Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2015

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

MV18

[AMF11]

WEDNESDAY 24 JUNE, MORNING

TIME

1 hour 30 minutes, plus your additional time allowance.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed at the end of each question indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Let the matrix $\mathbf{R} = \begin{pmatrix} 4 & 9 & 0 \\ 0 & -2 & 8 \\ 0 & 0 & 7 \end{pmatrix}$

(i) Calculate $\mathbf{R}\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$ [2 marks]

(ii) Explain fully the relationship between the matrix $\mathbf{R}$ and $\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$ [2 marks]

(iii) Hence, or otherwise, express $\mathbf{R}^2\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$ in the form $\alpha\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$, where α is an integer. [4 marks]

2 A circle has equation

$$x^2 + y^2 - 4x - 8y + 10 = 0$$

(i) Find the equation of the tangent to the circle at the point $(-1, 5)$. [5 marks]

(ii) Find the equation of the other tangent to this circle that is parallel to the tangent found in **(i)**. [5 marks]

3 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} a & 1 & -2 \\ 1 & 4 & 1 \\ -2 & 1 & a \end{pmatrix}$

Consider the matrix equation $\mathbf{M}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} b \\ 2 \\ 5 \end{pmatrix}$

(i) Find the values of a for which the matrix equation does not have a unique solution. [5 marks]

(ii) If $a = -2$ and $b = 6$ explain why the matrix equation has no solution. [3 marks]

(iii) If $a = -2$ and $b = 5$ find the general solution of the matrix equation. [4 marks]

4 S is the set of matrices $\begin{pmatrix} r & s \\ s & r \end{pmatrix}$, where r, s are real numbers such that $r^2 \neq s^2$

Prove that S forms a group under matrix multiplication.
You may assume that matrix multiplication is associative.
[12 marks]

5 **(a)** Write down the matrix which represents a rotation of 45° anticlockwise about the origin. [2 marks]

(b) The matrix $\mathbf{N} = \begin{pmatrix} 2 & 1 \\ -6 & 3 \end{pmatrix}$

Under the transformation represented by **N** the line $y = mx$ is reflected in the y-axis.

Find the possible values of m. [8 marks]

6 **(a)** **(i)** Given that $(a + bi)^2 = -5 + 12i$

find the real values of a and the corresponding values of b. [8 marks]

(ii) Hence find the complex roots of the quadratic equation

$$z^2 - (4 - i)z + (5 - 5i) = 0$$ [6 marks]

(b) **(i)** Sketch on an Argand diagram the locus of those points w which satisfy

$$|w - (3 + 3i)| = \frac{3}{\sqrt{2}}$$ [3 marks]

(ii) For any point w on the locus described in **(b)(i)**, show that

$$\frac{\pi}{12} \leqslant \arg w \leqslant \frac{5\pi}{12}$$

A solution by scale drawing will not be accepted.
[6 marks]

THIS IS THE END OF THE QUESTION PAPER