



Rewarding Learning

ADVANCED

General Certificate of Education

2016

Mathematics

Assessment Unit M2

assessing

Module M2: Mechanics 2



[AMM21]

THURSDAY 2 JUNE, AFTERNOON

TIME

1 hour 30 minutes, plus your additional time allowance.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all seven** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

Answers should include diagrams where appropriate and marks may be awarded for them.

Take $g = 9.8 \text{ m s}^{-2}$, unless specified otherwise.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

- 1** A particle, P, of mass 4 kg is acted on by a force $(-2\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$ N.
Initially, P is at rest at a point A.
After 6 seconds, P reaches the point B.

(i) Find the velocity of P at B. [4]

A is $(10\mathbf{i} + 12\mathbf{k})$ m from a fixed point O.

(ii) Find the displacement vector \vec{OB} . [4]

(iii) Find the unit vector in the direction of P's motion. [4]

- 2 **Fig. 1** below, shows a cricket ball being hit from a height of 0.9 m above horizontal ground with a speed of 25 m s^{-1} at an angle of θ above the horizontal, where $\tan \theta = \frac{7}{24}$

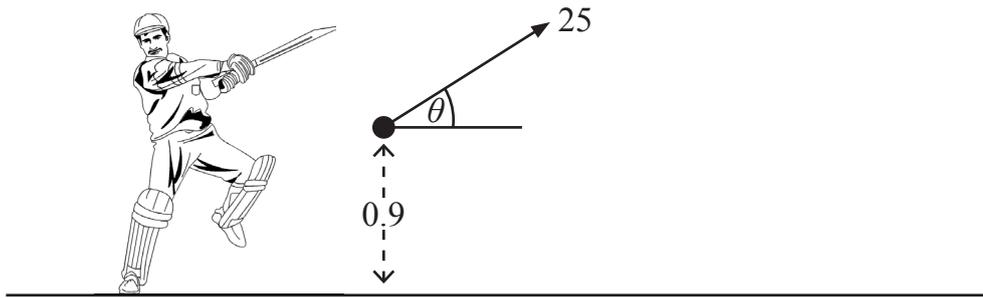


Fig. 1

The motion of the cricket ball can be modelled as that of a particle moving freely under gravity.

- (i) Find the length of time for which the ball is at least 3 m above the ground. [5]

The ball is caught by a boy who is 33 m horizontally from the point where it was struck.

- (ii) Find the vertical height above the ground, of the ball when it is caught. [5]

- (iii) State, briefly, one way in which the above model could be refined in order to make it more realistic. [1]

- 3 A particle P moves so that at time t seconds, its velocity \mathbf{v} is given by

$$\mathbf{v} = [(3t^2 - 6)\mathbf{i} - 6t^2\mathbf{j}] \text{ m s}^{-1}$$

At time $t = 0$, the displacement of P from a fixed point O is $(15\mathbf{i} + 75\mathbf{j})\text{m}$.

- (i) Find the position vector of P from O at time t . [4]

The position vector of a second particle Q from O at the same time t is given by

$$\vec{\text{OQ}} = [(t^3 - 3t)\mathbf{i} + (3t^2 - 2t^3)\mathbf{j}] \text{ m}$$

- (ii) Find the value of t when P and Q meet. [4]

- (iii) Find the velocity of Q when the particles meet. [3]

- 4 **Fig. 2** below shows two particles P and Q of mass 3 kg and m kg respectively, connected by a light inextensible string of length 6 m.
 The string passes through a small, smooth ring fixed at O.
 P hangs in equilibrium 4 m vertically below O and Q moves with a constant speed in a horizontal circle about OP.
 The string OQ makes an angle of θ with the horizontal, where $\theta = \sin^{-1}\left(\frac{2}{5}\right)$

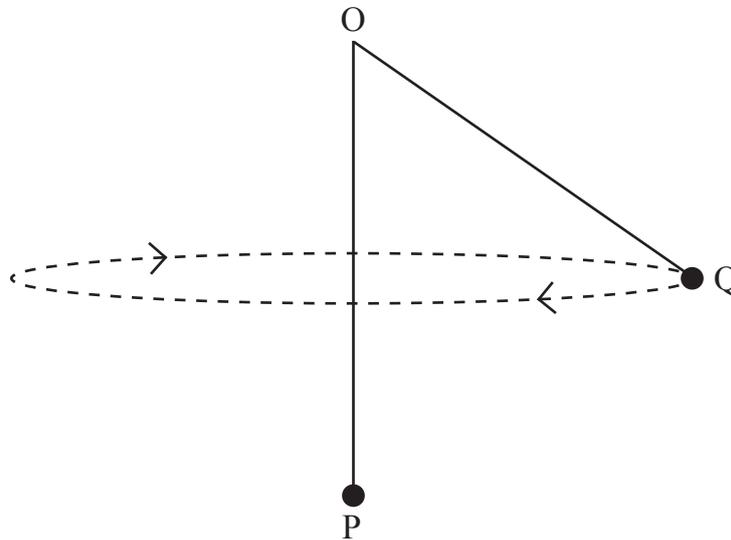


Fig. 2

(i) Draw a diagram showing the external forces acting on P and Q. [2]

(ii) Find m . [4]

Q rotates with an angular velocity of ω rad s^{-1}

(iii) Find ω . [5]

- 5 A van is travelling up a straight road inclined at an angle θ to the horizontal, where $\sin \theta = \frac{1}{12}$. The resistances to the motion of the van can be modelled as a constant force of 1.2 kN. Initially, the speed of the van is 30 m s^{-1} and the engine of the van is working at a rate of 60 kW. At this point, the van is decelerating at 0.3 m s^{-2} .
- (i) Show that the mass of the van is 1548 kg. [6]

While travelling up the same road, the rate of working of the van's engine is now increased to 78 kW. Assume resistances remain as before.

- (ii) Find the maximum speed of the van at this rate of working. [4]
- (iii) State why in a more realistic model, the resistances to motion would not be constant. [1]

- 6 A pump takes water from a tank and the water comes out from the end of a hose that is 8 m vertically above the level from where the water has been taken. The cross-sectional area of the hose is $A \text{ m}^2$ and the water leaves the end of the hose at a speed of 16 m s^{-1} . The pump works at a rate of 500 W. The density of water is 1000 kg m^{-3} .

Find A . [7]

- 7 At time $t = 0$ seconds, a parachutist of mass m kg jumps from rest from the top of a cliff and falls freely under gravity.
The air resistance, at speed v m s⁻¹, can be modelled as kmv newtons, where k is a constant.
Model the parachutist as a particle.
The terminal velocity of the parachutist, V m s⁻¹, is her theoretical maximum speed during her fall.

(i) Show that $V = \frac{g}{k}$ [3]

(ii) Find an expression, in terms of k , for the time taken for her to reach a speed of $\frac{V}{2}$ [9]

THIS IS THE END OF THE QUESTION PAPER
