 2.4 as a first approximation to the x-coordinate of the turning point, use the Newton Raphson method once to find a better approximation. [5 marks]

10 (a) Find

$$\int x^{-\frac{1}{2}} \ln x \, dx$$ [7 marks]

(b) Using the substitution $u^2 = x^2 + 4$, or otherwise, find the exact value of

$$\int_0^{\sqrt{5}} \frac{x^3}{\sqrt{x^2 + 4}} \, dx$$

[8 marks]

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(Questions continue overleaf)

11 The graphs of the curves

$y = \sin 2x$ and $y = \cos 2x$

are shown in **Fig. 3** below.

The curves intersect at the points **A** and **B**.

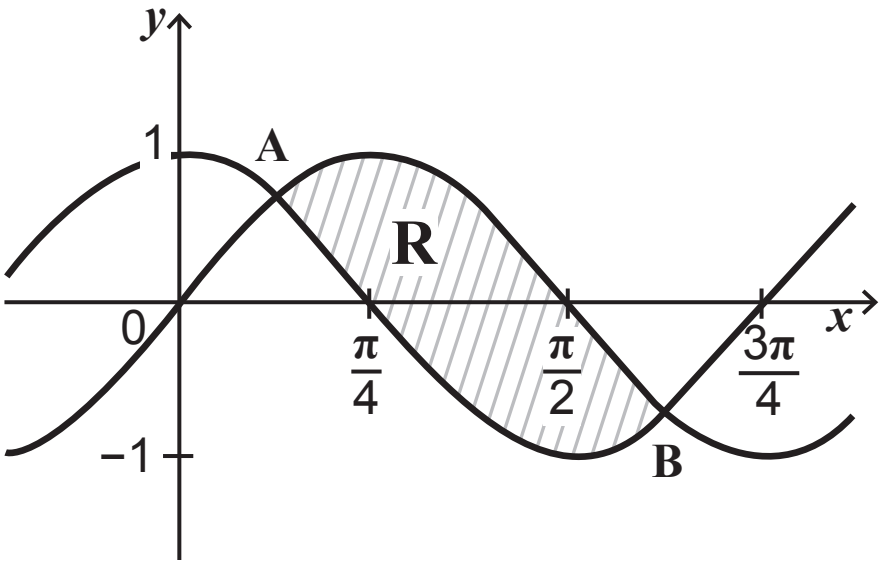

Fig. 3

(i) Show that the x-coordinates of **A** and **B** are $\frac{\pi}{8}$ and $\frac{5\pi}{8}$ [4 marks]

The top section of a trophy is a flat metal sheet modelled in the shape of the shaded region **R**.

(ii) Calculate the area of this region. [6 marks]

A circle has the equation

$x^2 + y^2 = 4$

The base of the trophy can be modelled as the solid formed when the area bounded by this circle, the y-axis and the line $x = 1$ is rotated through 2π radians about the x-axis.

(iii) Find the exact volume of the trophy base. [6 marks]

12 (a) (i) Prove that the sum of n terms of an arithmetic progression with first term a and last term l is

$$S_n = \frac{1}{2}n(a + l)$$ [4 marks]

The first term of an arithmetic progression is 7 and the last term is 79
The sum of the progression is 1075

(ii) Find the number of terms. [3 marks]

(iii) Find the common difference. [3 marks]

(b) A salesman receives a bonus at the end of each year and decides to invest this money in a savings account.

At the end of Year 1 he invests **£**400

At the end of Year 2 he invests a further **£**400 and receives 2**%** interest on the first year's **£**400

At the end of Year 3 he invests a further **£**400 and 2**%** interest is added to the total sum of money which he has accumulated during the first two years.

(i) Show that he has **£**1,224.16 in his account at the end of Year 3 [4 marks]

(ii) Assuming that the man continues to invest in this way, form and sum a series to prove that he will have

£20 000$(1.02^n - 1)$

in his account at the end of n years. [6 marks]

(iii) Hence find the least number of years until his investment exceeds **£**7,000 [4 marks]

This is the end of the question paper

For Examiner's use only	
Question Number	**Marks**
1	
2	
3	
4	
5	
6	
7	
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12	
Total Marks	