



ADVANCED
General Certificate of Education
2013

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

THURSDAY 6 JUNE, MORNING

MV18

TIME

1 hour 30 minutes, plus your additional time allowance.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all eight** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed at the end of each question indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables** booklet is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

- 1** The motion of a toy car on a race track can be modelled by the equations

$$x = 3 \cos t \quad \text{and} \quad y = \sin 2t$$

Find $\frac{dy}{dx}$ when $t = 2$ [5 marks]

- 2** \mathbf{a} and \mathbf{b} are two vectors where

$$\mathbf{a} = t\mathbf{i} + 5\mathbf{j} + \mathbf{k}$$

$$\mathbf{b} = 2t\mathbf{i} + t\mathbf{j} + 2\mathbf{k}$$

- (i) State the value of t for which the vectors are parallel.
[1 mark]
- (ii) Find the values of t for which the vectors are perpendicular. [6 marks]

3 (a) (i) Use partial fractions to find A and B where

$$\frac{4}{x(2-x)} = \frac{A}{x} + \frac{B}{2-x} \quad [4 \text{ marks}]$$

(ii) Hence find

$$\int \frac{4}{2x-x^2} dx \quad [3 \text{ marks}]$$

(b) Use integration by parts to find

$$\int x e^{3x} dx \quad [5 \text{ marks}]$$

4 The area under the curve

$$y = 2\sqrt{x} + 1$$

between $x = 0$ and $x = 4$ is rotated through 360° about the x -axis.

Find the volume of the body so formed. [7 marks]

- 5 (a) Sketch the graph of $y = \cos^{-1} x$, stating its domain.
[3 marks]

(b) Solve the equation

$$\sin (\theta + 30^\circ) - \cos (\theta + 60^\circ) = 1$$

where $-180^\circ \leq \theta \leq 180^\circ$ [8 marks]

- 6 The rate at which a body loses speed $S \text{ m s}^{-1}$ as it travels through a resistive medium at time t seconds is proportional to the square of its speed at that time.

(i) Model this by a differential equation. [2 marks]

(ii) If its initial speed is 100 m s^{-1} and after 3 seconds its speed is 60 m s^{-1} , find how much longer it will take to reduce its speed to 30 m s^{-1} [9 marks]

7 The function h is defined as

$$h(x) = \frac{x+2}{x-3} \quad x \in \mathbb{R} \quad x \neq 3$$

(i) Find the inverse function $h^{-1}(x)$ stating its domain.
[6 marks]

(ii) Rewrite $\frac{x+2}{x-3}$ in the form $a + \frac{b}{x-3}$ [3 marks]

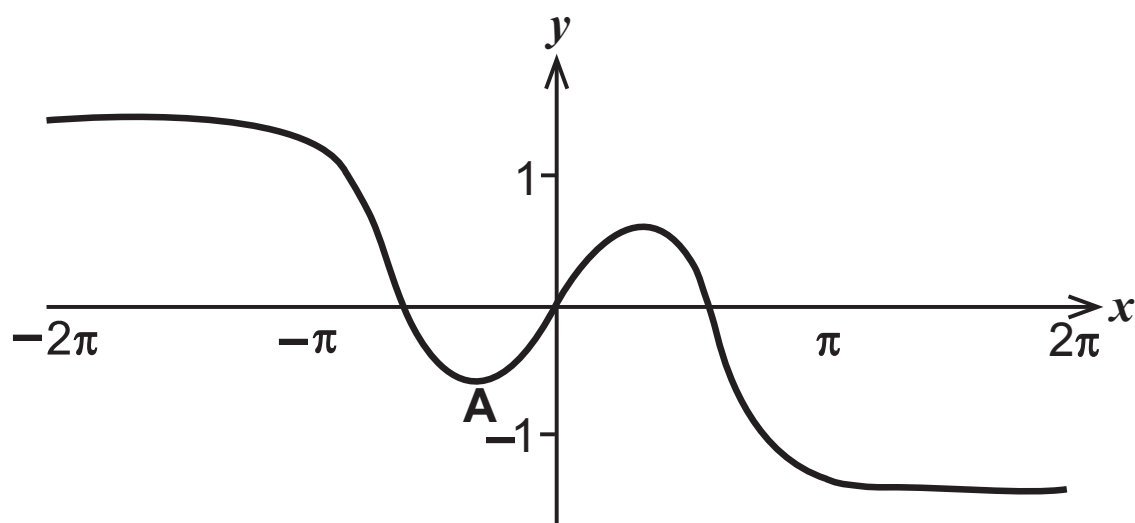
(iii) Write down two functions $f(x)$ and $g(x)$ such that
 $h(x) = fg(x)$.
Neither $f(x)$ nor $g(x)$ may be the identity function.
[2 marks]

8 Part of the graph of

$$2 \sin x - x = \tan y$$

is shown in **Fig. 1** below.

Fig. 1



(i) Find $\frac{dy}{dx}$ [4 marks]

(ii) Hence find the coordinates of the turning point labelled **A** in **Fig. 1** above. [5 marks]

(iii) State the equations of the 2 horizontal asymptotes. [2 marks]

THIS IS THE END OF THE QUESTION PAPER
