Rewarding Learning

ADVANCED
General Certificate of Education
2013

Mathematics

Assessment Unit C3
assessing
Module C3: Core Mathematics 3

[AMC31]

MV18

FRIDAY 17 MAY, MORNING

TIME

1 hour 30 minutes, plus your additional time allowance.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed at the end of each question indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Differentiate:

(i) $x \cos x$ [3 marks]

(ii) $\dfrac{\tan 2x}{x}$ [5 marks]

(iii) $\ln (x^2 + 3)$ [2 marks]

2 **(a)** Simplify as far as possible [5 marks]

$$\frac{(2x^2 + 7x - 15)(2x - 14)}{(x^2 - 2x - 35)(4x^2 - 9)}$$

(b) Solve the inequality [5 marks]

$$|2x - 3| < 1$$

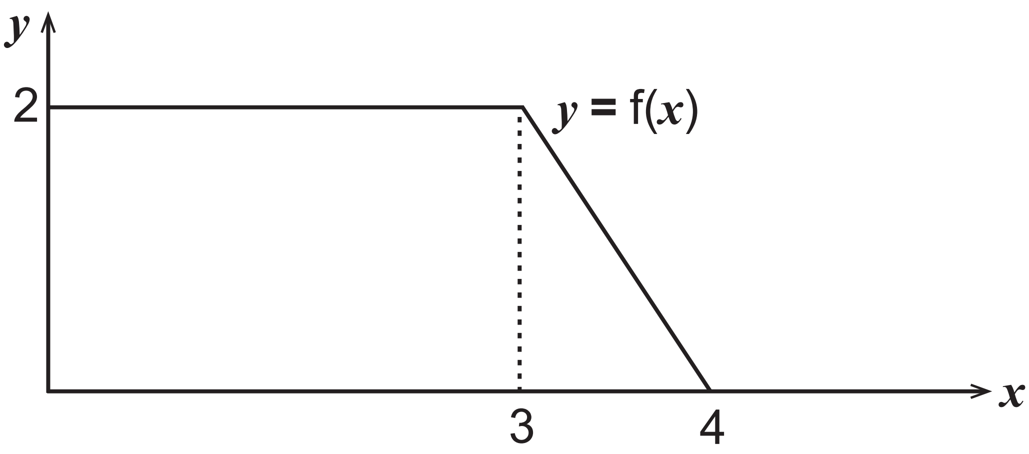
3 **(i)** Find the first 3 terms in the binomial expansion of

$(1-2x)^{\frac{1}{3}}$ [4 marks]

(ii) **Hence**, clearly showing your method, find an approximation to $\sqrt[3]{0.88}$ without using a calculator. [2 marks]

4 The graph of a function $y = \mathrm{f}(x)$ is sketched below in **Fig. 1**

Fig. 1



On separate diagrams sketch the graphs of:

(i) $y = 1 - \mathrm{f}(x)$ [3 marks]

(ii) $y = \frac{1}{2}\mathrm{f}(x+2)$ [3 marks]

labelling clearly where the graphs cross or touch the axes.

5 **(a)** The temperature, C, of an ingot of cooling metal can be modelled by

$$C = 12 + 80e^{\frac{-t}{30}}$$

where t is measured in minutes.

Find C when $t = 20$ [2 marks]

(b) Find a Cartesian equation of the curve defined parametrically by

$x = e^{-t}$ $y = 3 + e^{2t}$ [4 marks]

6 **(a)** **(i)** Use partial fractions to rewrite

$$\frac{x+10}{(2x-5)x^2} \text{ in the form } \frac{A}{2x-5} + \frac{B}{x} + \frac{C}{x^2}$$

where A, B and C are integers. [6 marks]

(ii) Hence find

$$\frac{d}{dx}\left(\frac{x+10}{(2x-5)x^2}\right)$$ [3 marks]

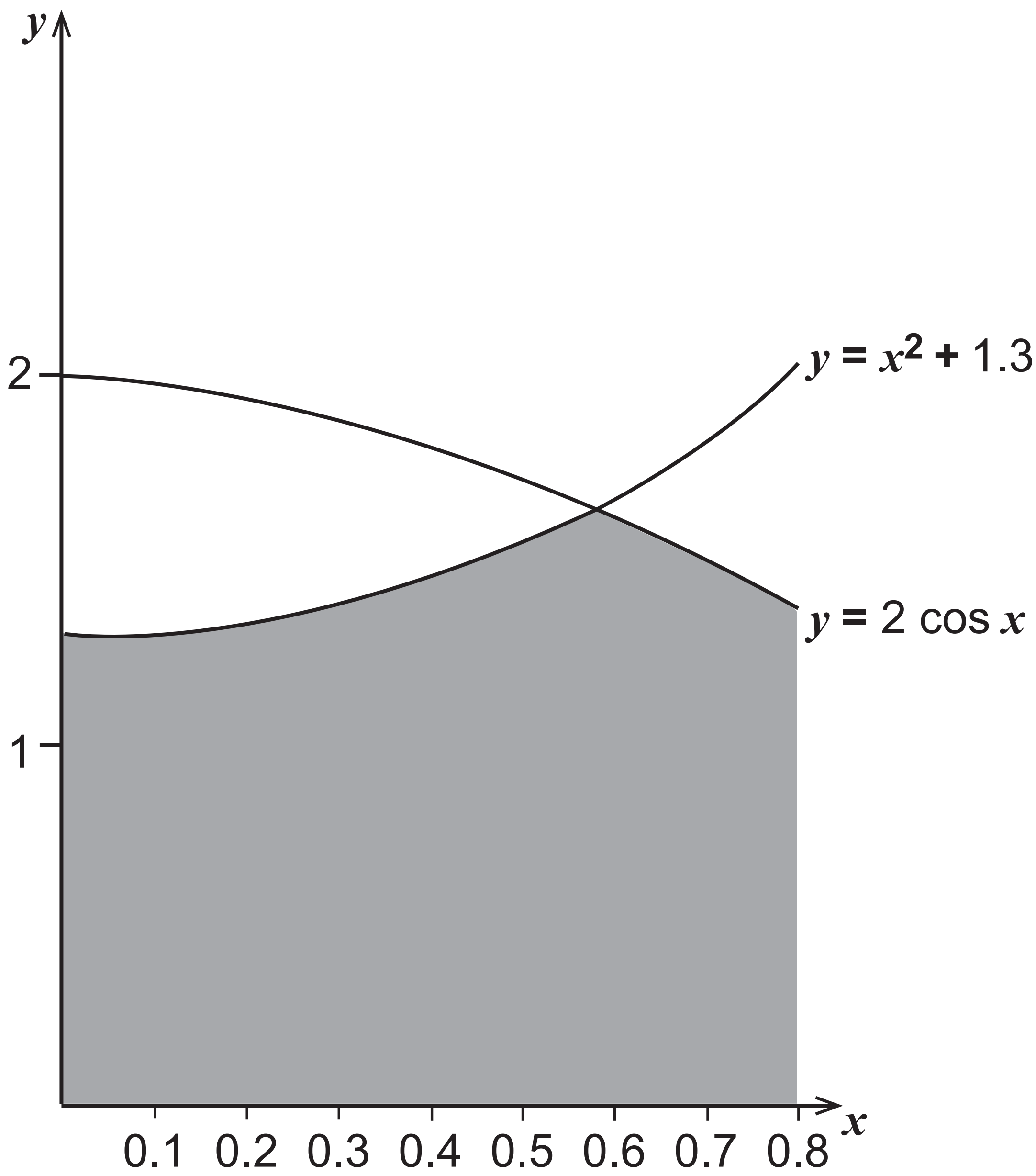
(b) Solve

$$\cot 2\theta = -\frac{4}{3}$$

where $0 \leqslant \theta \leqslant 2\pi$ [4 marks]

7 **Fig. 2** below shows the graph of $y = x^2 + 1.3$ and $y = 2\cos x$.

Fig. 2



(i) Taking $x_0 = 0.5$ as a first approximation, use the Newton-Raphson method twice to find a better approximation to the x-coordinate of the point of intersection of the curves. [8 marks]

A glazier designs a pane of a stained glass window identical to the shaded area in **Fig. 2** opposite, i.e. the area between the curves, the axes and the line $x = 0.8$

(ii) Use Simpson's rule with 5 ordinates to find an approximation to the area of the pane. [6 marks]

8 **(i)** Prove the identity

$$\tan A \sec A + \frac{1}{1 + \sin A} \equiv \sec^2 A \quad \text{[5 marks]}$$

(ii) Hence find

$$\int \left(\tan 2x \sec 2x - \frac{3}{x} + e^{2x} + \frac{1}{1 + \sin 2x} \right) dx \quad \text{[5 marks]}$$

THIS IS THE END OF THE QUESTION PAPER